Visualization of Branch-and-bound Algorithms

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Collaborators

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Lehigh University

Outline

- Reasons to monitor the progress of branch-and-bound
- Current measures
- Our approach
- Visualization tools
- Current and future work

Why monitor the progress of B&B algorithms?

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- How much longer do we have to wait until the algorithm terminates?
- How likely is it that a better solution will be found, and how much better will it be?
- Should we change any algorithm strategies? (branching, node selection,...)

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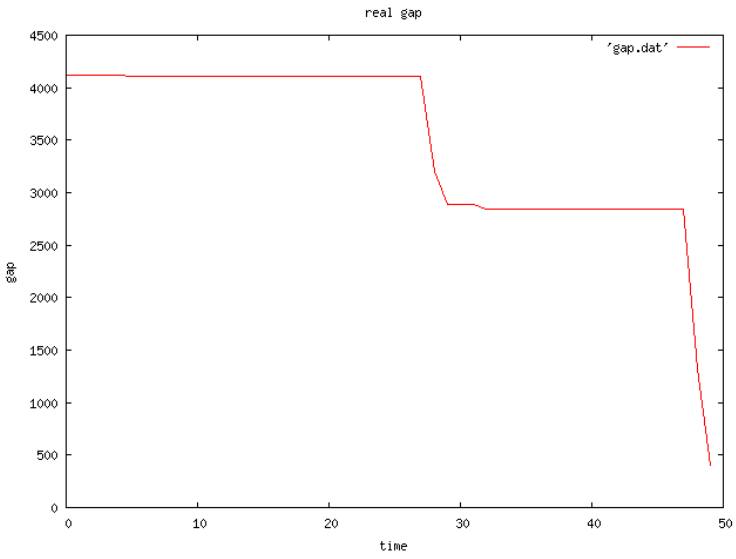
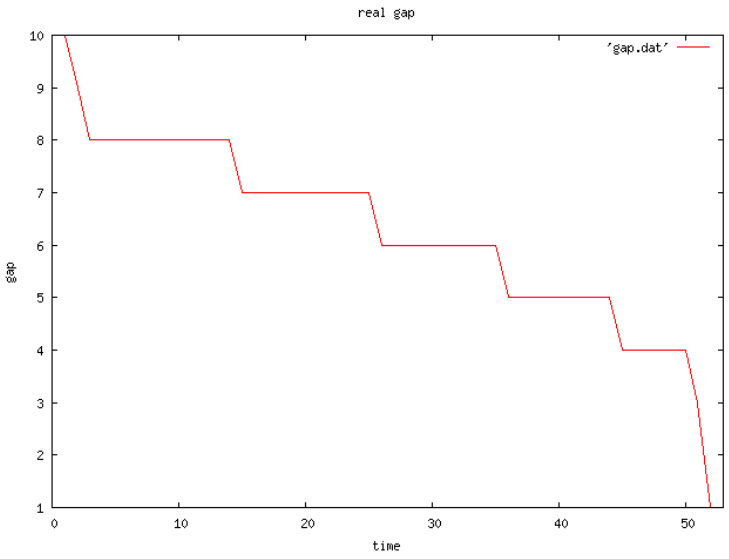
  - some internal measures for guiding the algorithm
- Each of these methods have some strengths and weaknesses

Current methods: optimality gap





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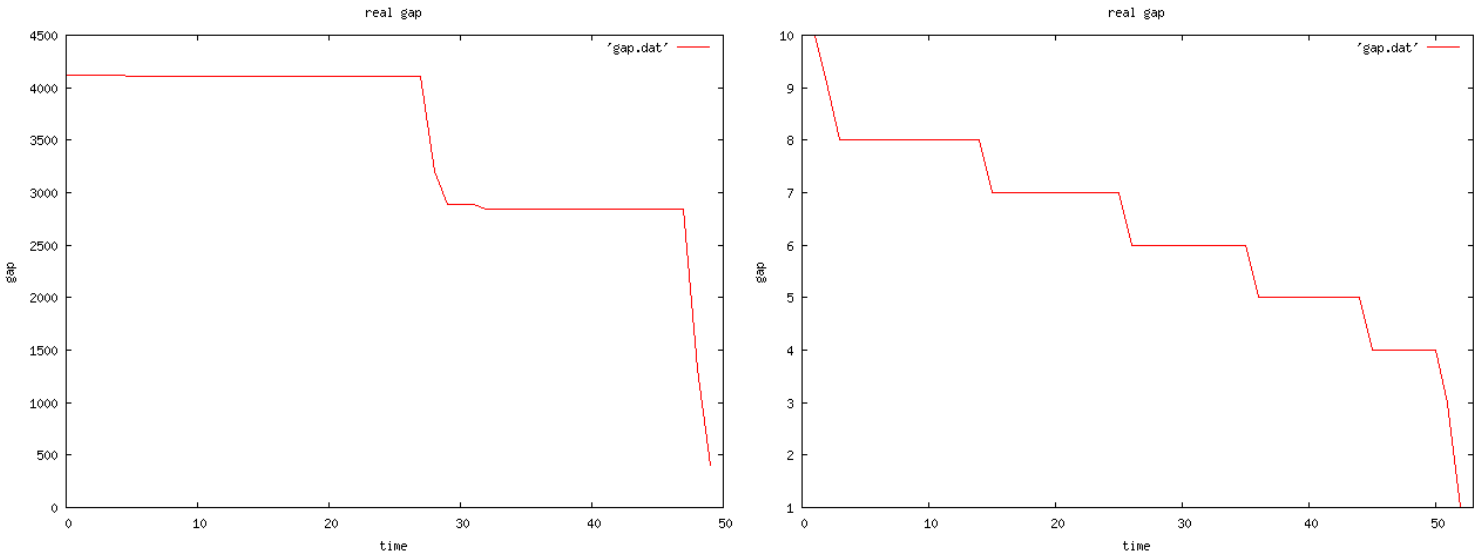

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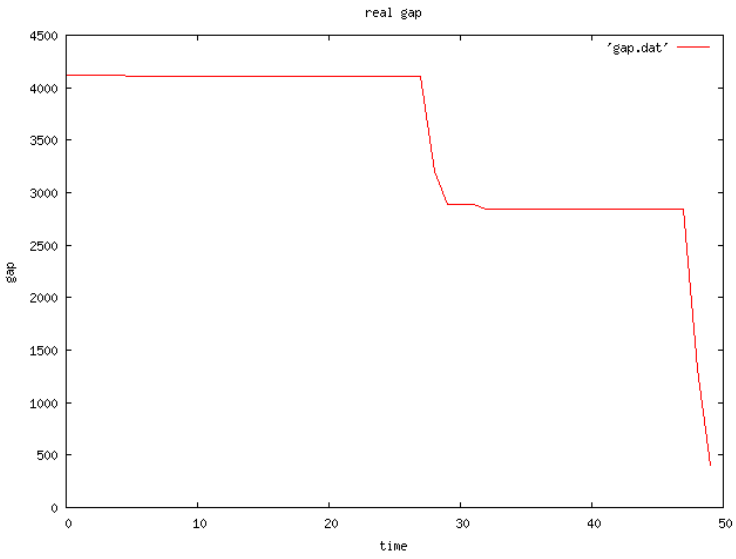
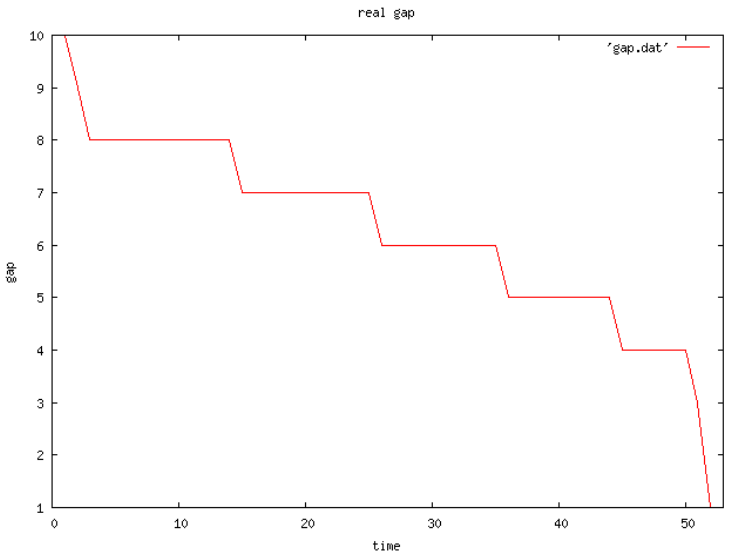
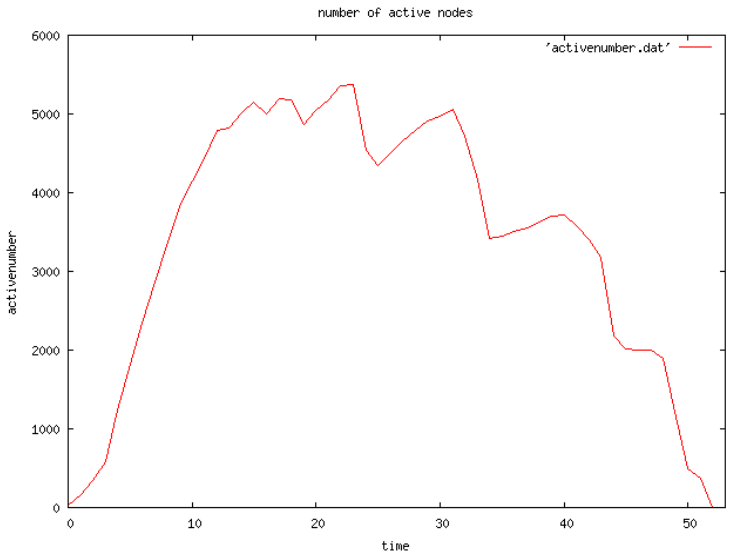
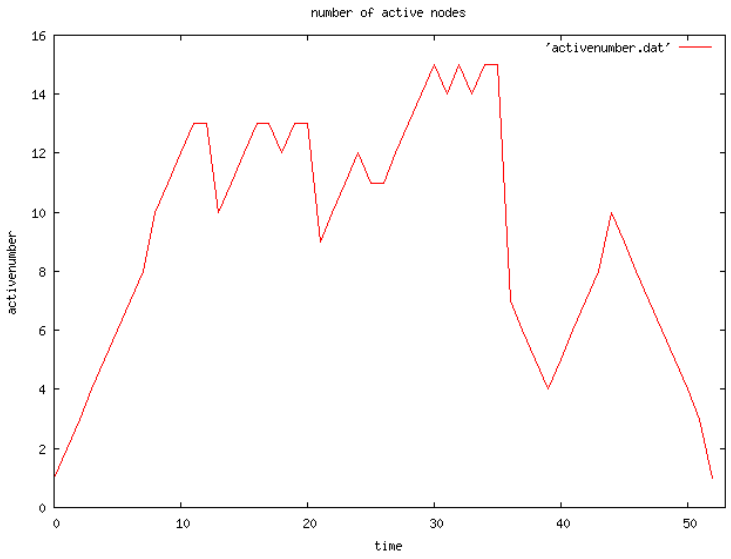




- Strength: guarantee on quality of solution
- Strength: nonincreasing
- Weakness: may remain constant for long periods, then drop suddenly

Current methods: number of active nodes





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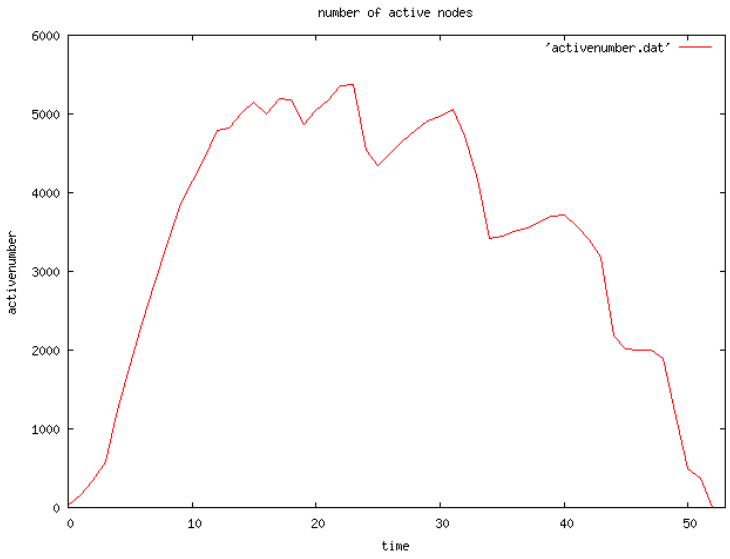

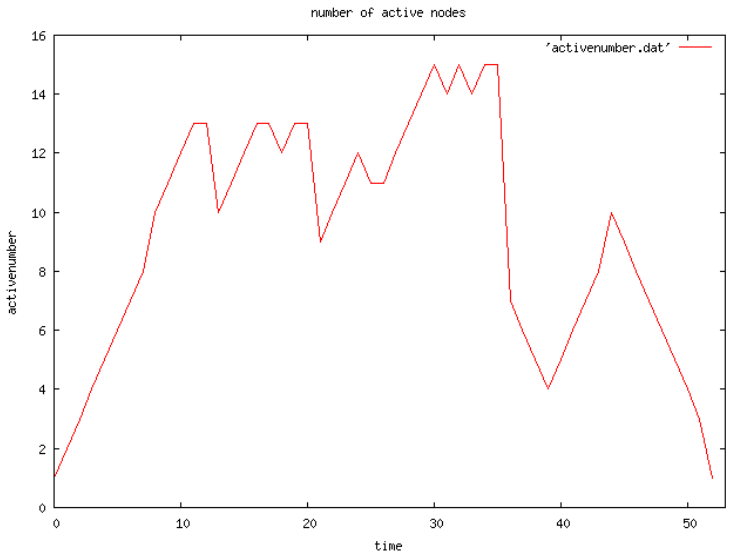

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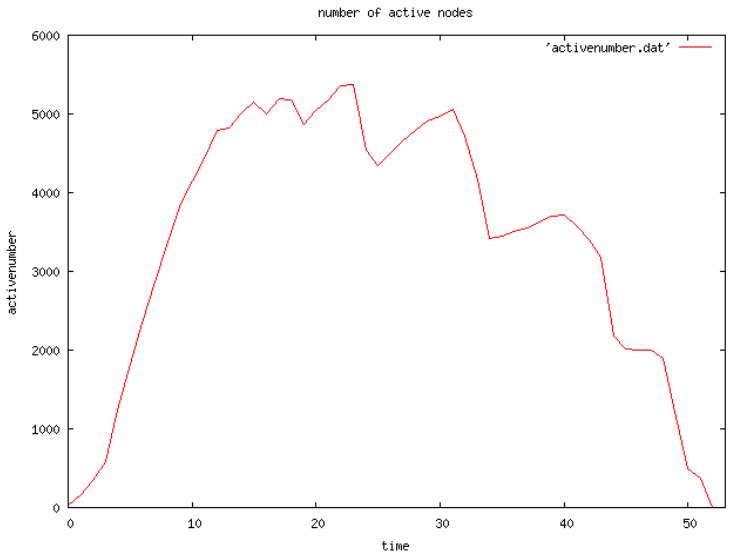

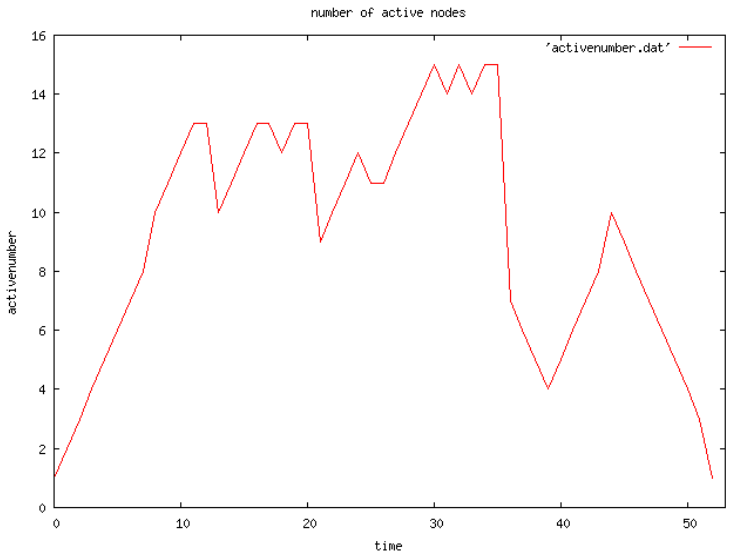

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2. Current methods use only a small amount of this data
3. Most prior work only considers one type of information at a time
4. Develop tools that consider as much data as possible!

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  - Output file can be processed at any point during the solving process
  - Parsing is done with Perl; images are created with Gnuplot

Example of output from solver

```
# CBC
0.040003 heuristic -28.000000
2.692169 branched 0 -1 N -39.248099 16 0.169729
2.692169 pregnant 2 0 R -39.248063 14 105.991922
2.708170 pregnant 3 0 L -38.939929 6 0.105246
2.764173 pregnant 5 2 R -39.244862 12 49.115388
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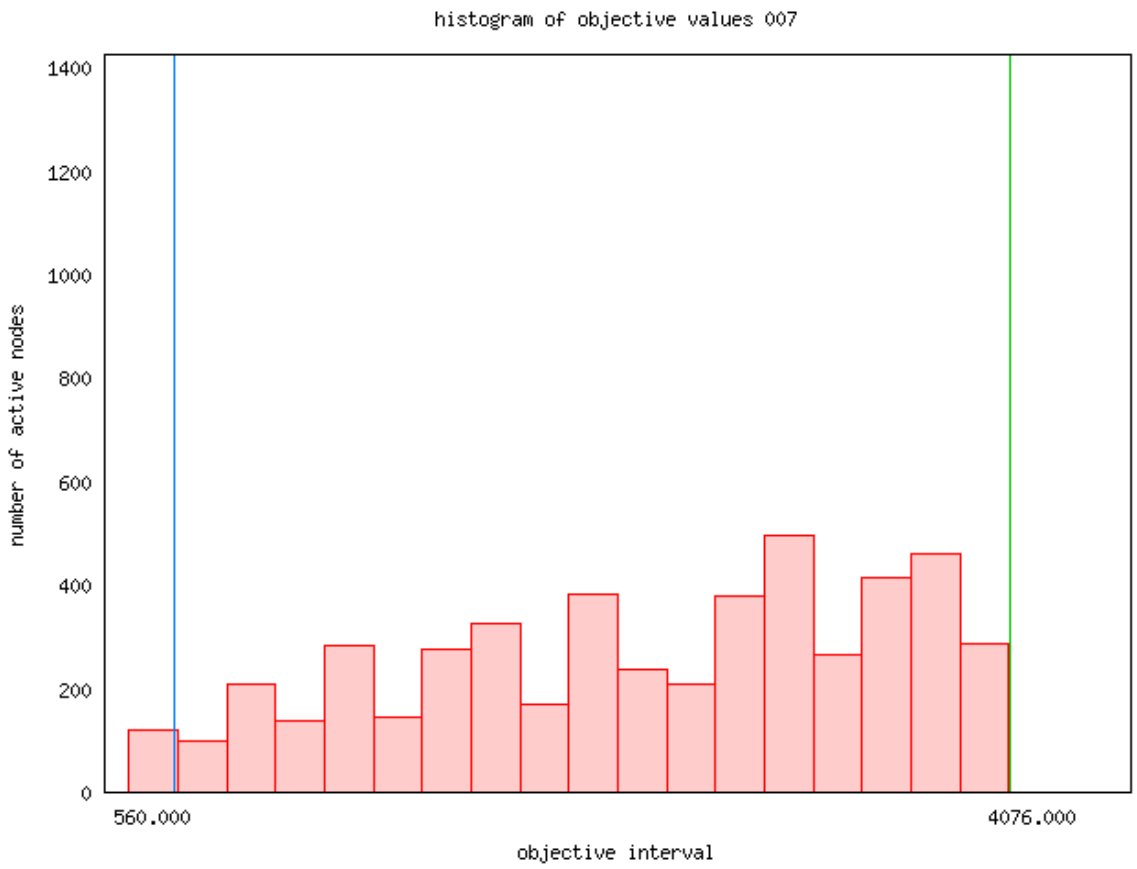
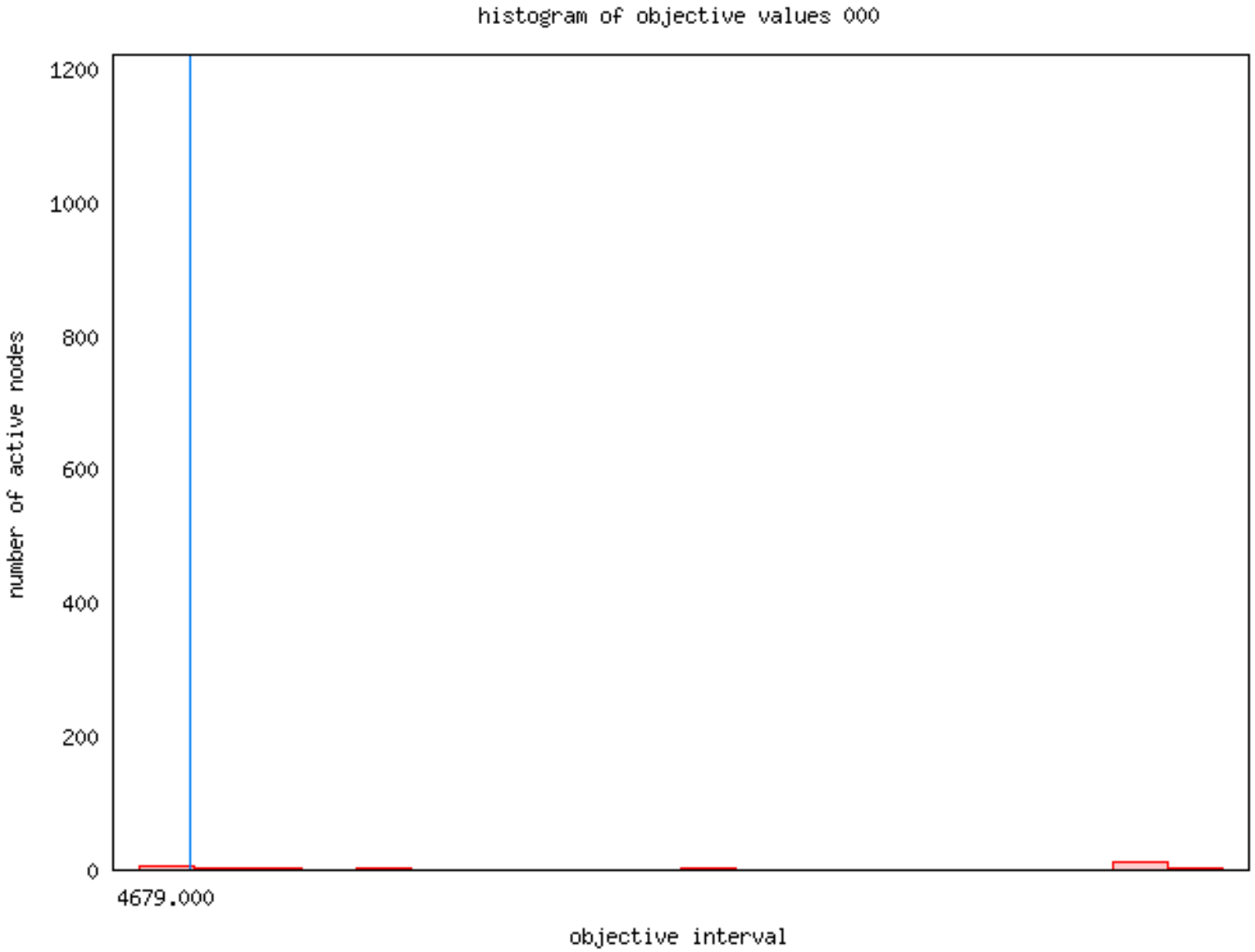
  - Scatter plot of active node LP bounds & integer infeasibility
  - Incumbent node history in scatter plot
  - B&B trees showing the LP bound of each node

Visualization tools: Histogram of active node LP bounds



- Horizontal axis is the LP bound
- Vertical axis is number of active nodes
- Green vertical line shows the current incumbent value and the blue one shows the overall LP bound

Example histogram series 1: l152lav (MIPLIB 2003)



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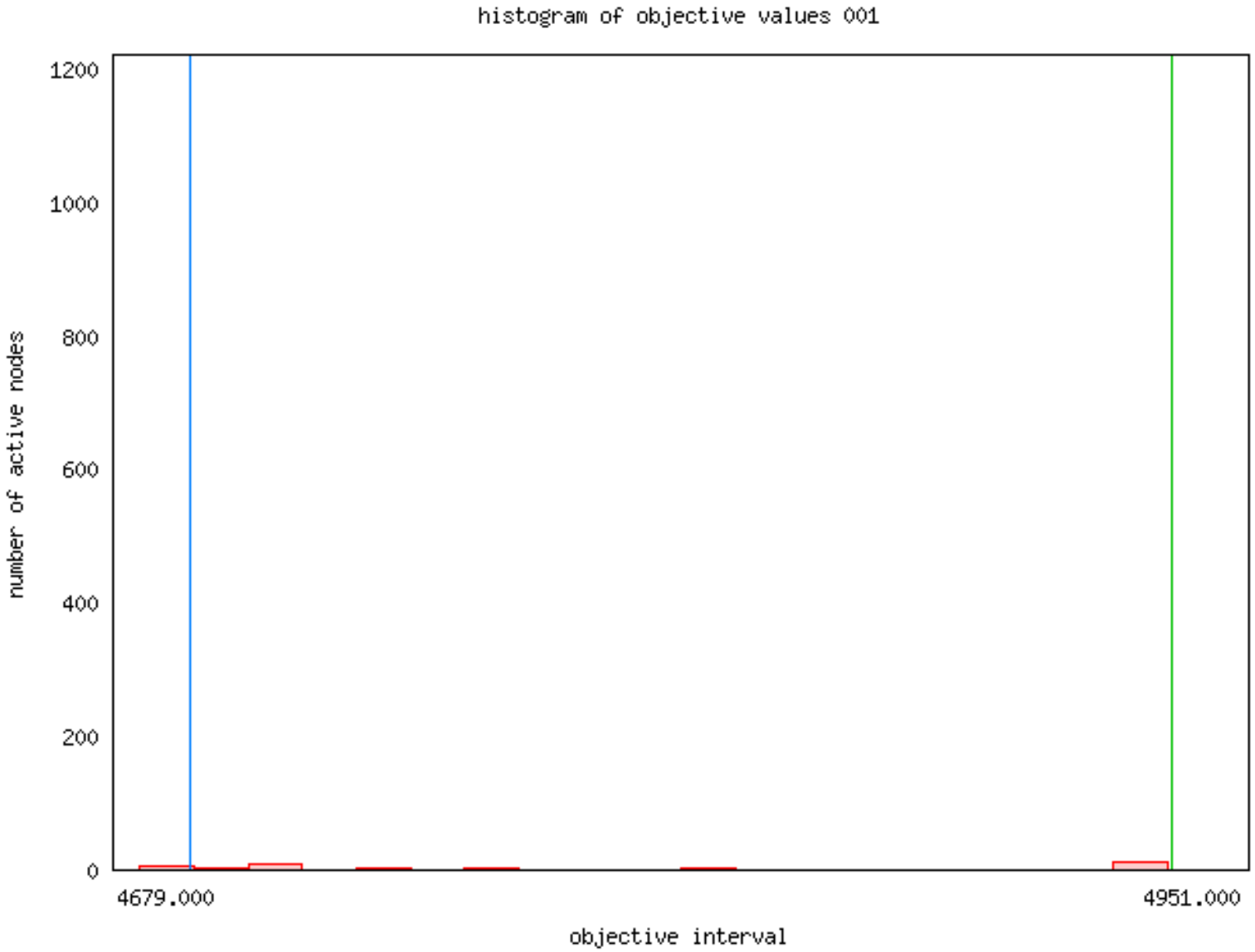

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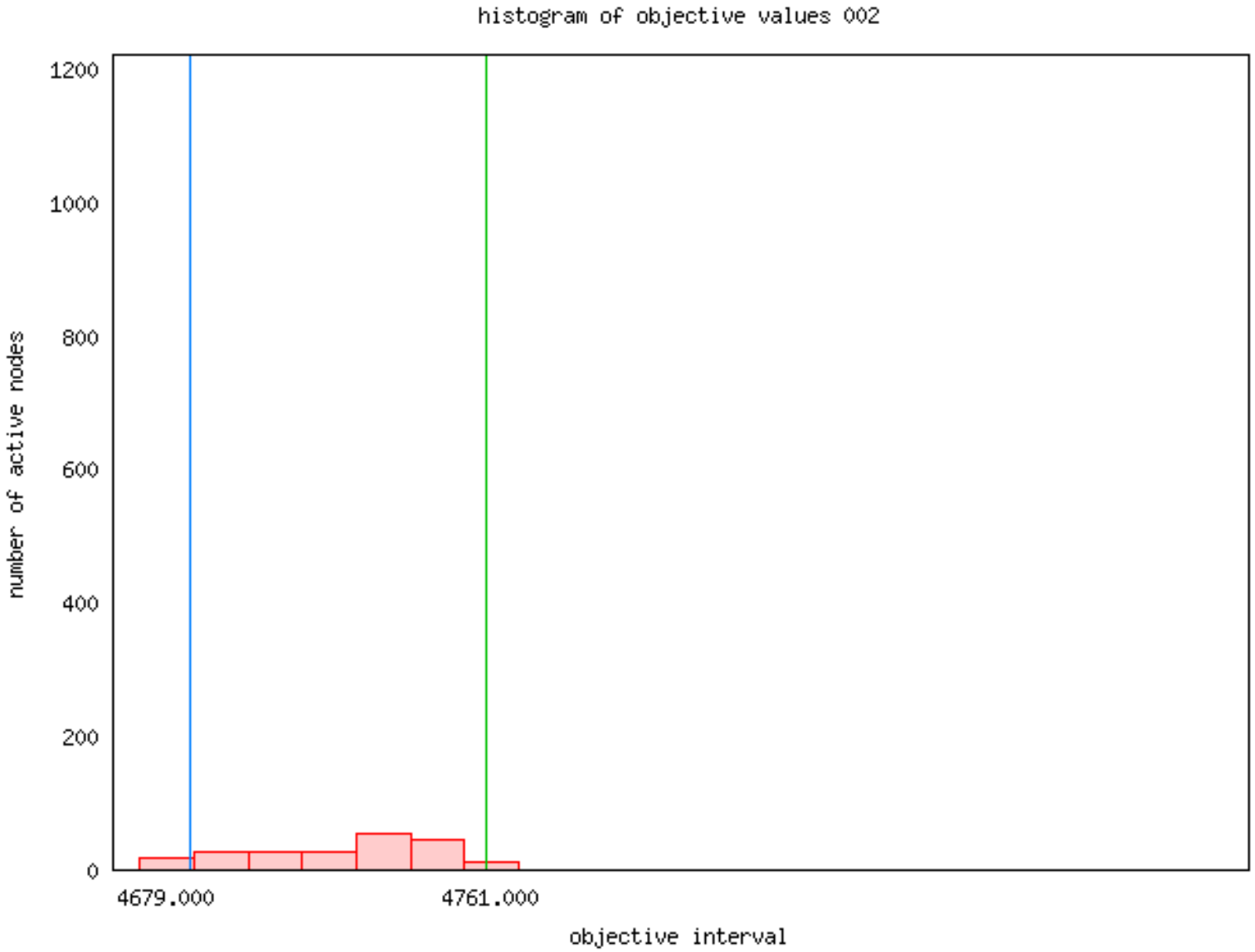

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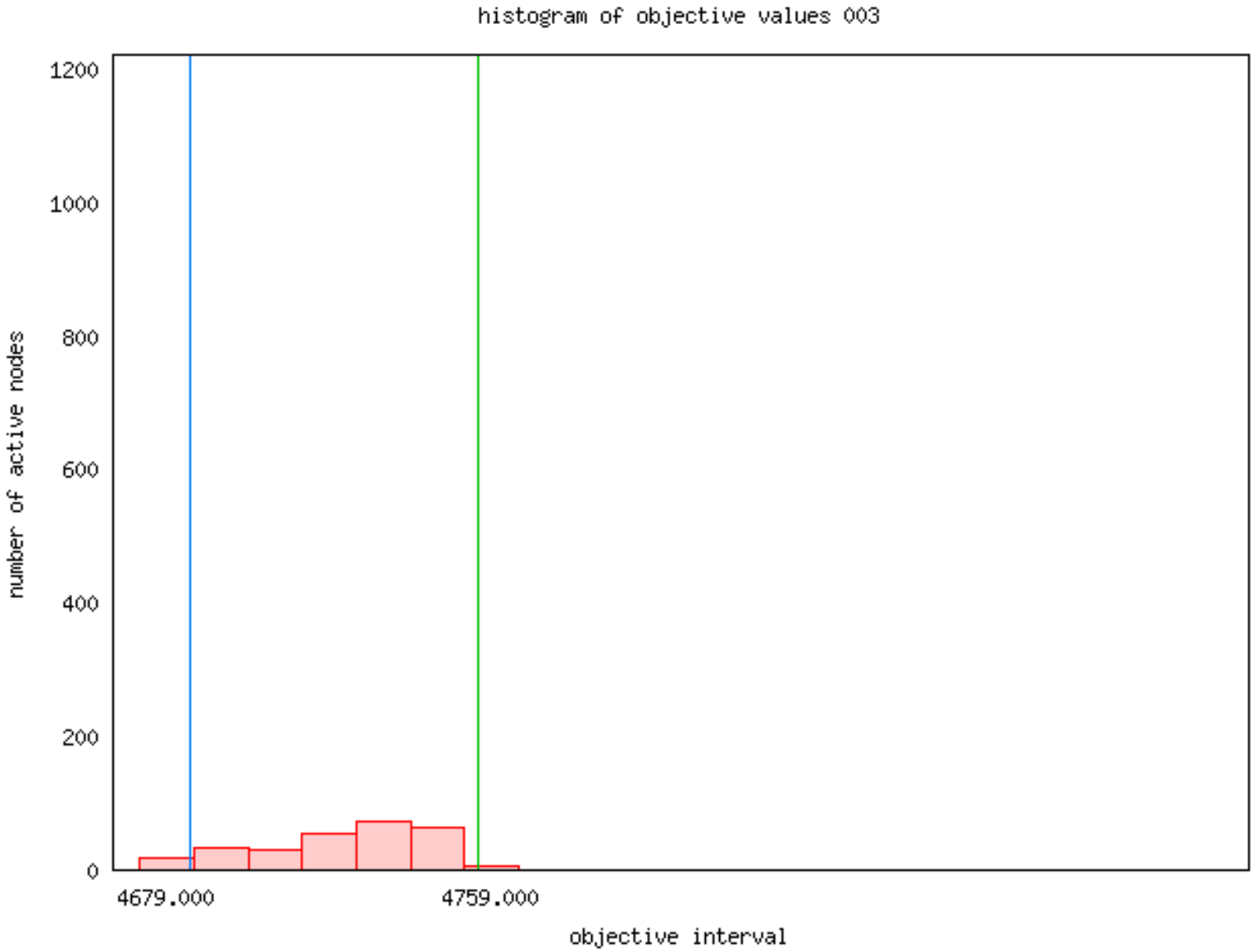

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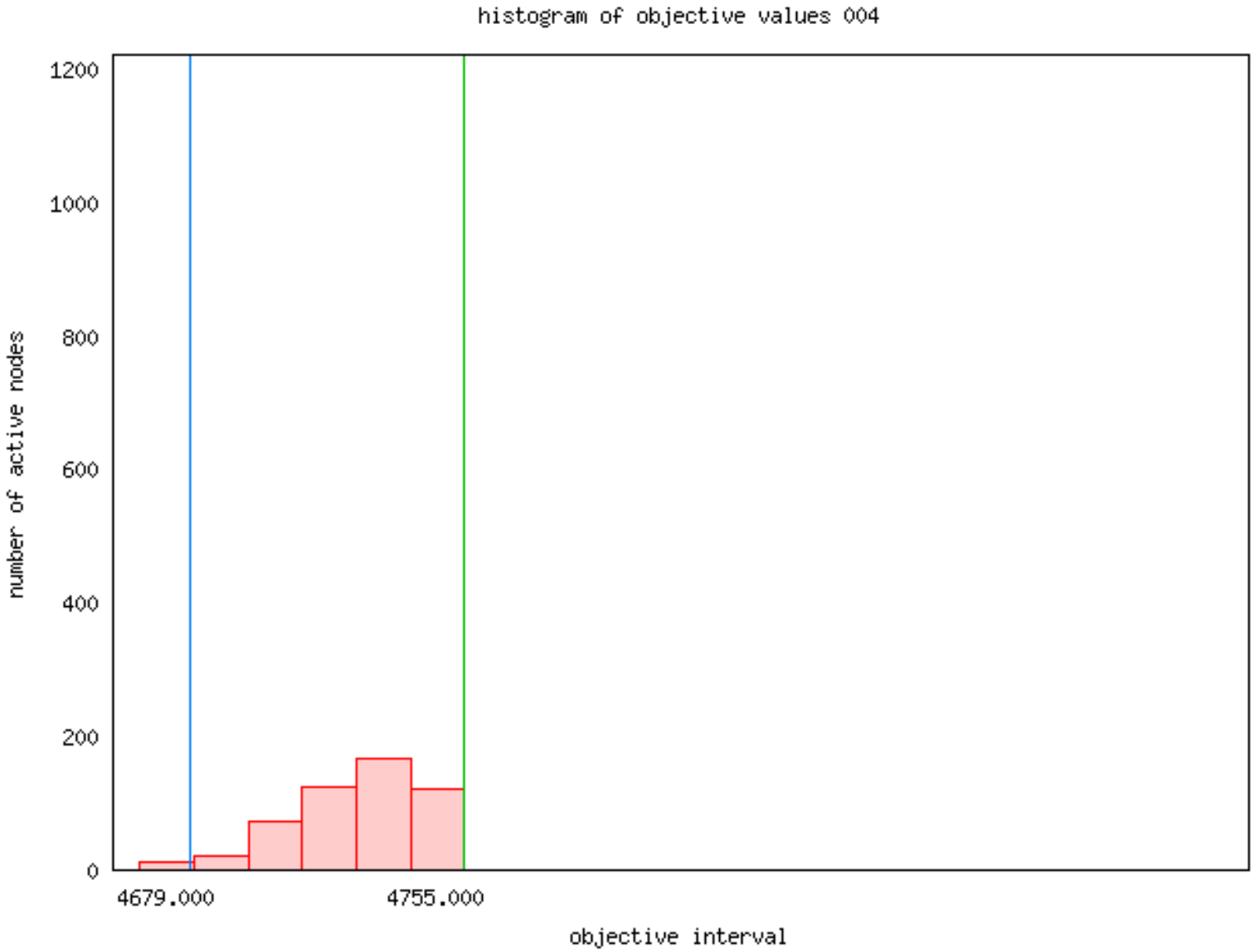

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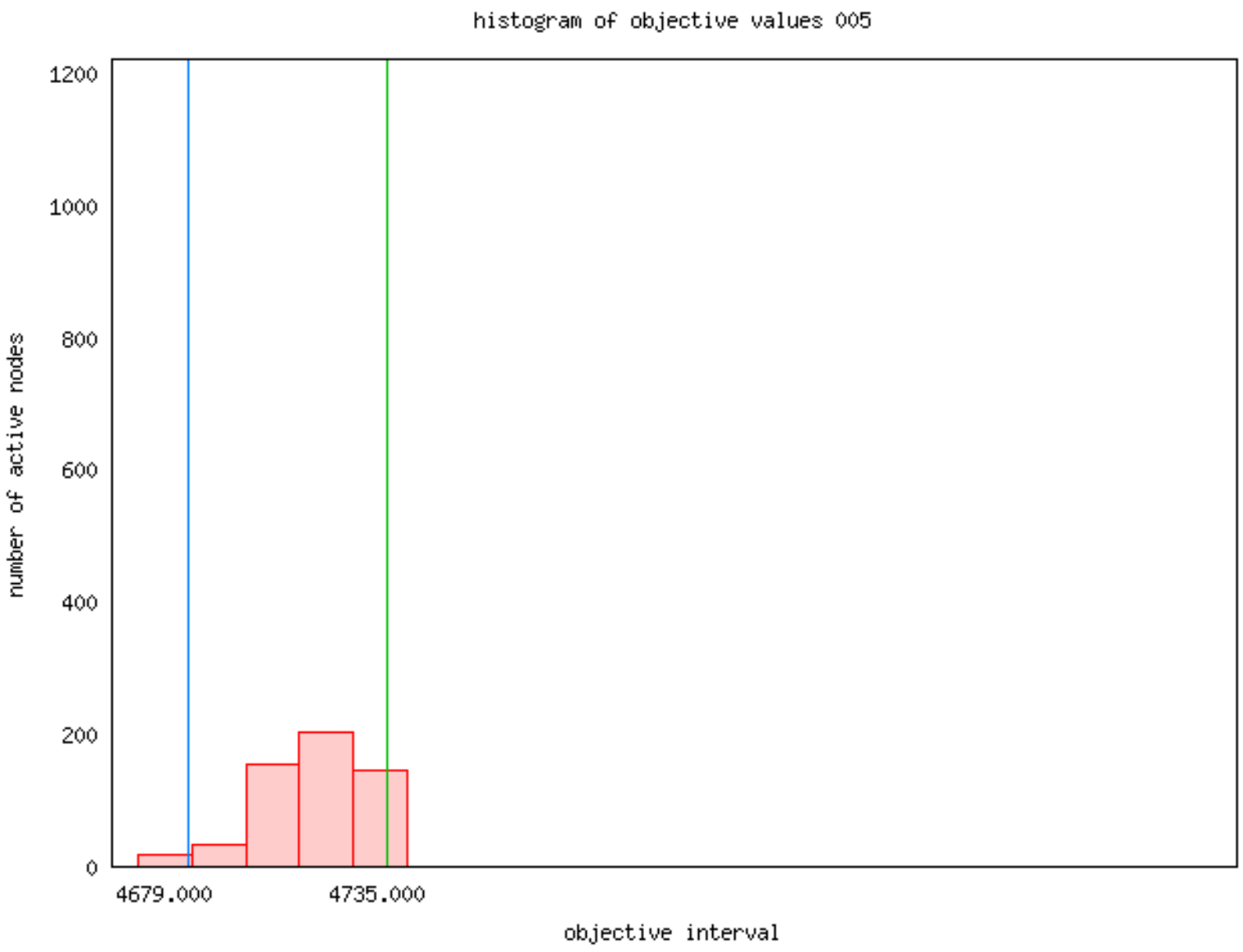

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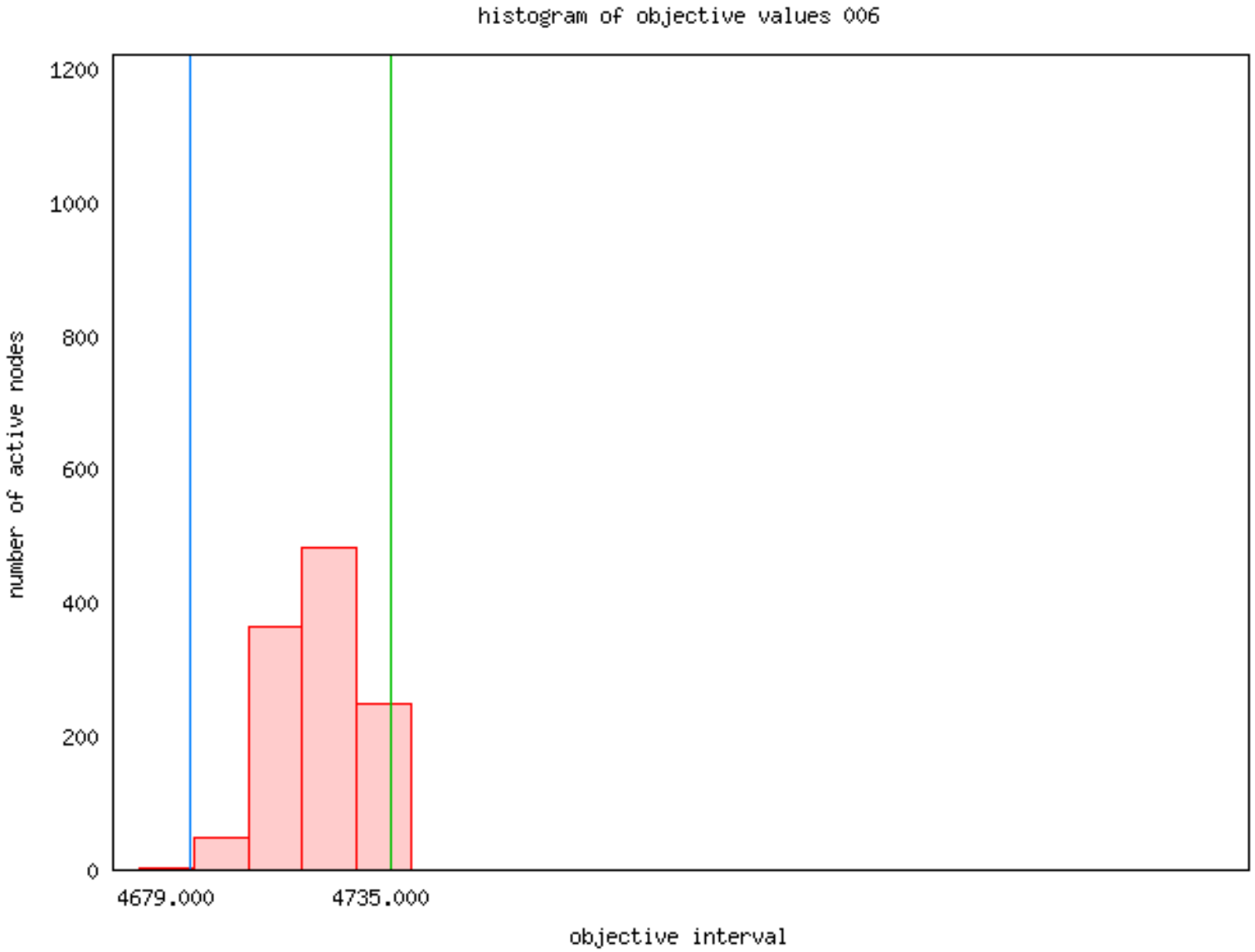

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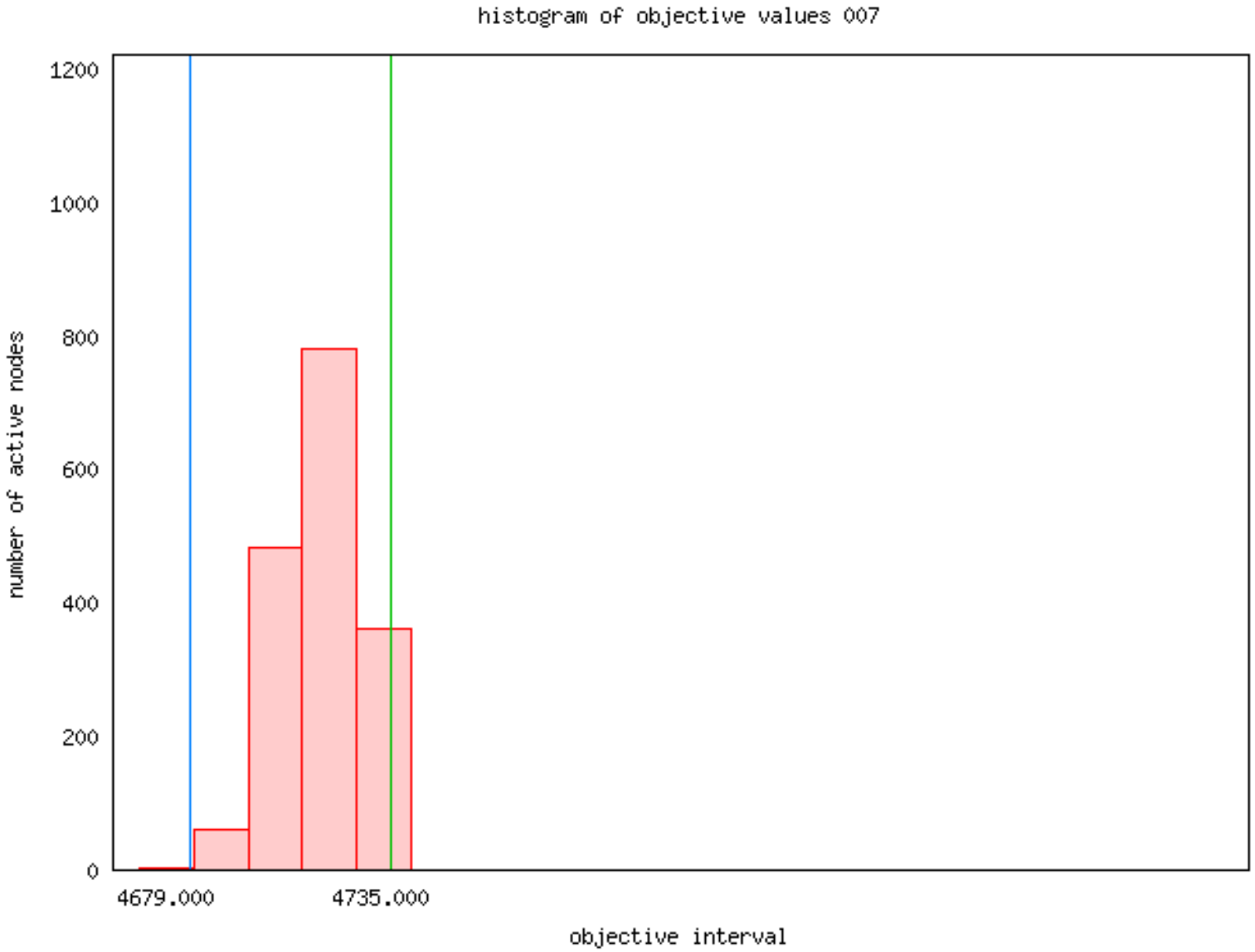

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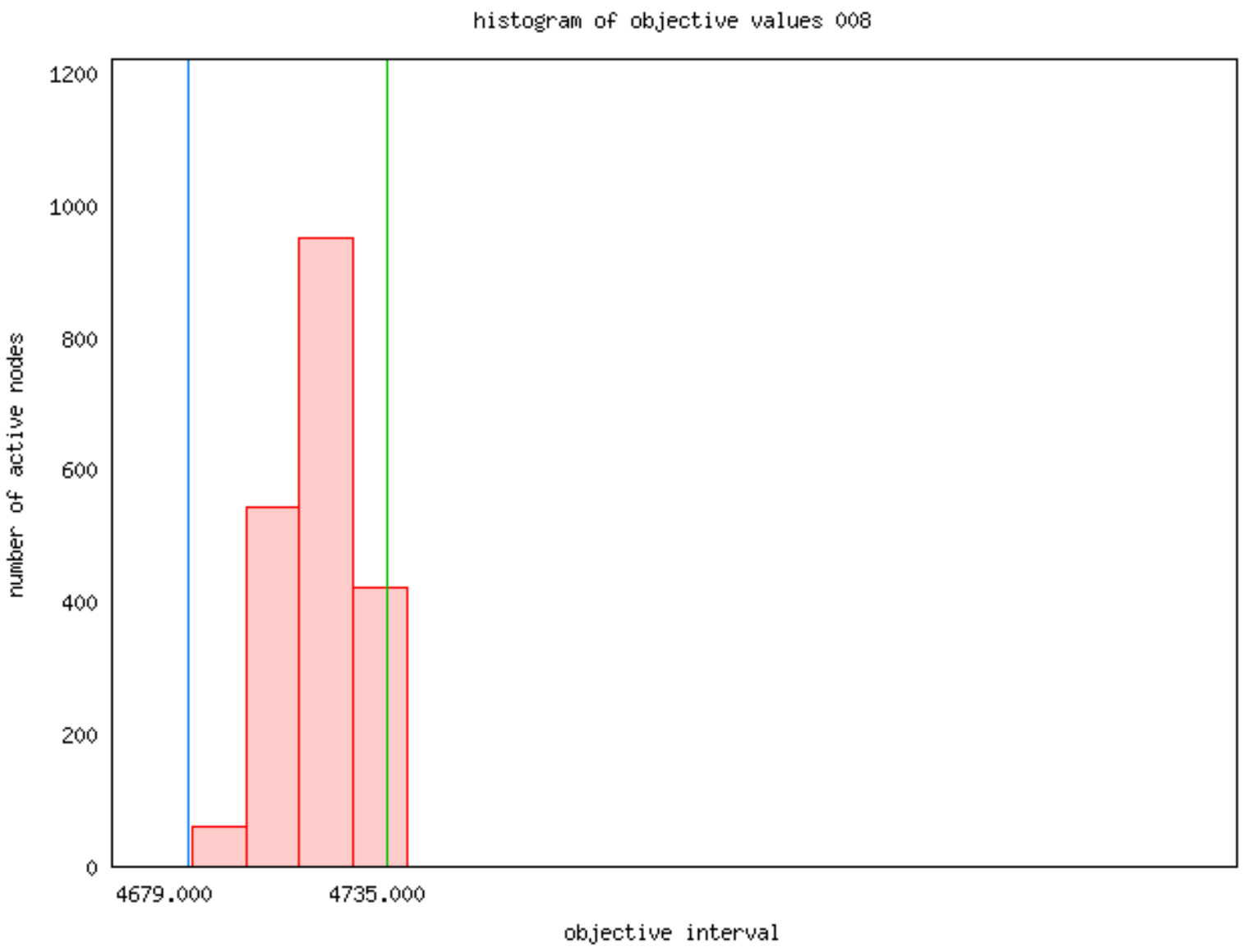

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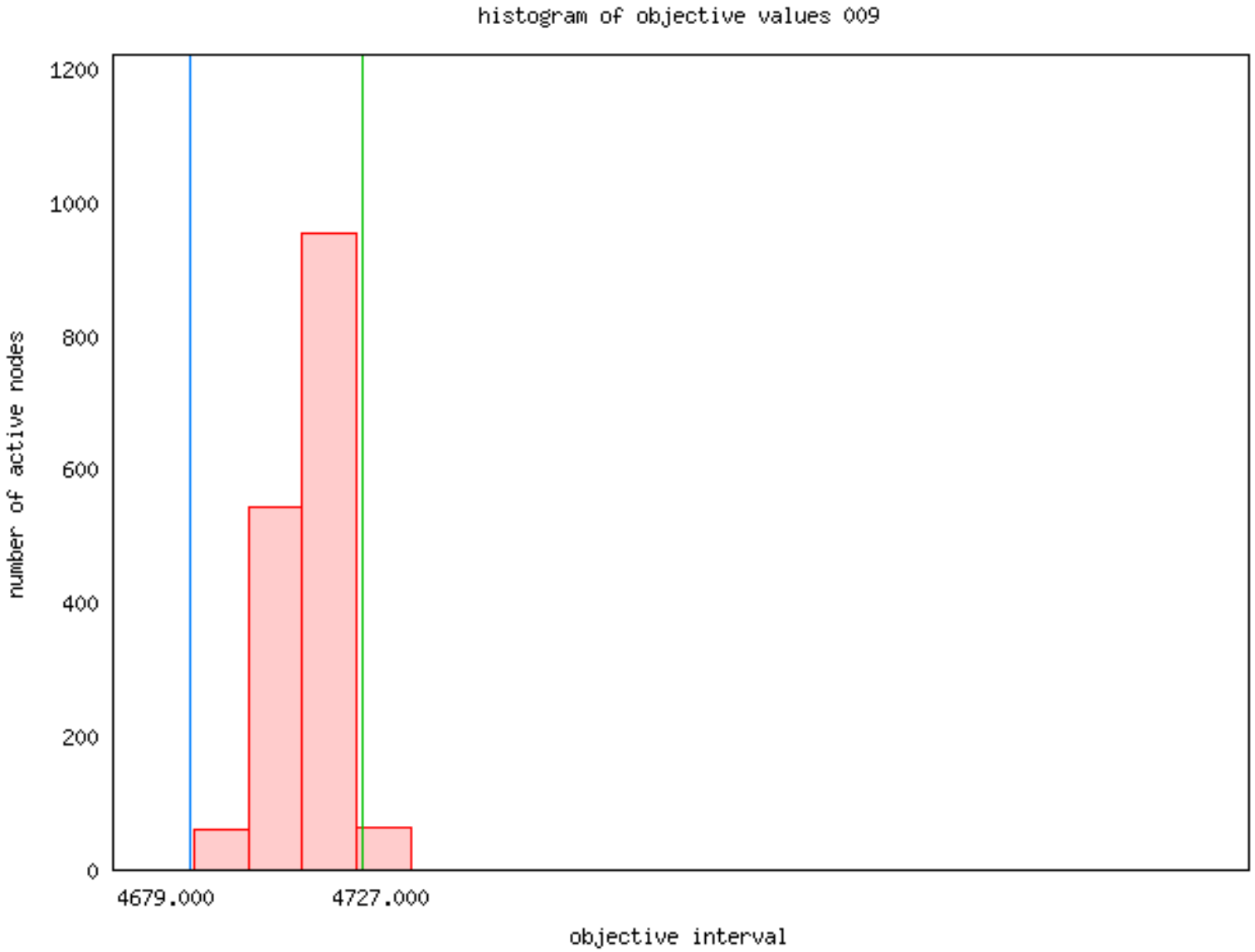

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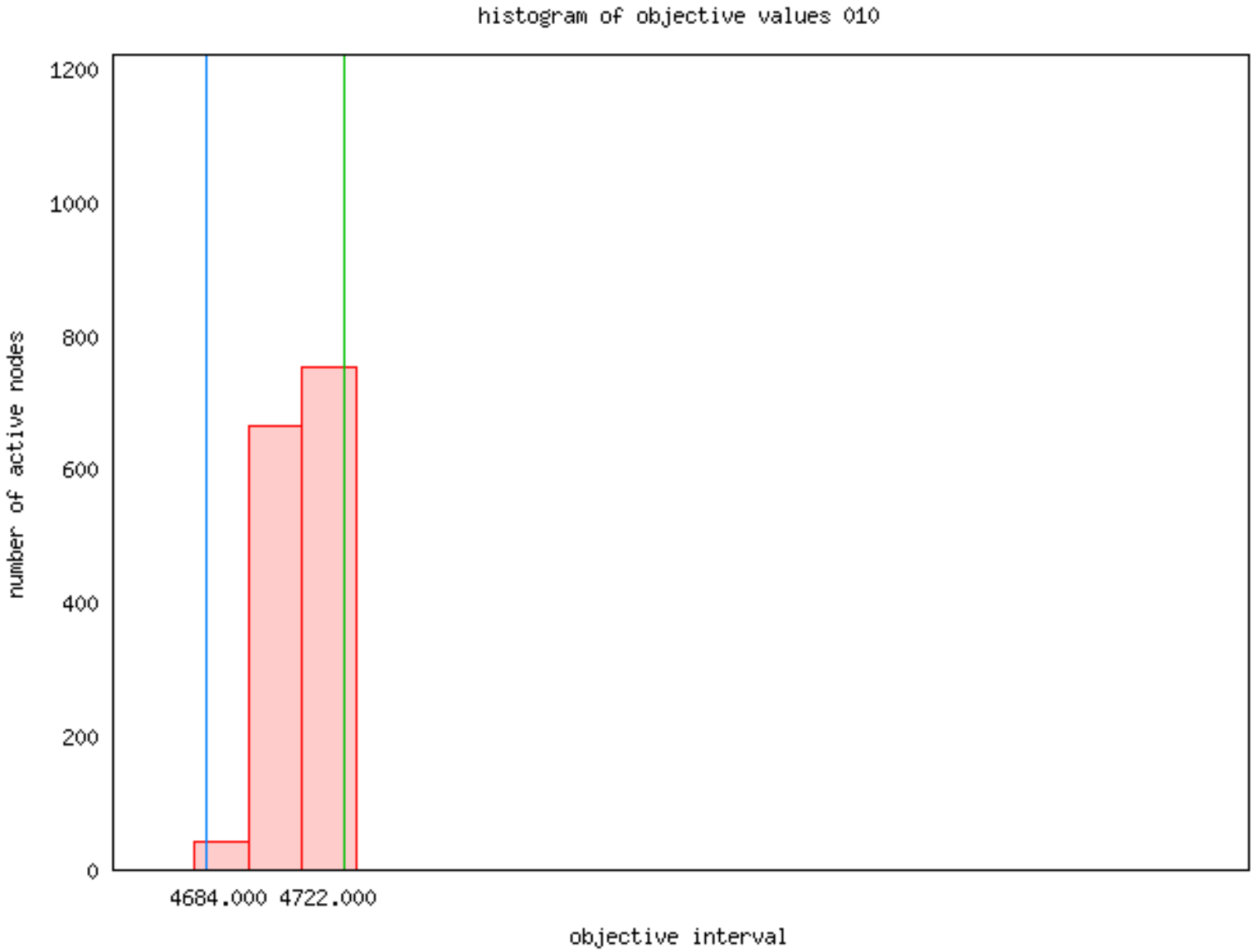

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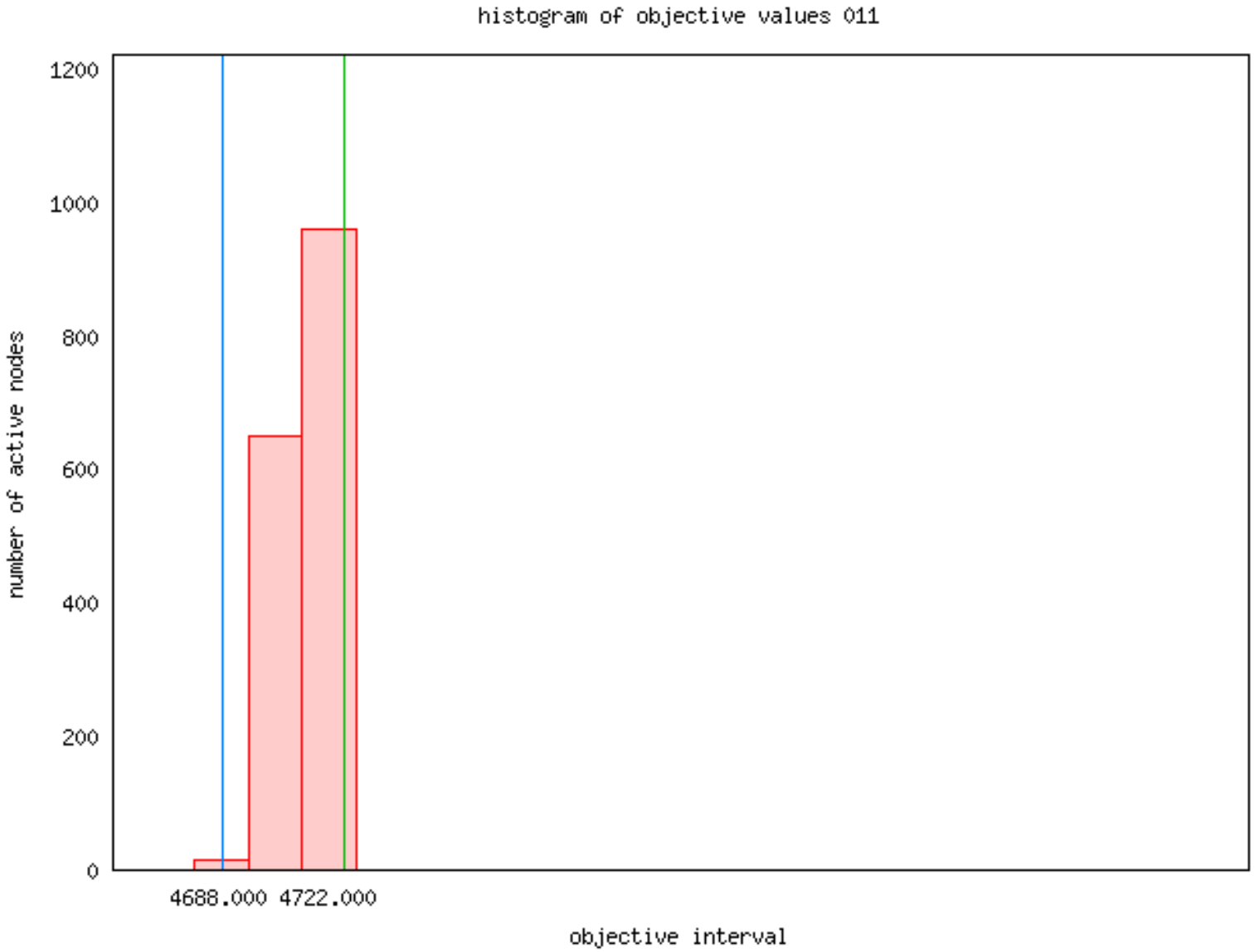

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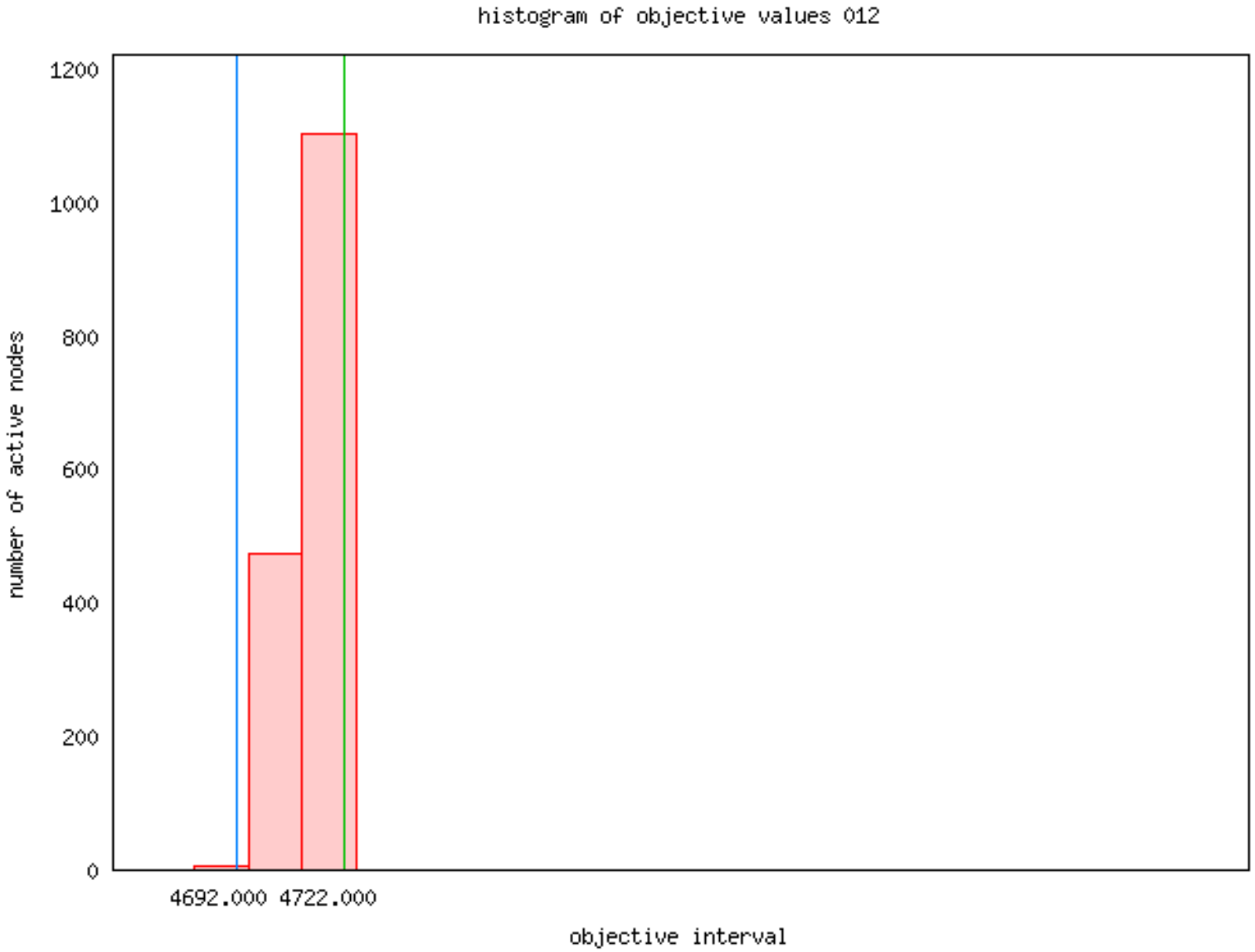

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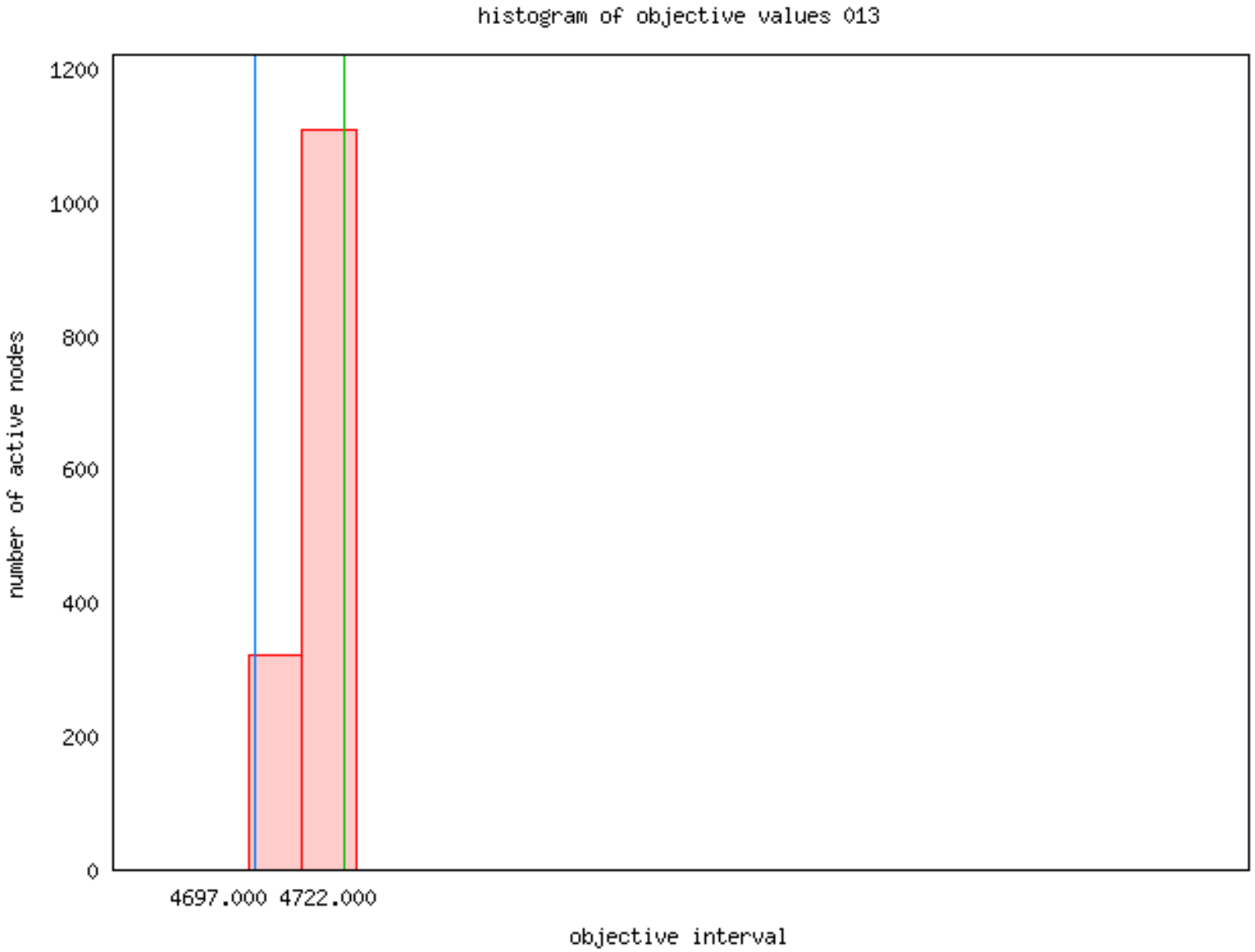

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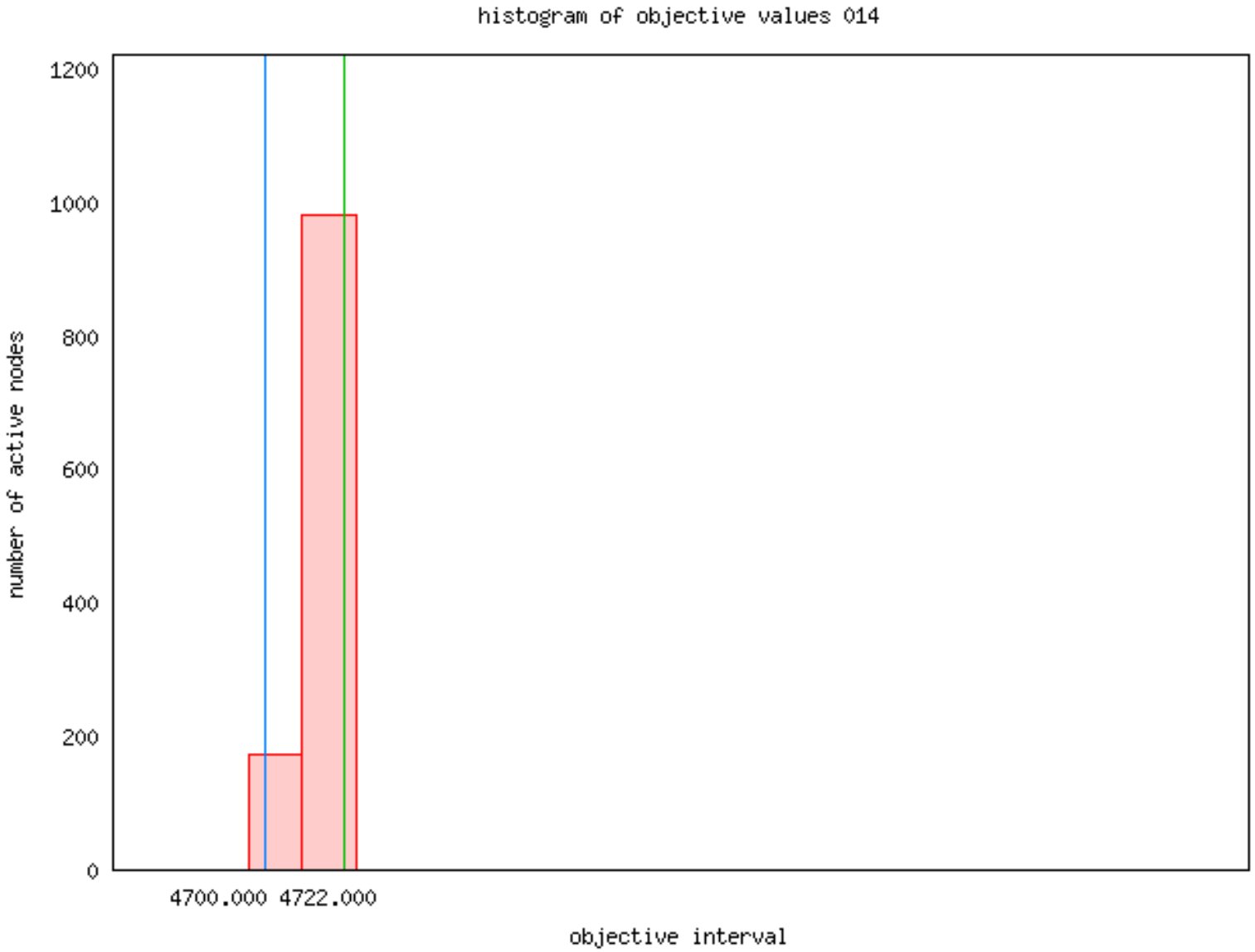

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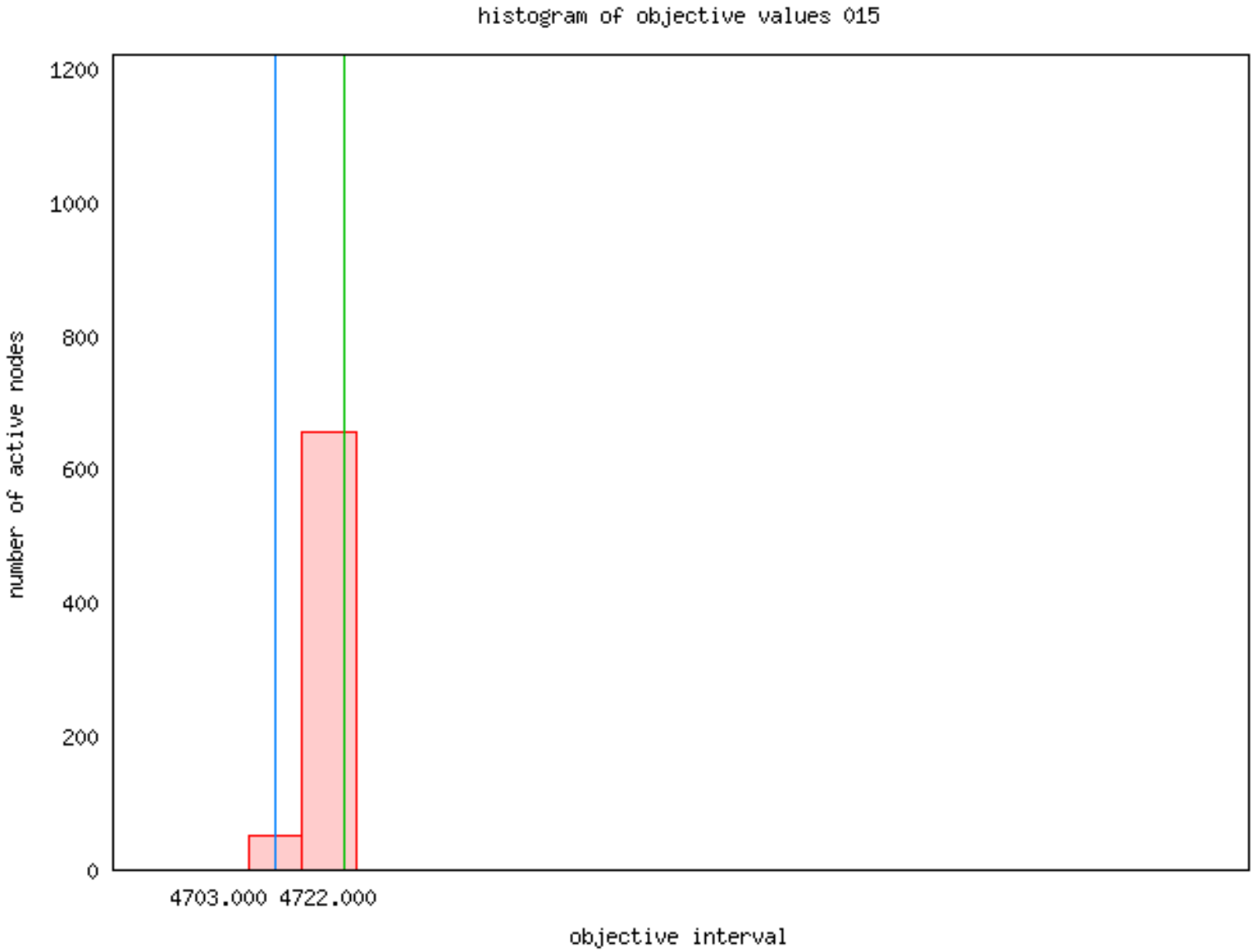

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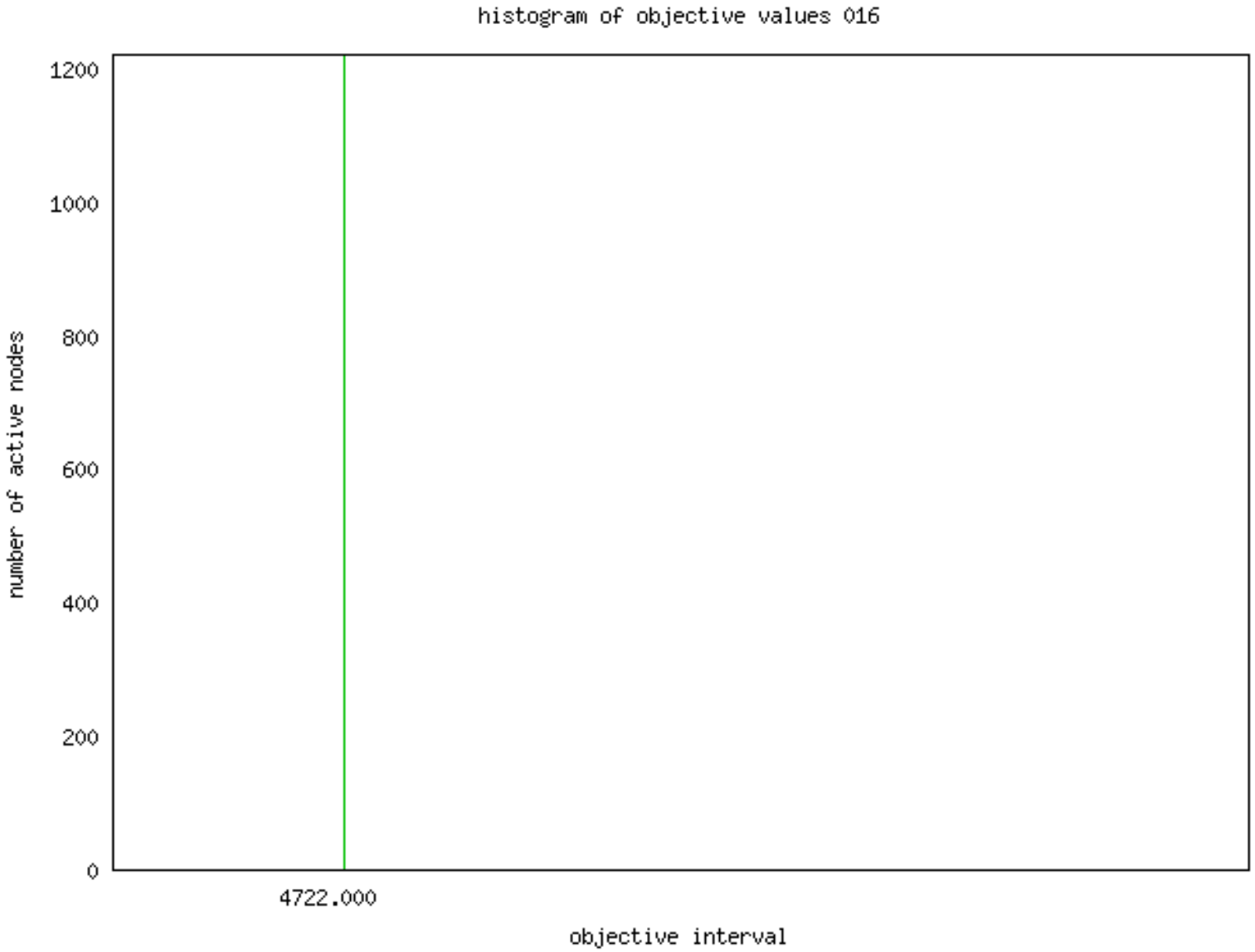

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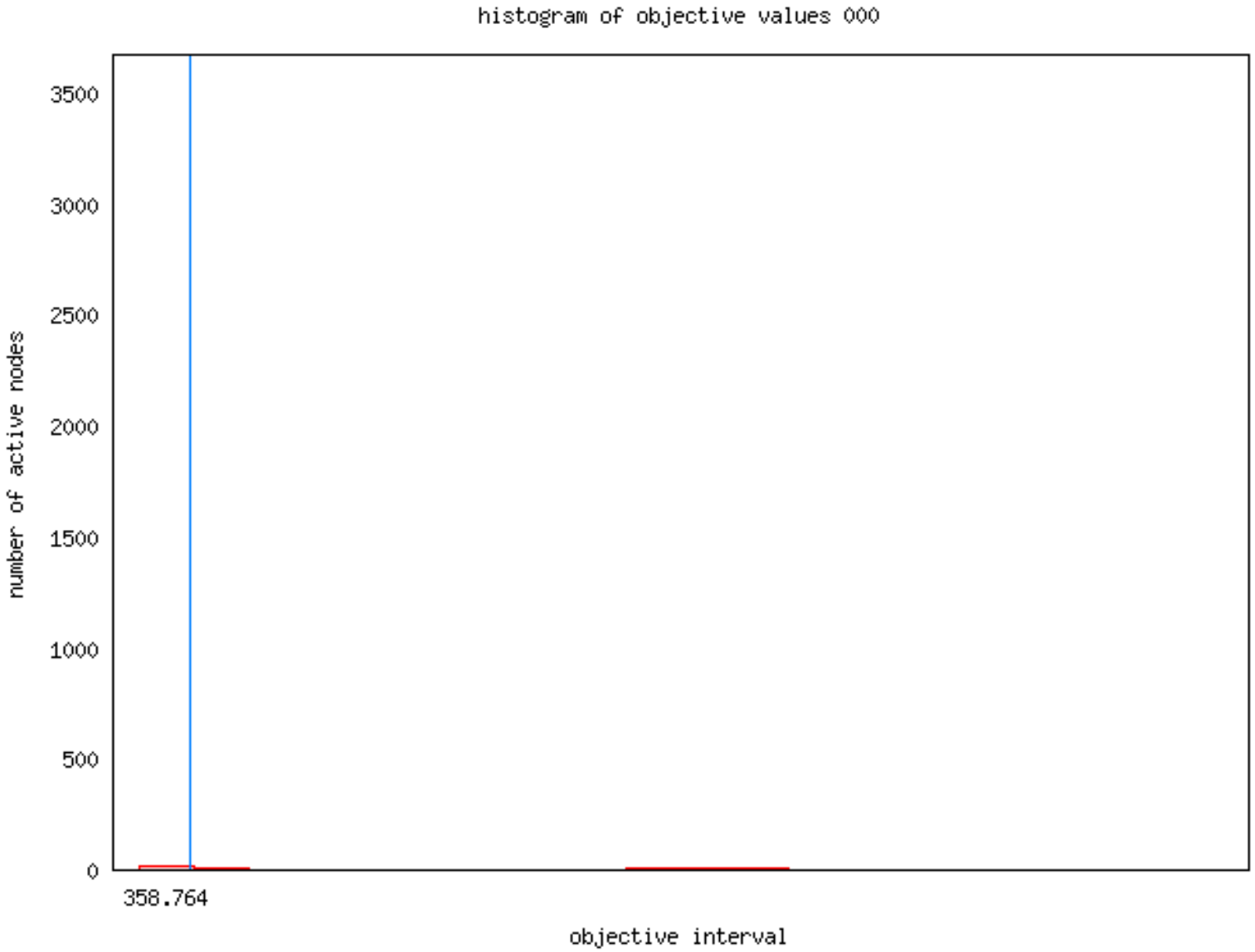

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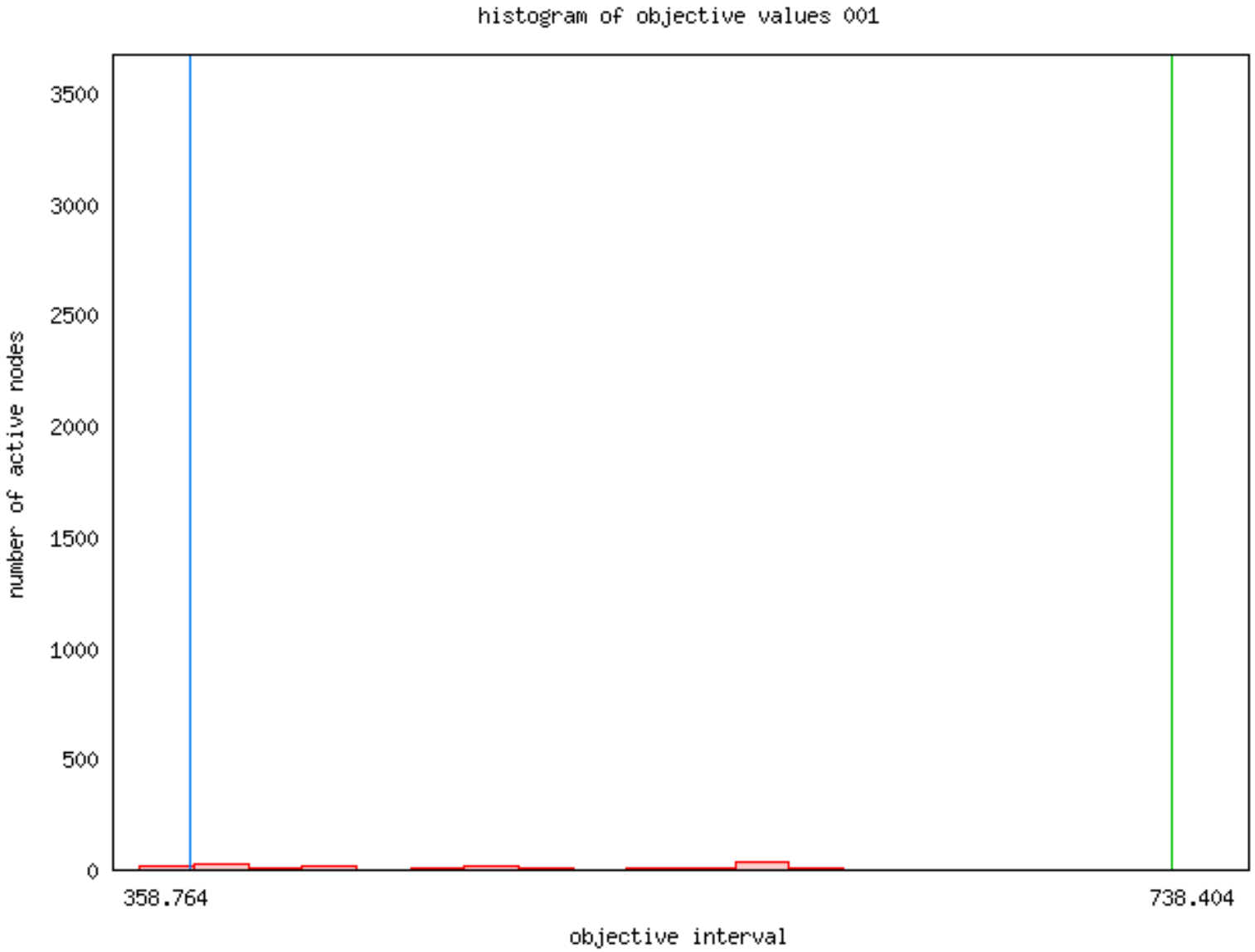

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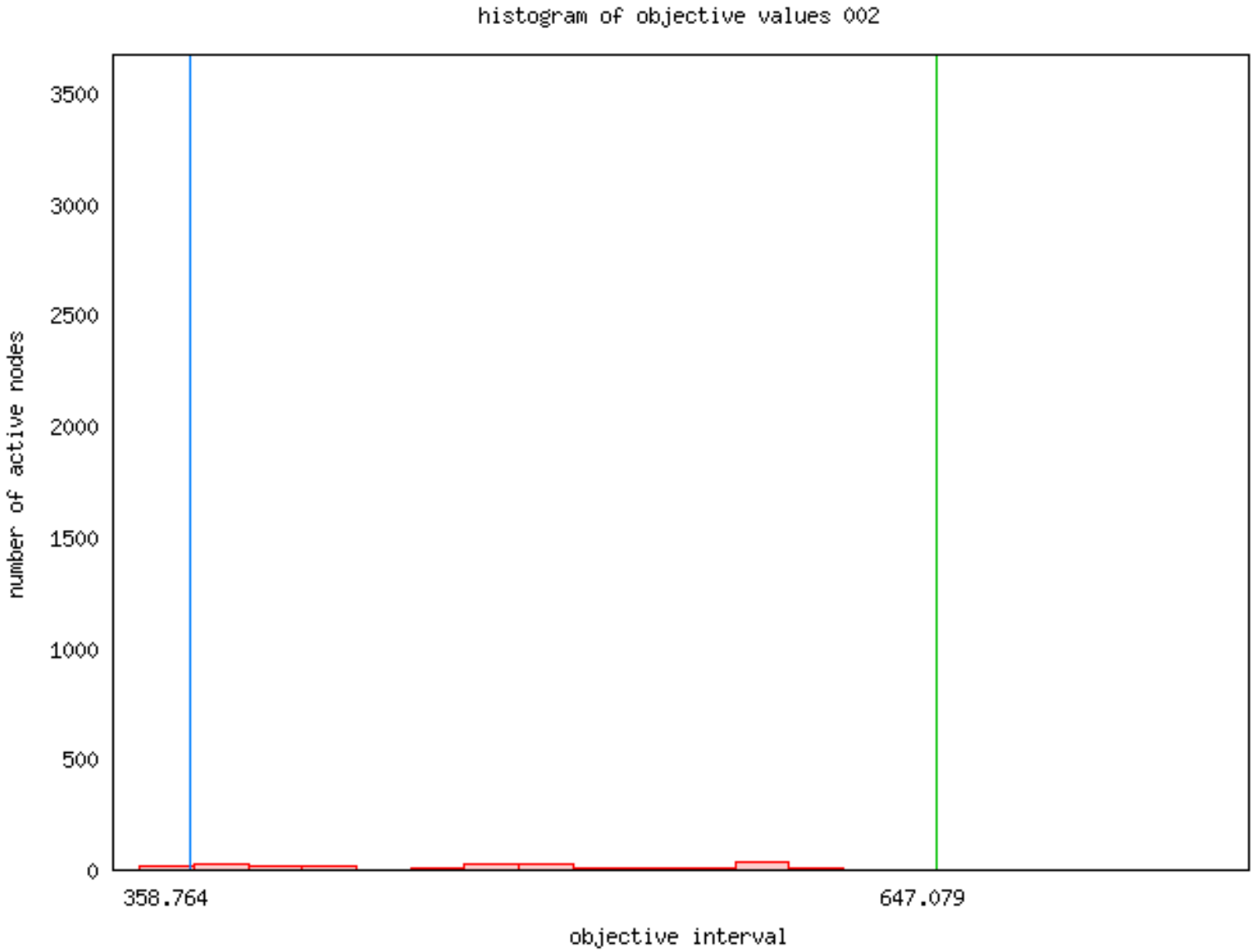

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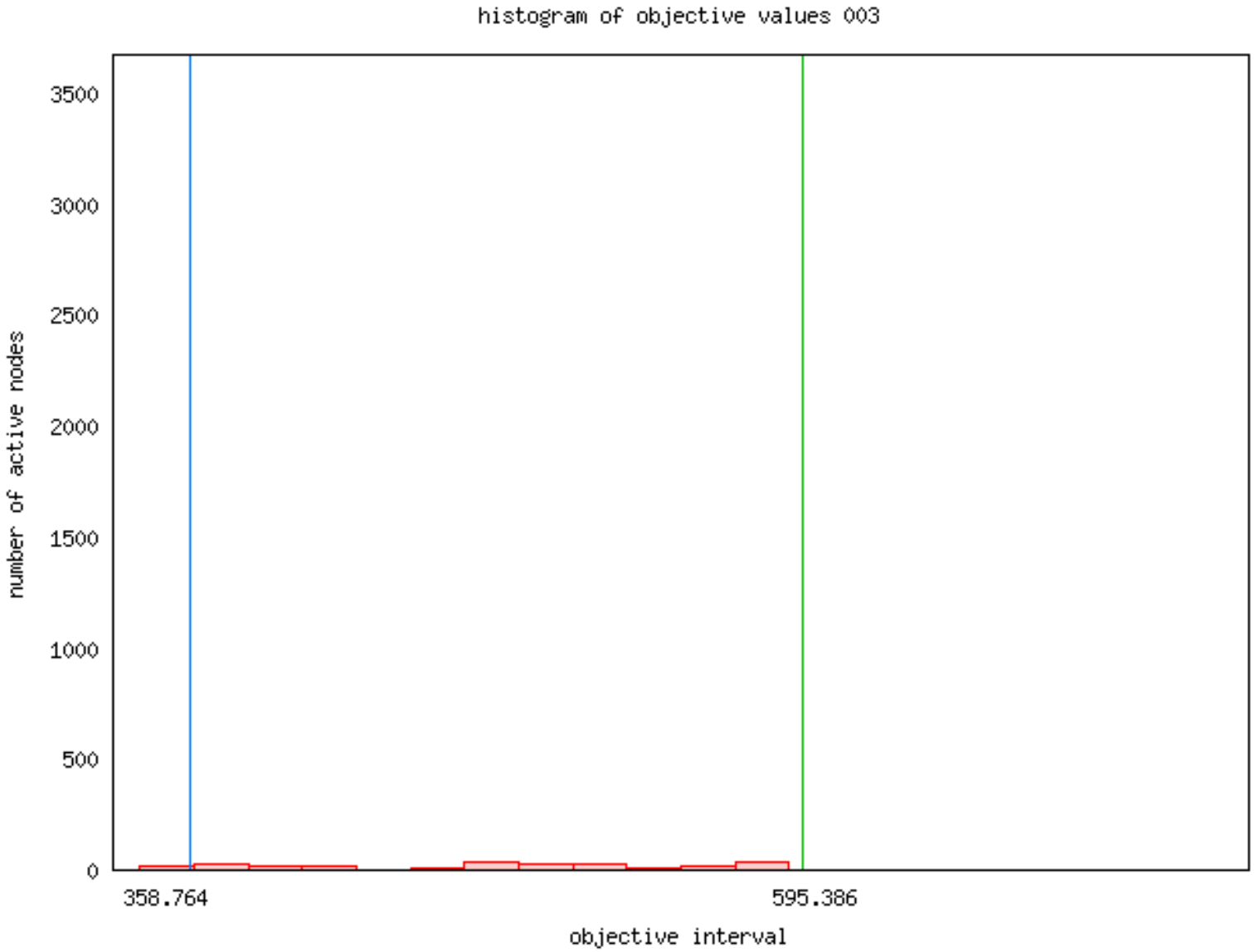

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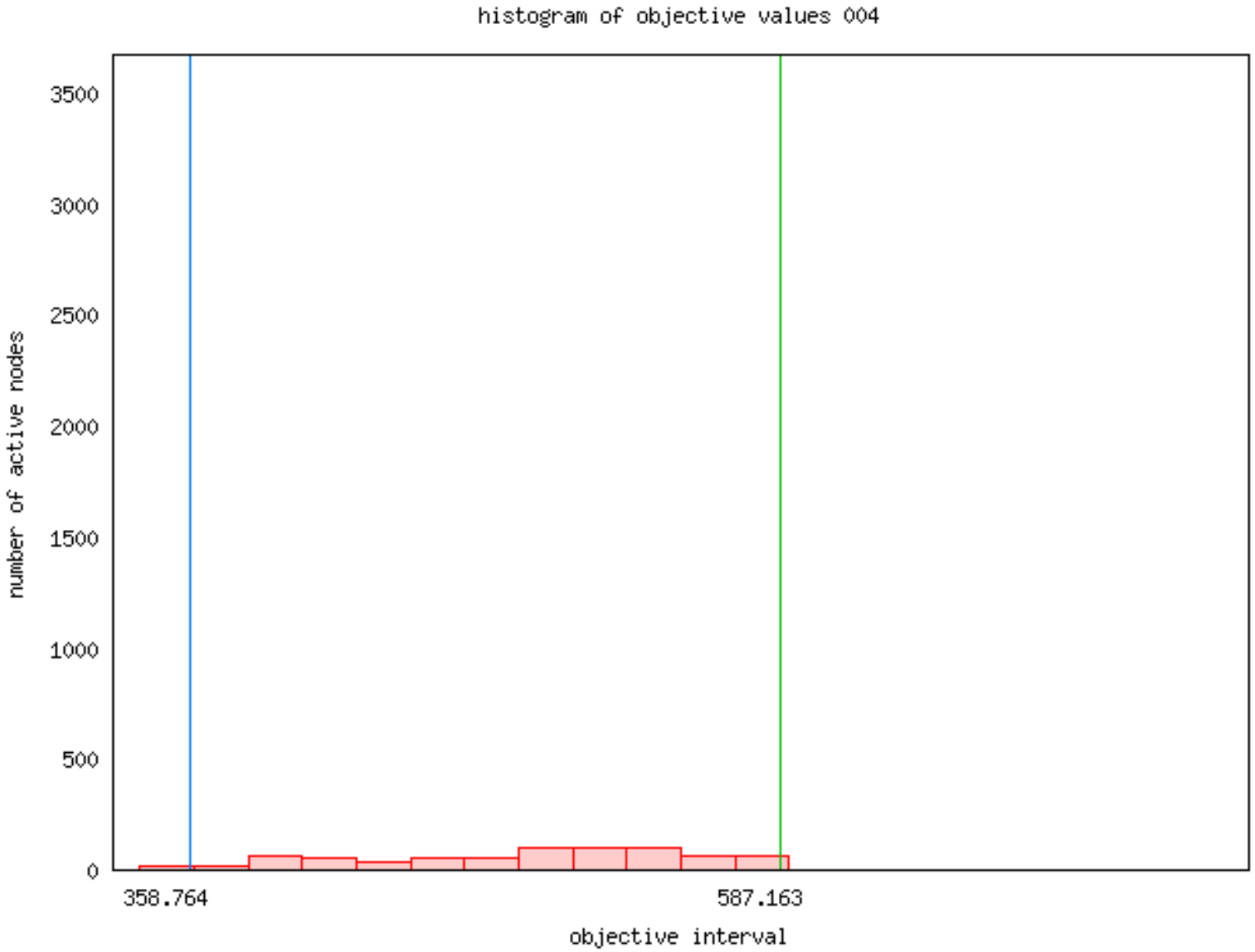

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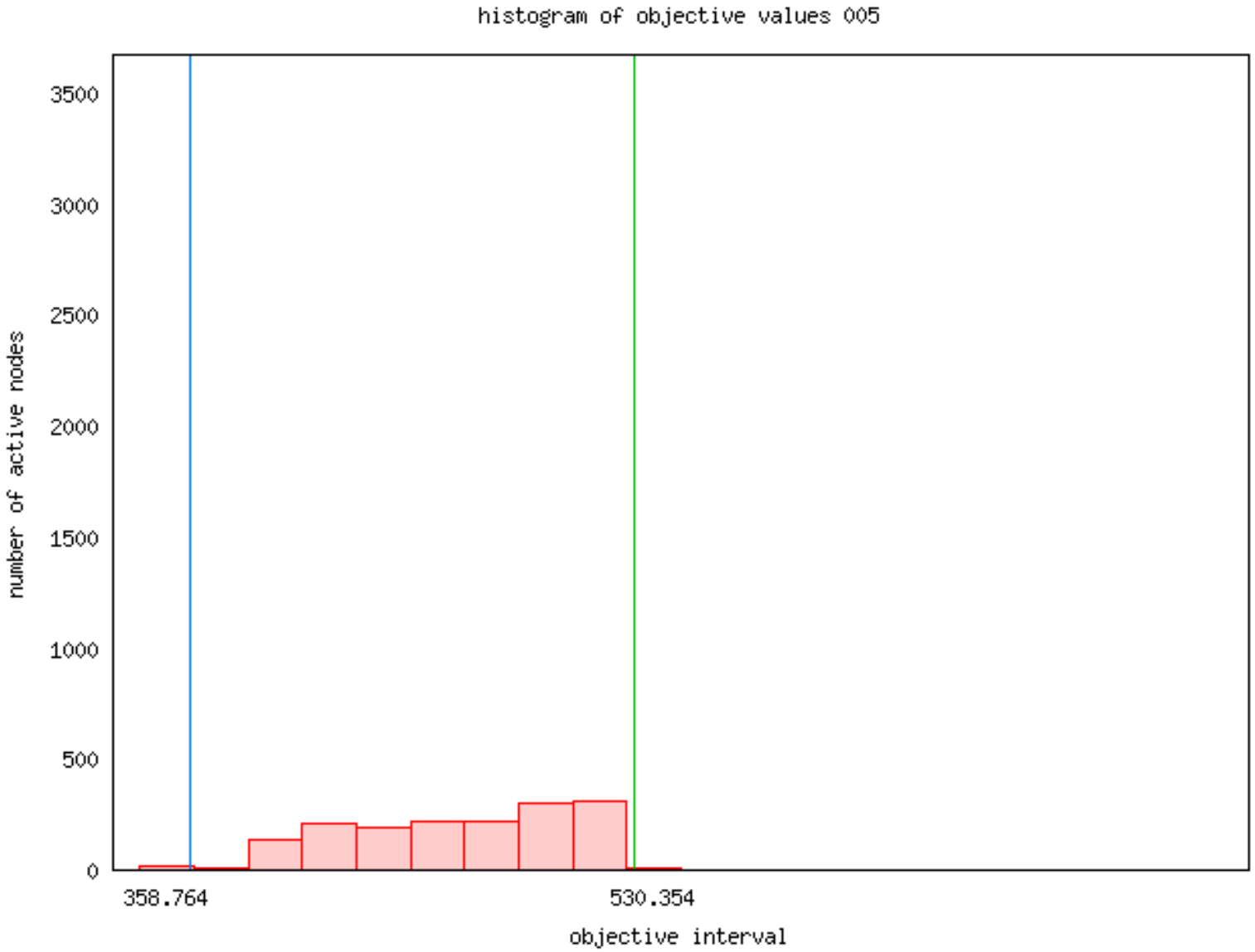

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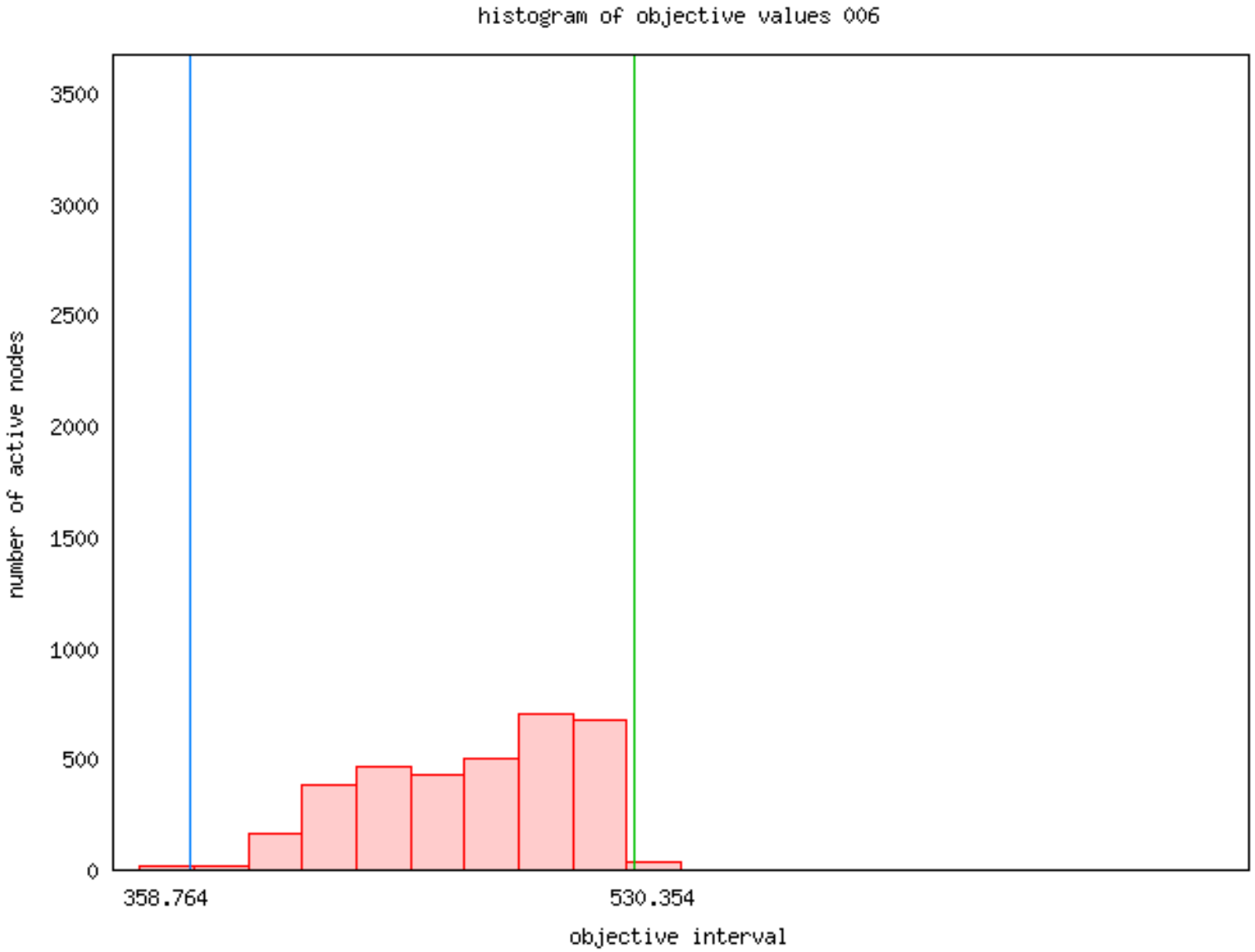

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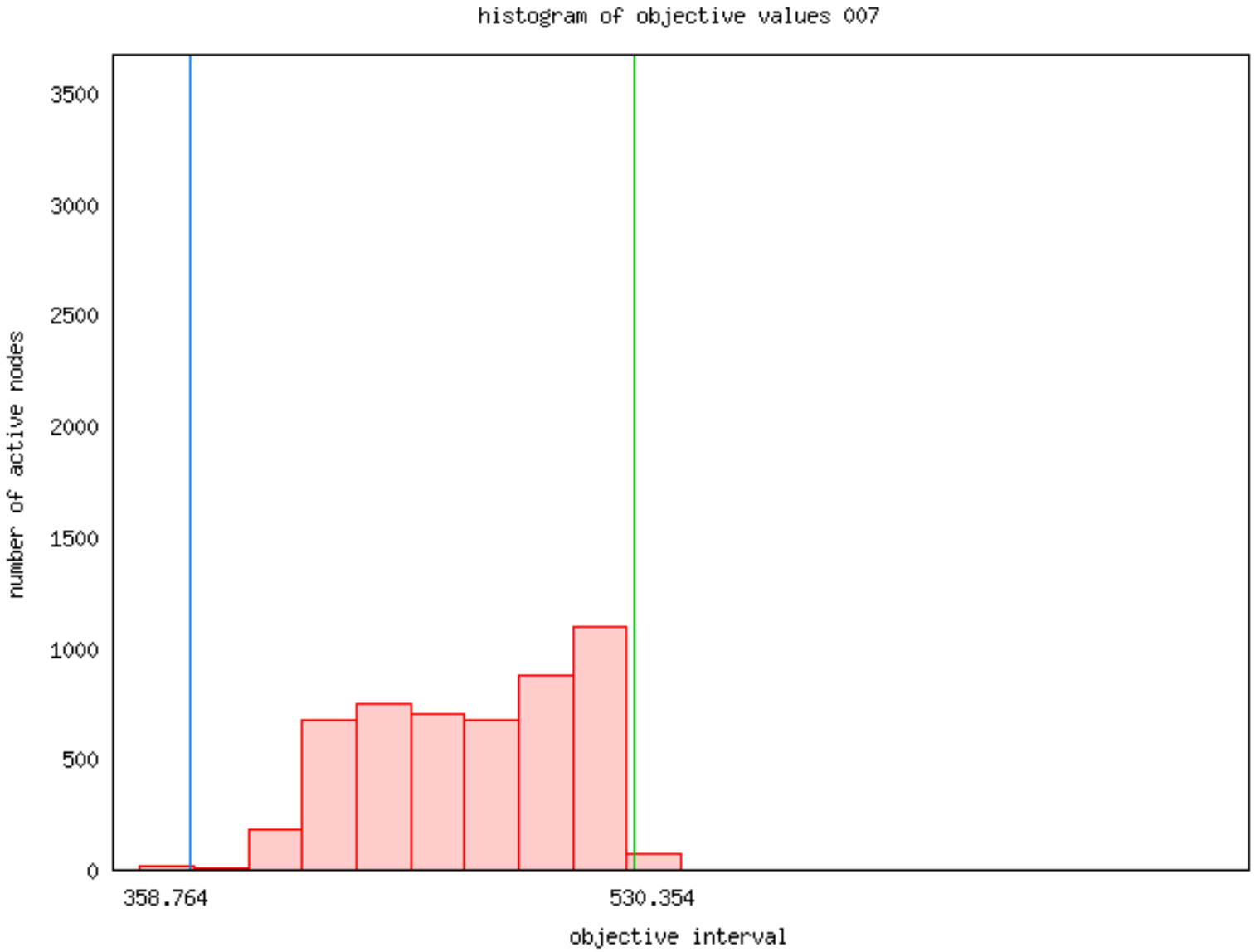

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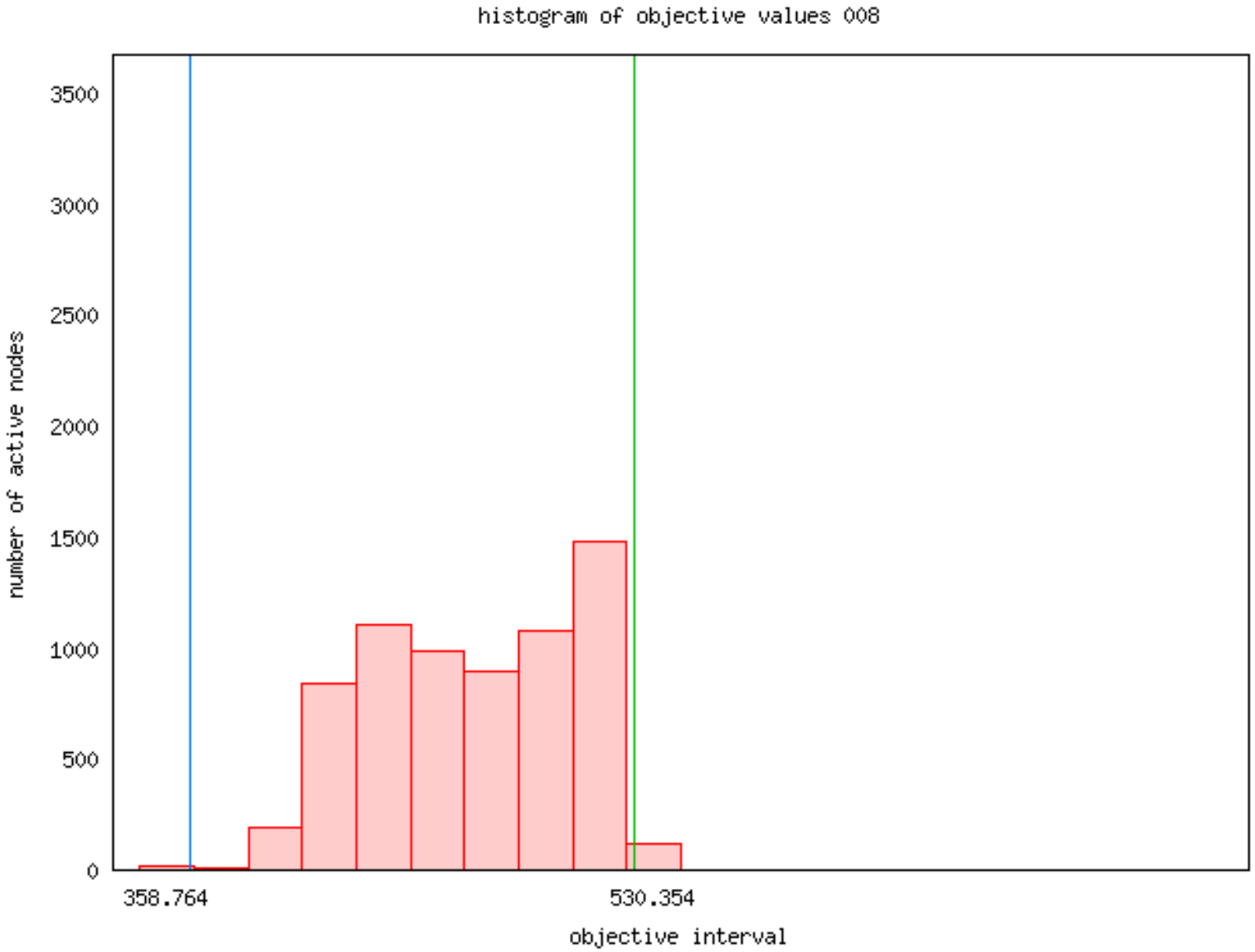

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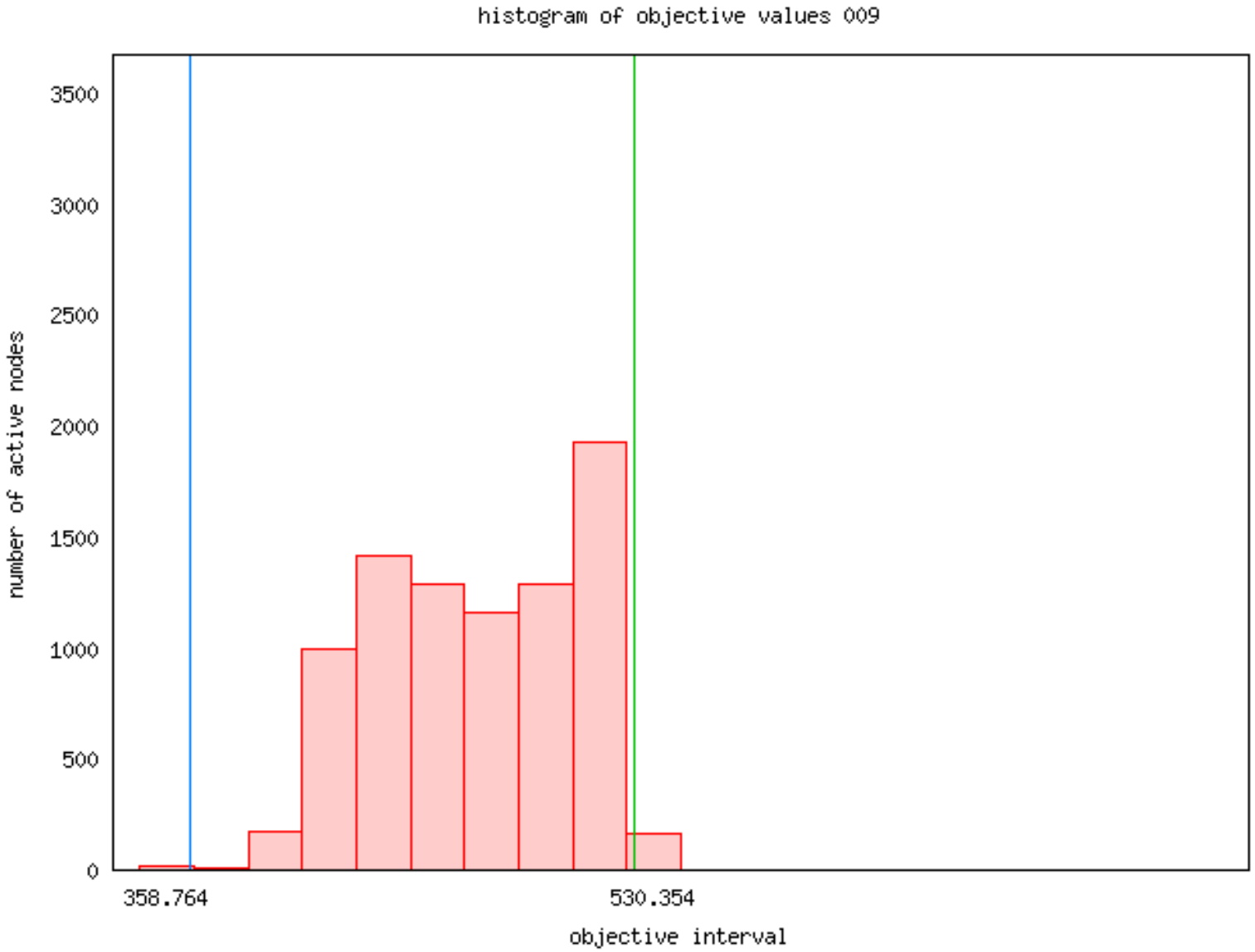

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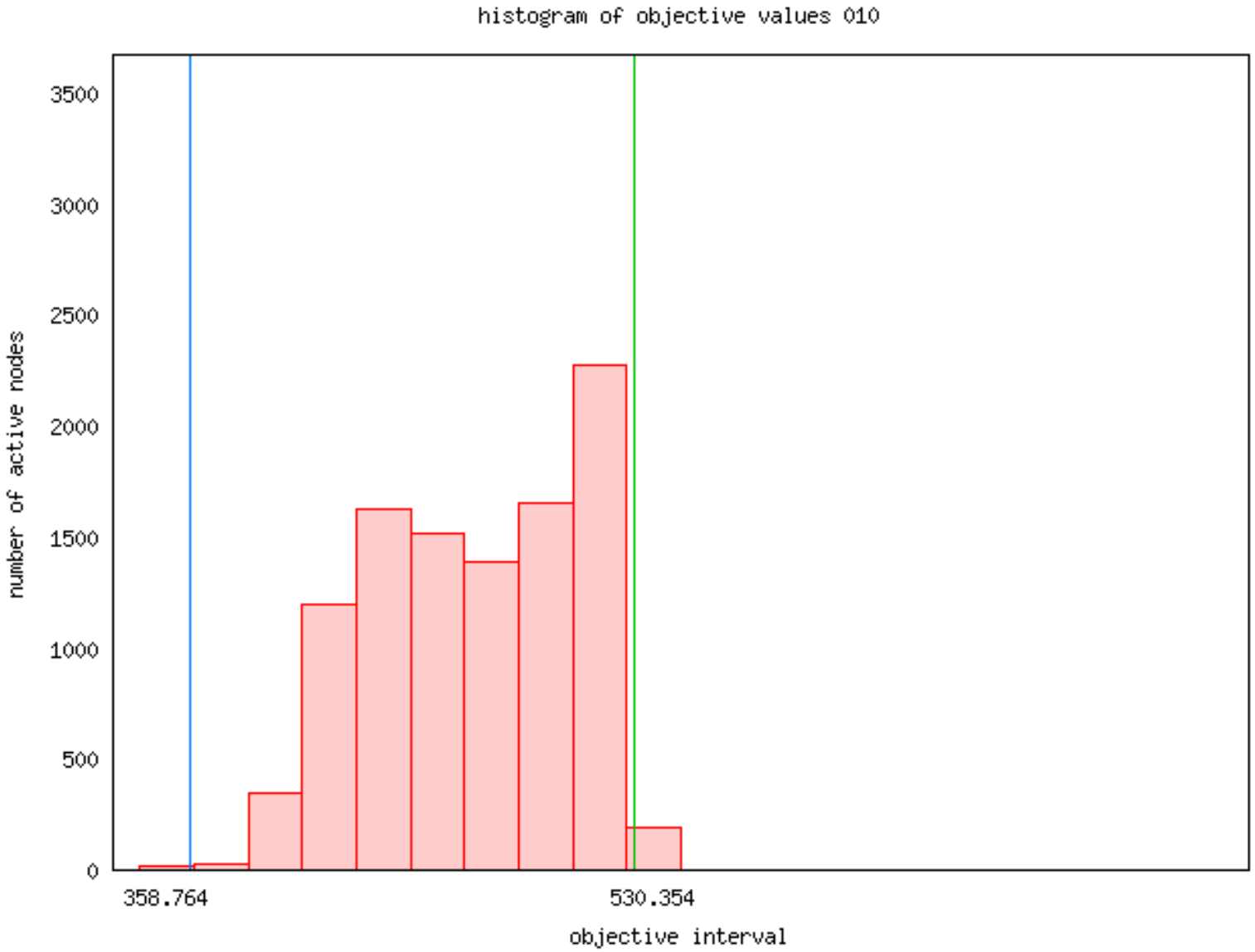
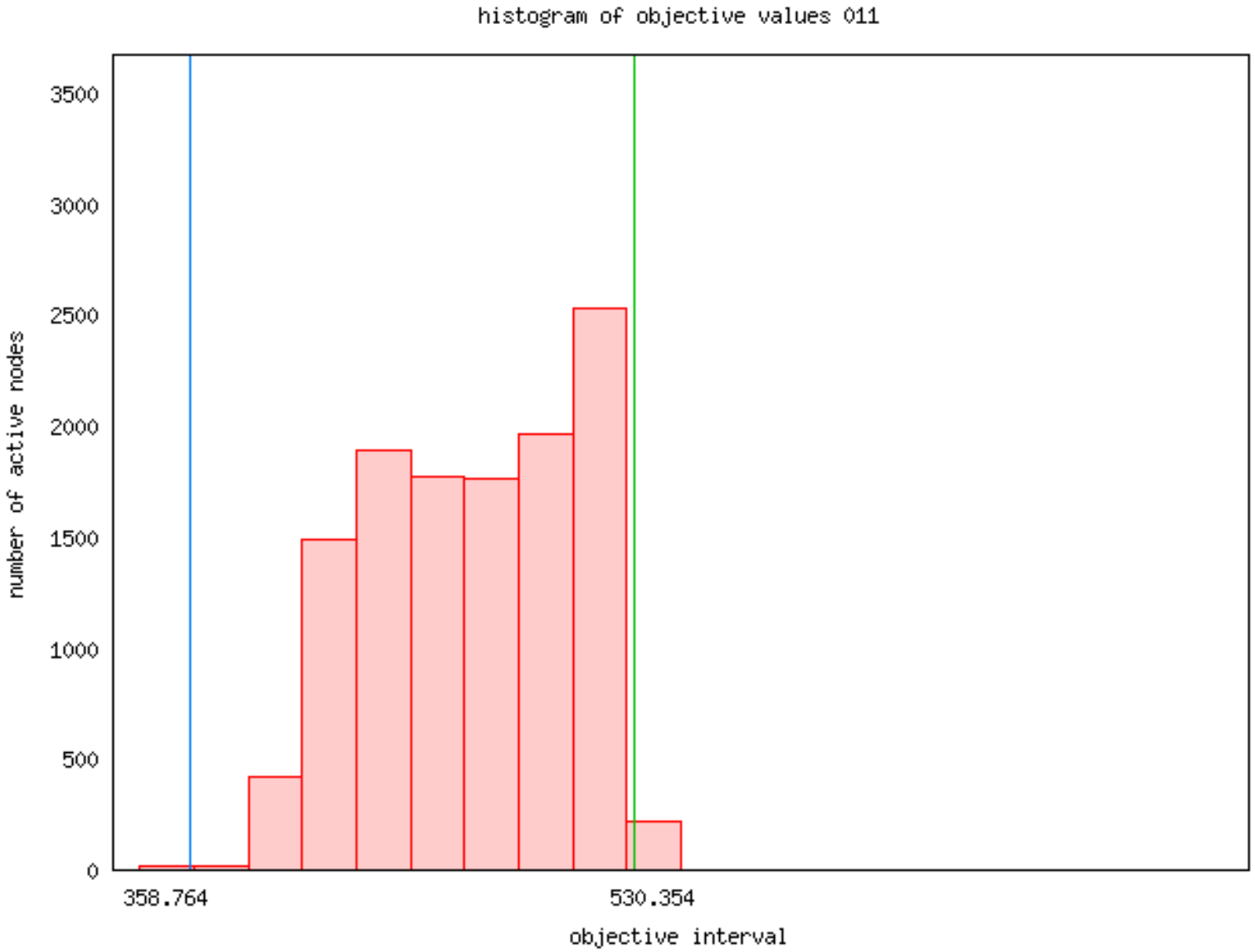
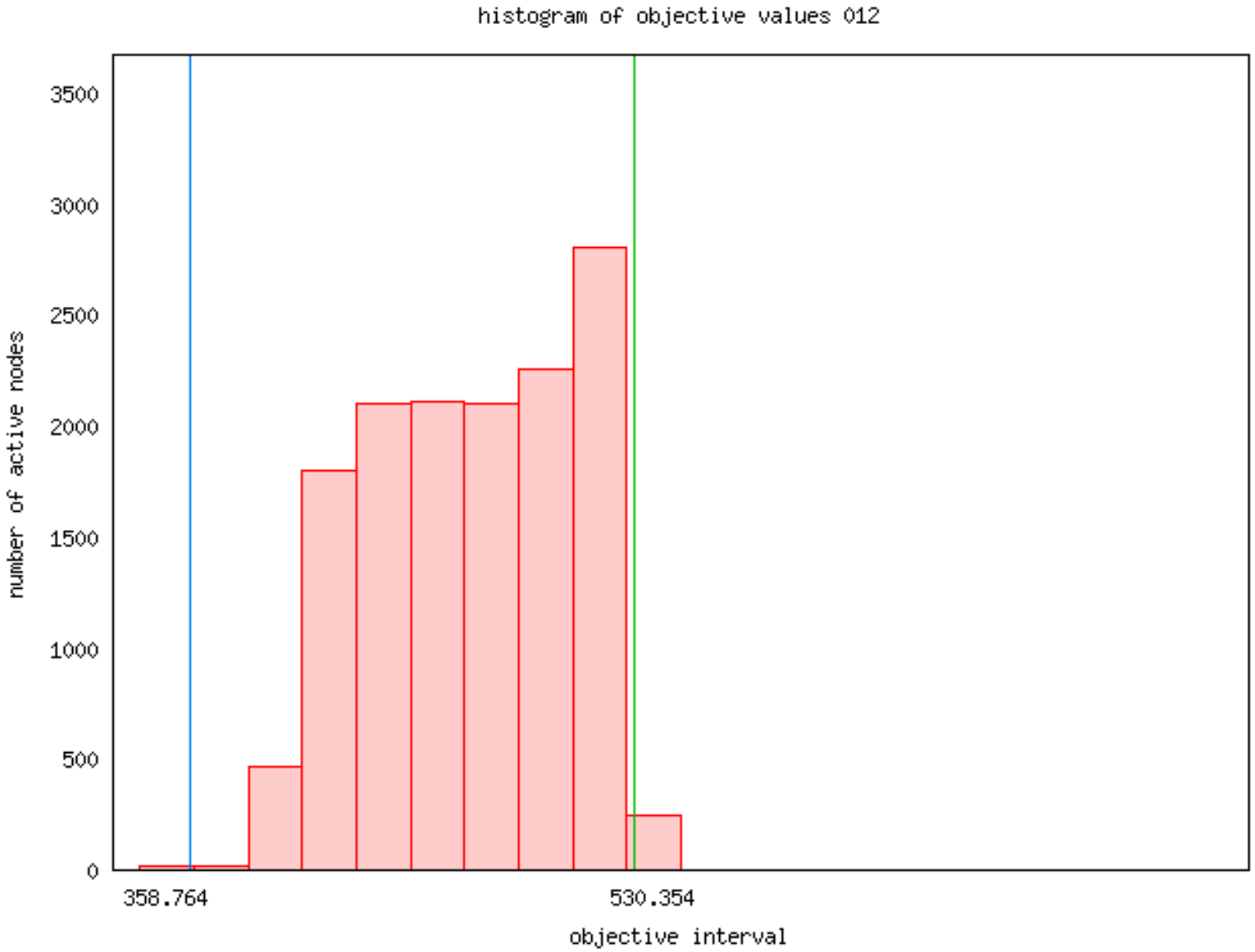


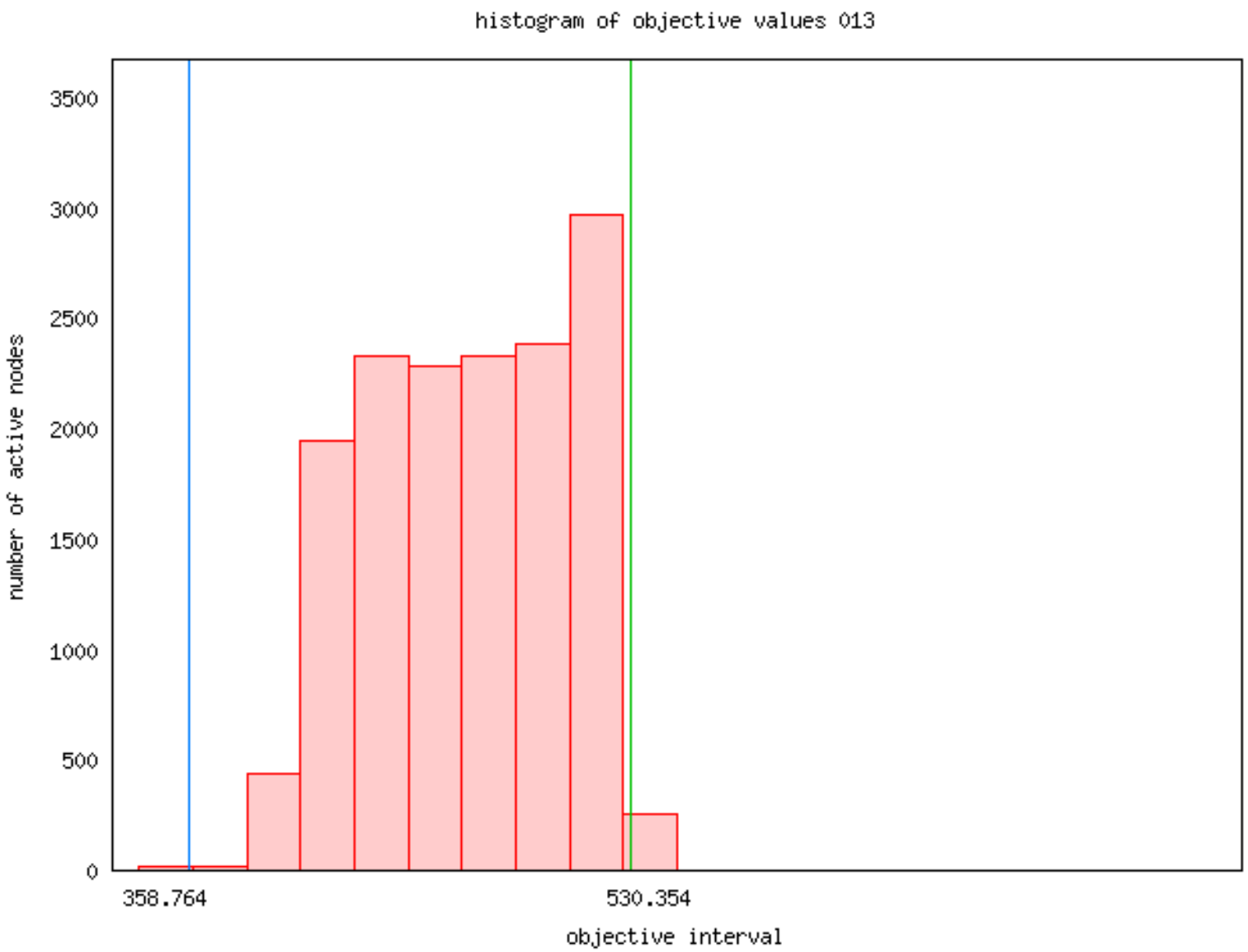
Example histogram series 2: swath (MIPLIB 2003)



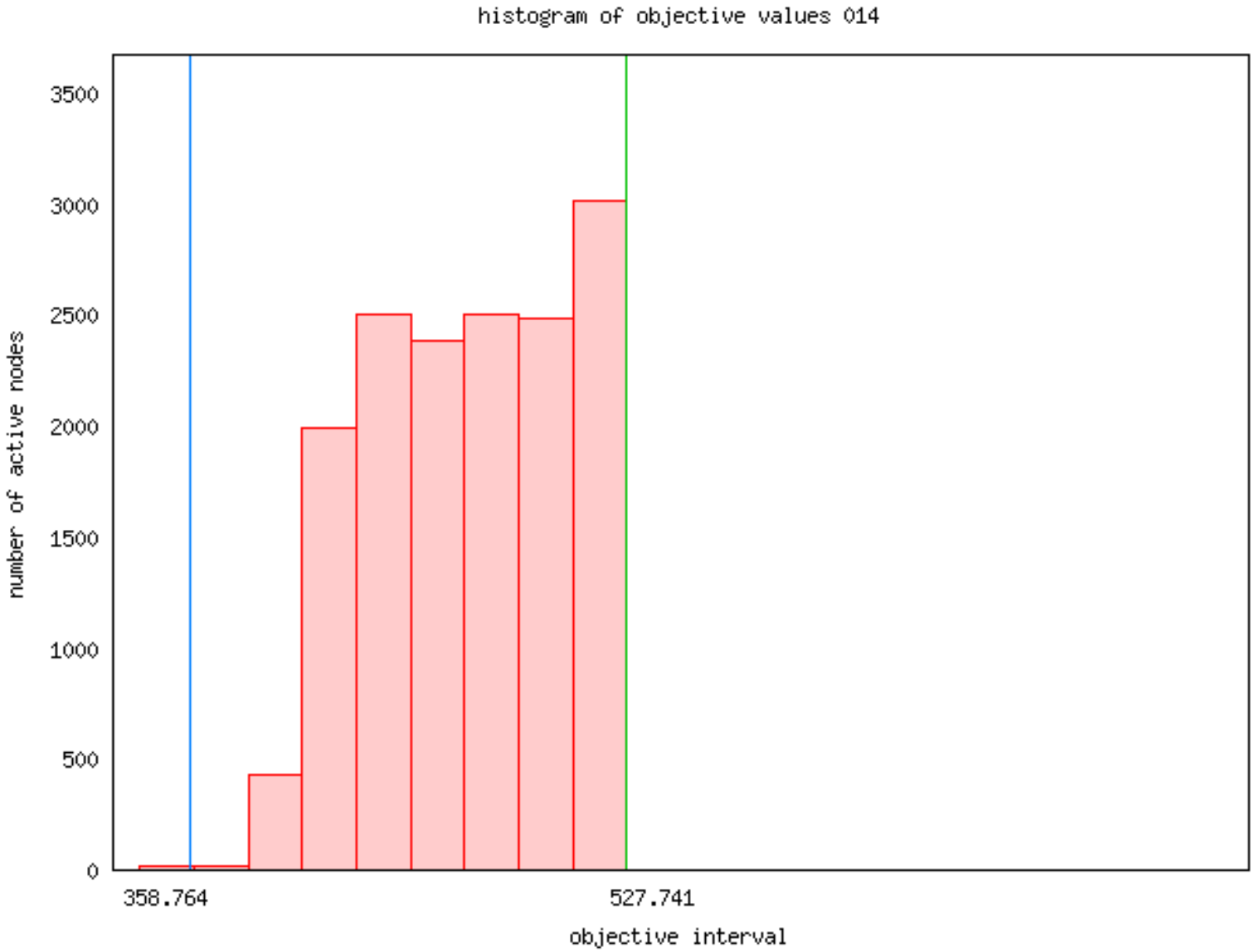
Example histogram series 2: swath (MIPLIB 2003)



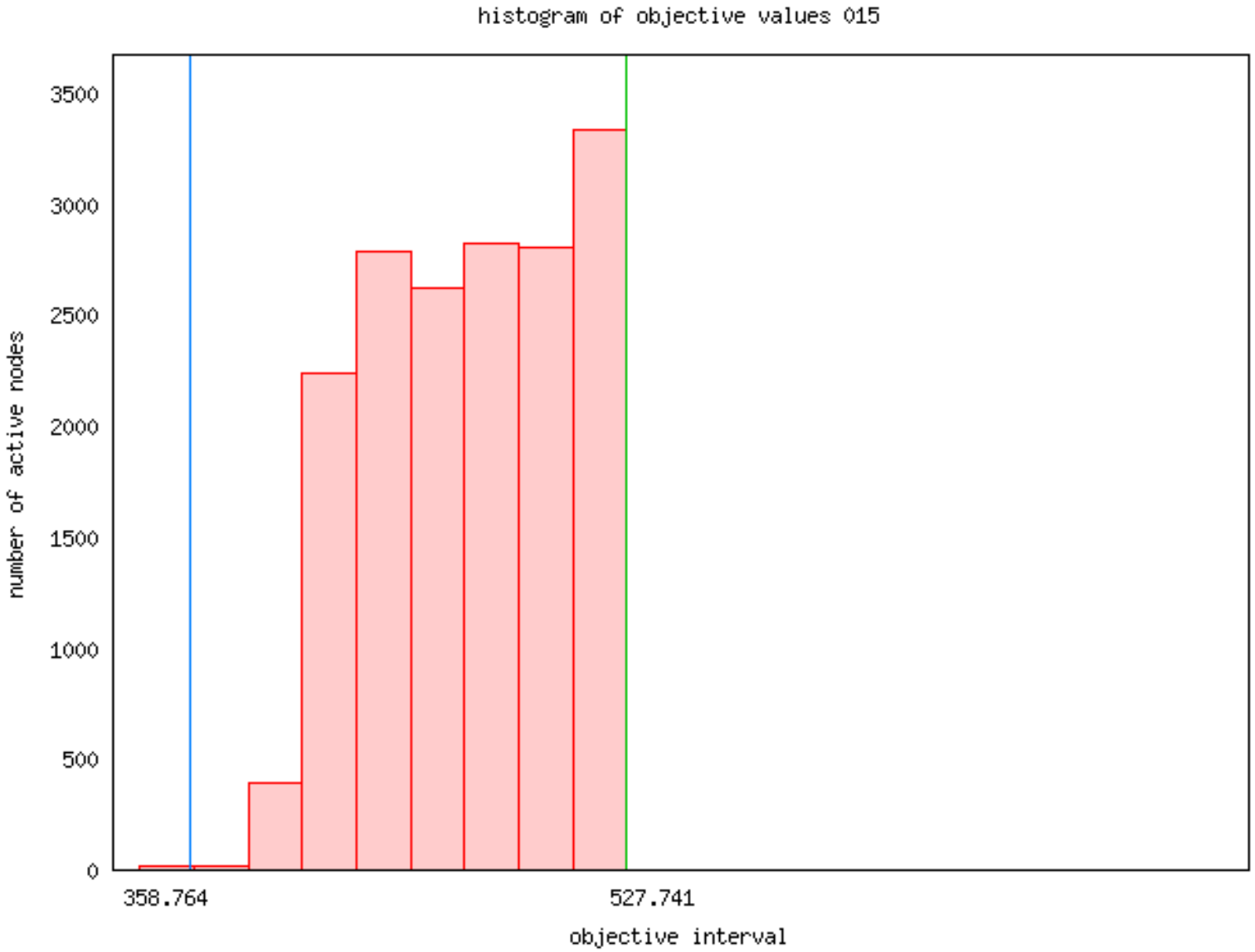
Example histogram series 2: swath (MIPLIB 2003)



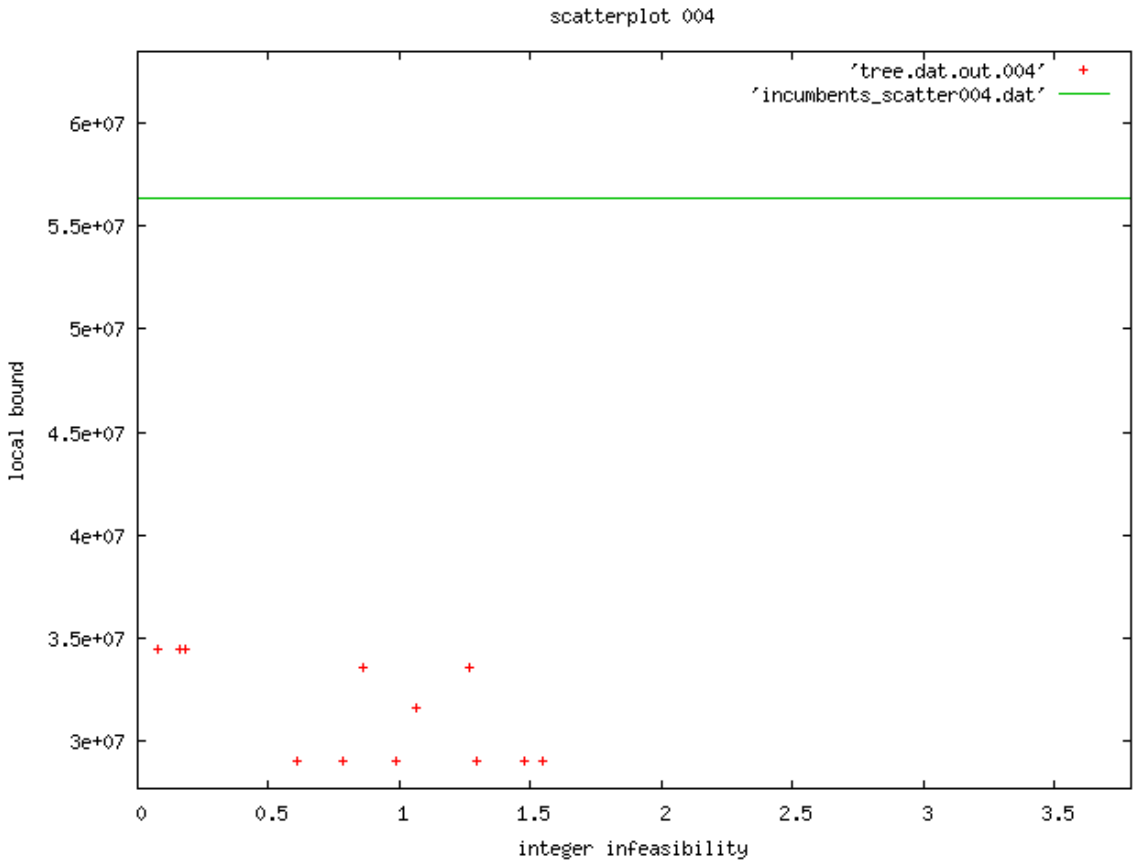
Example histogram series 2: swath (MIPLIB 2003)



Example histogram series 2: swath (MIPLIB 2003)

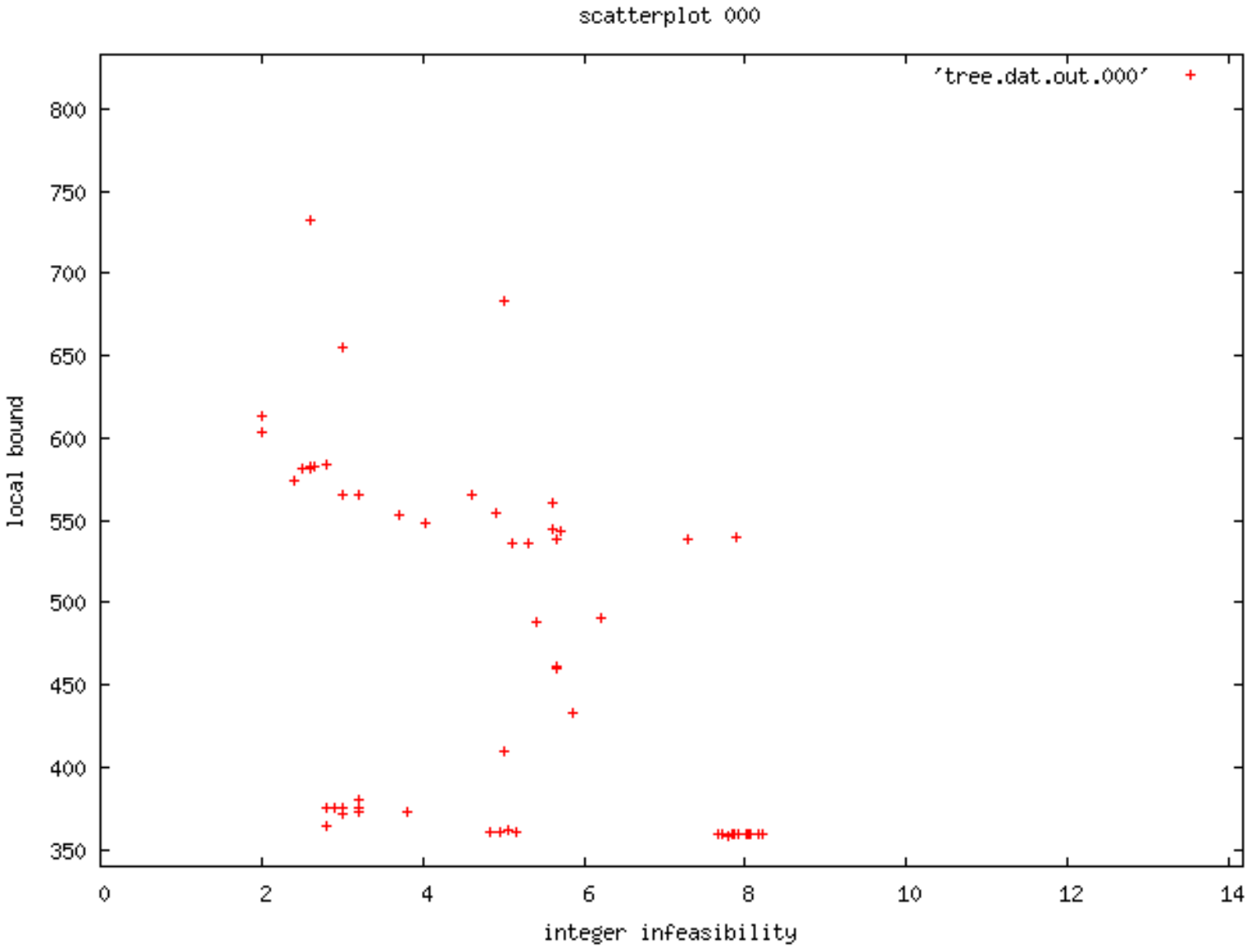


Visualization tools: Scatter plot of active node LP bounds & integer infeasibility

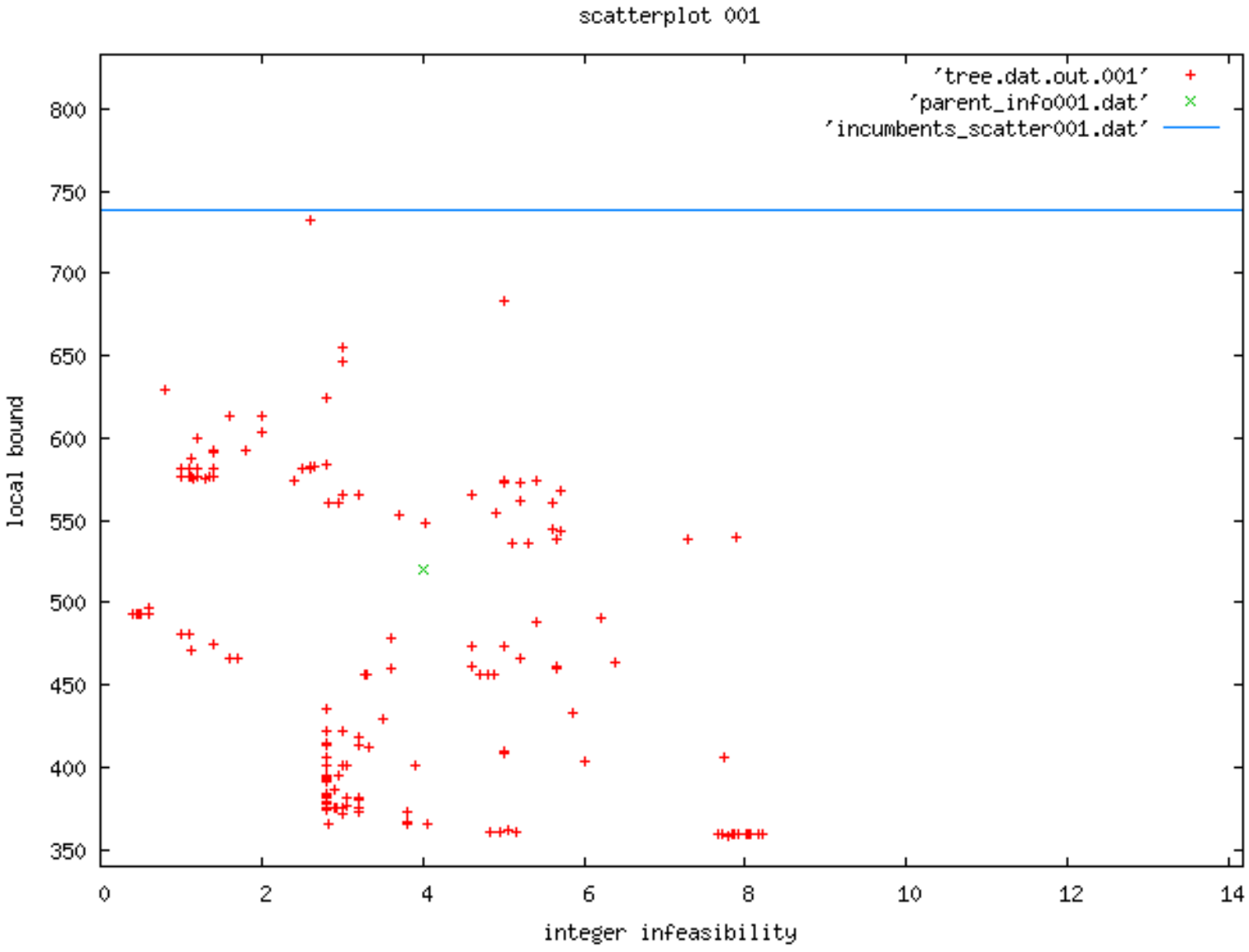


- Horizontal axis is the integer infeasibility
- Vertical axis is the LP bound
- Green horizontal line is the current incumbent value

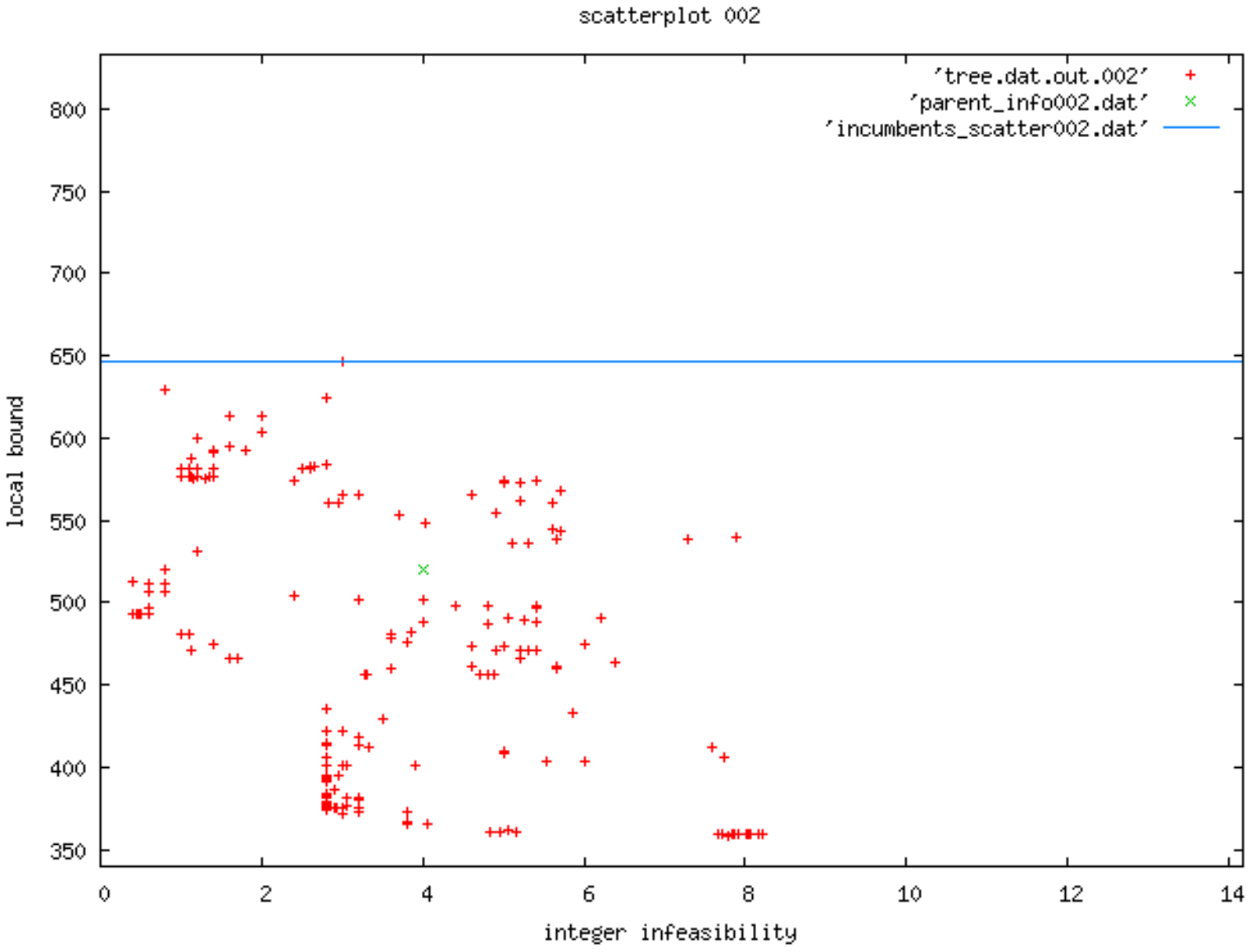
Example scatter plot series 1: swath (MIPLIB 2003)



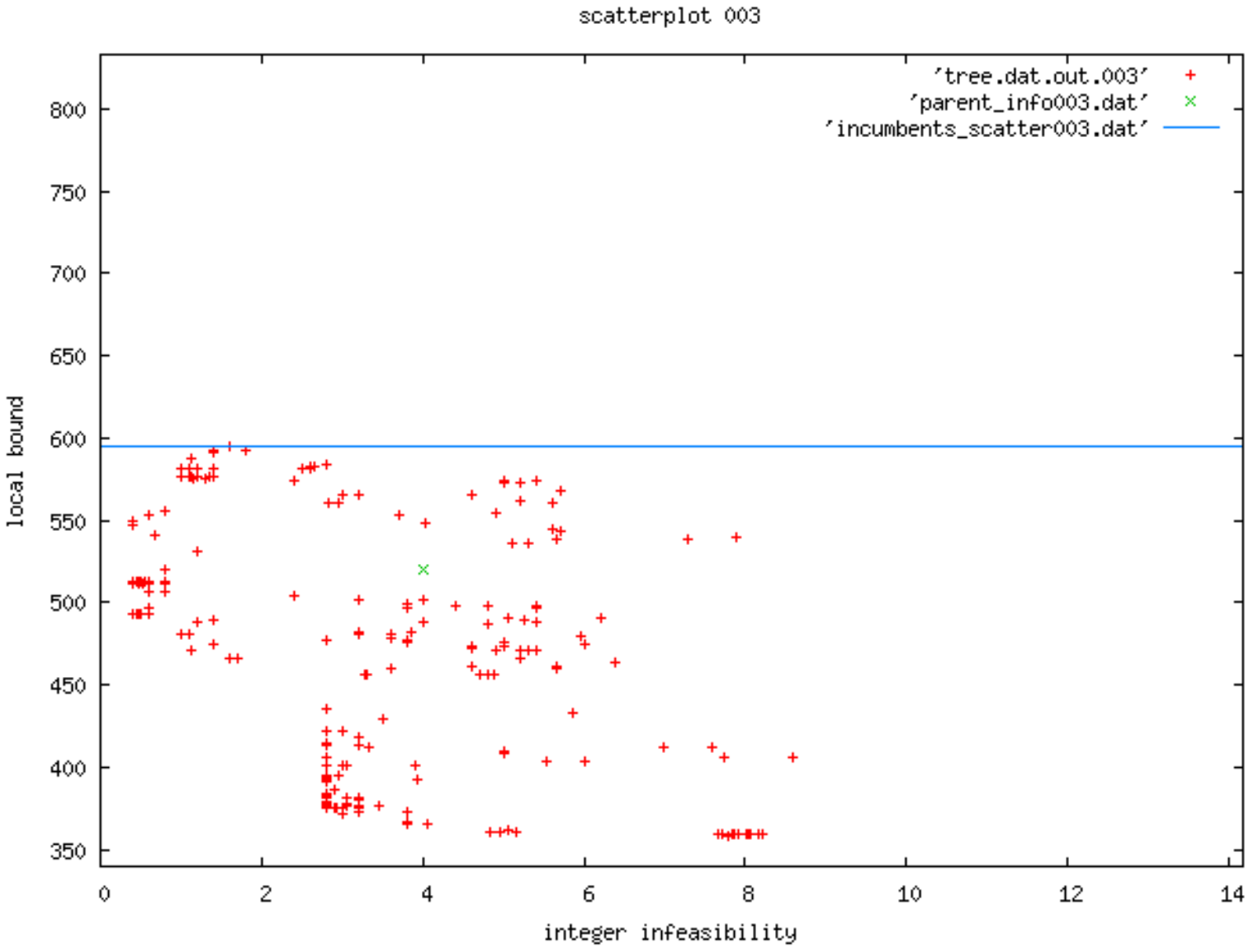
Example scatter plot series 1: swath (MIPLIB 2003)



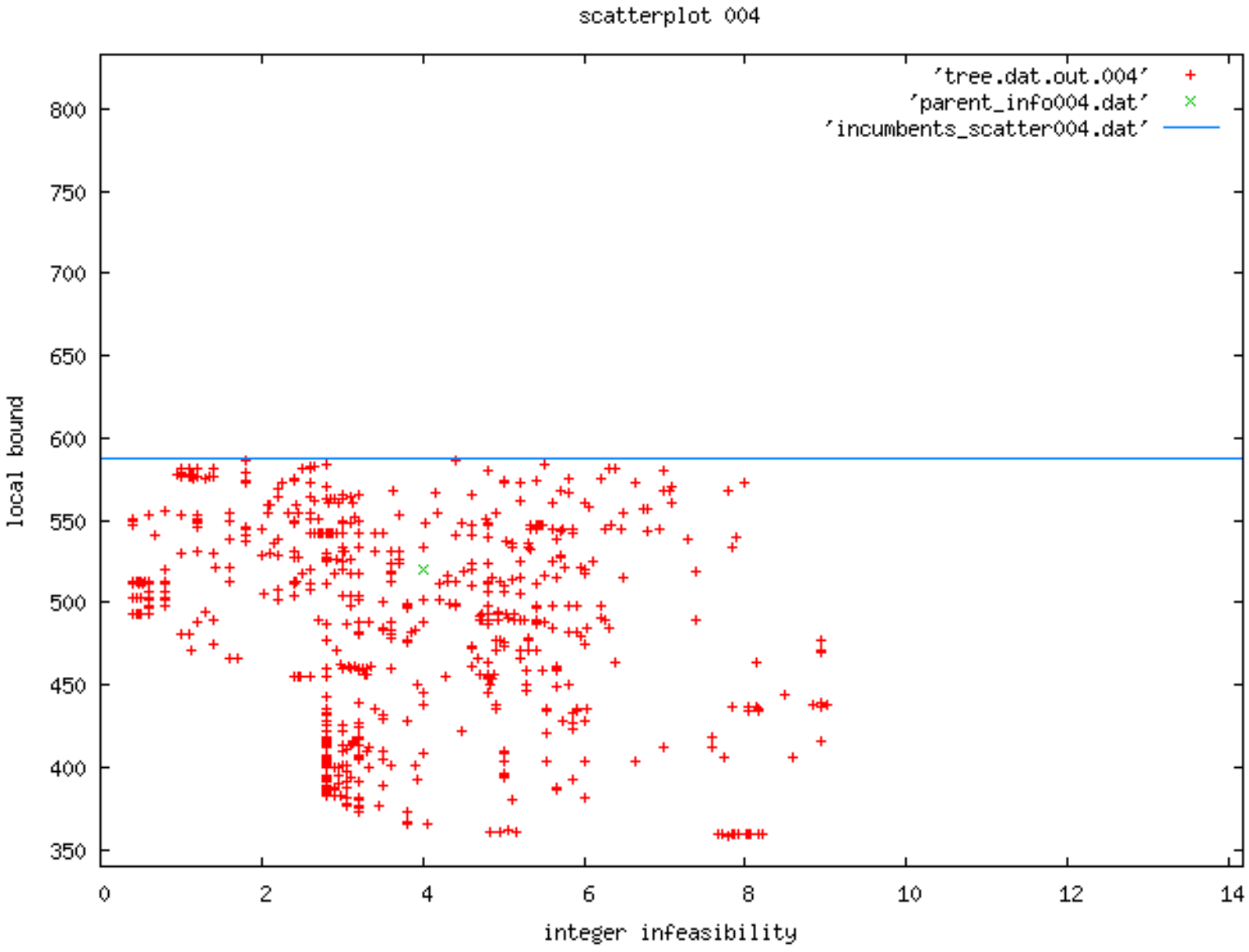
Example scatter plot series 1: swath (MIPLIB 2003)



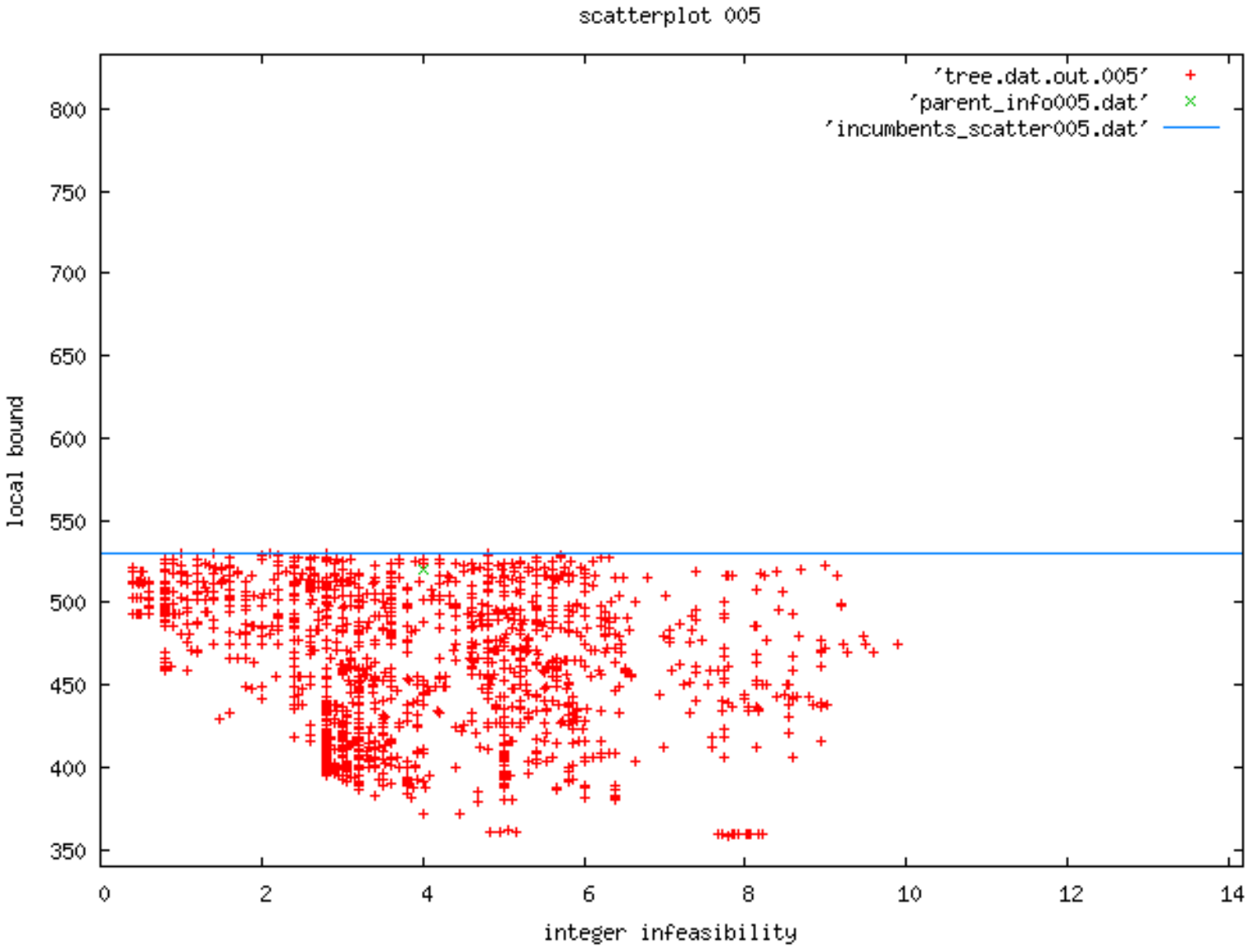
Example scatter plot series 1: swath (MIPLIB 2003)



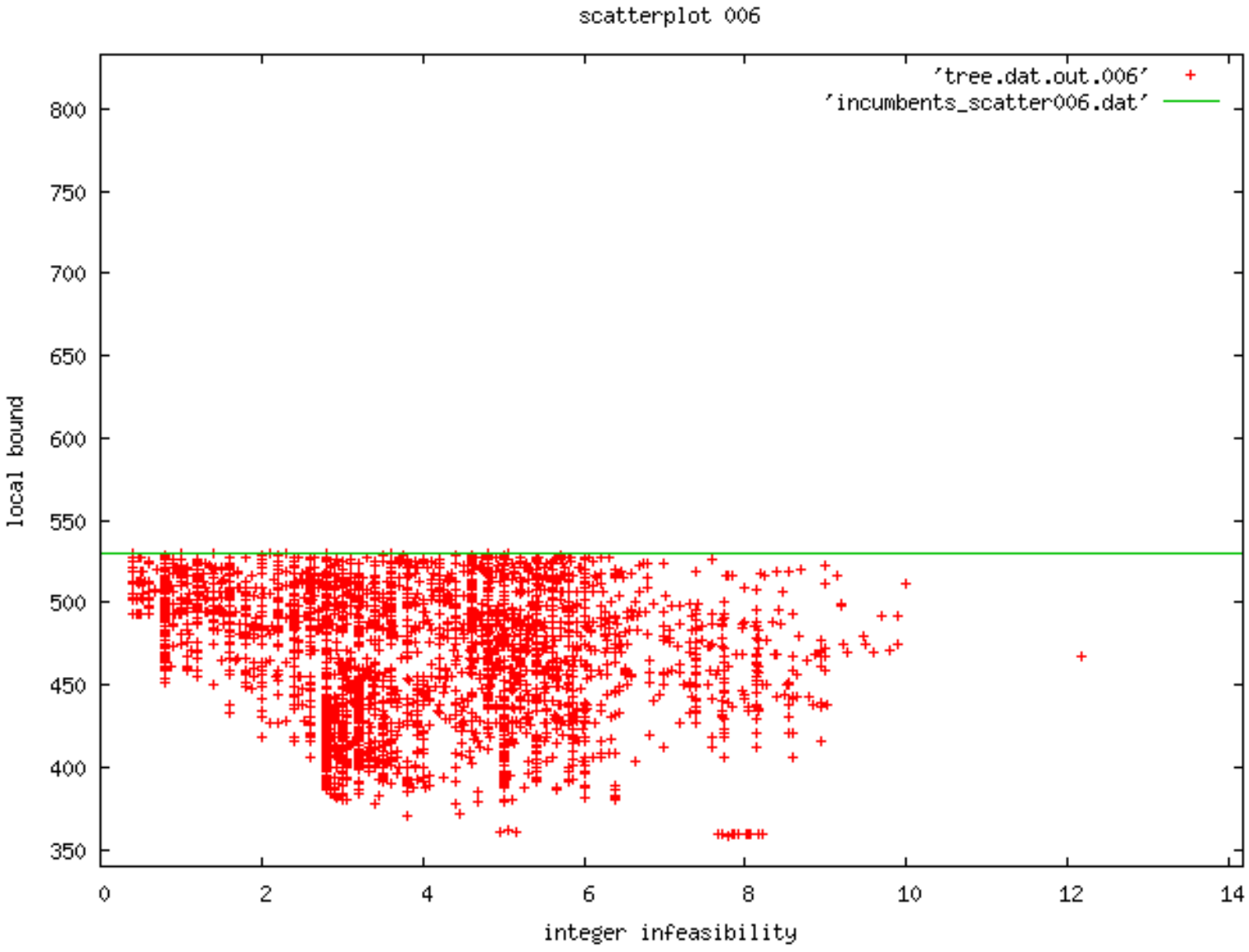
Example scatter plot series 1: swath (MIPLIB 2003)



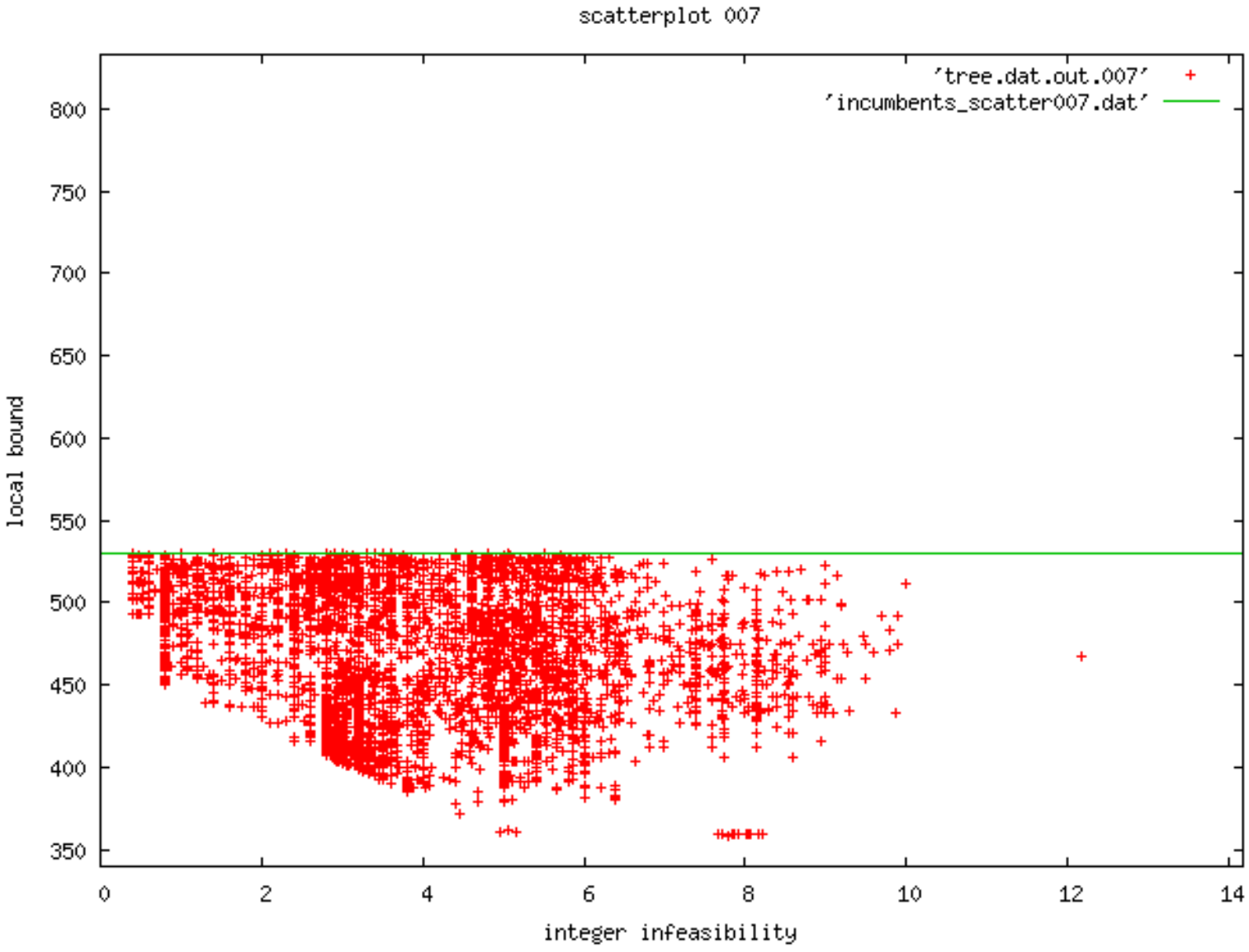
Example scatter plot series 1: swath (MIPLIB 2003)



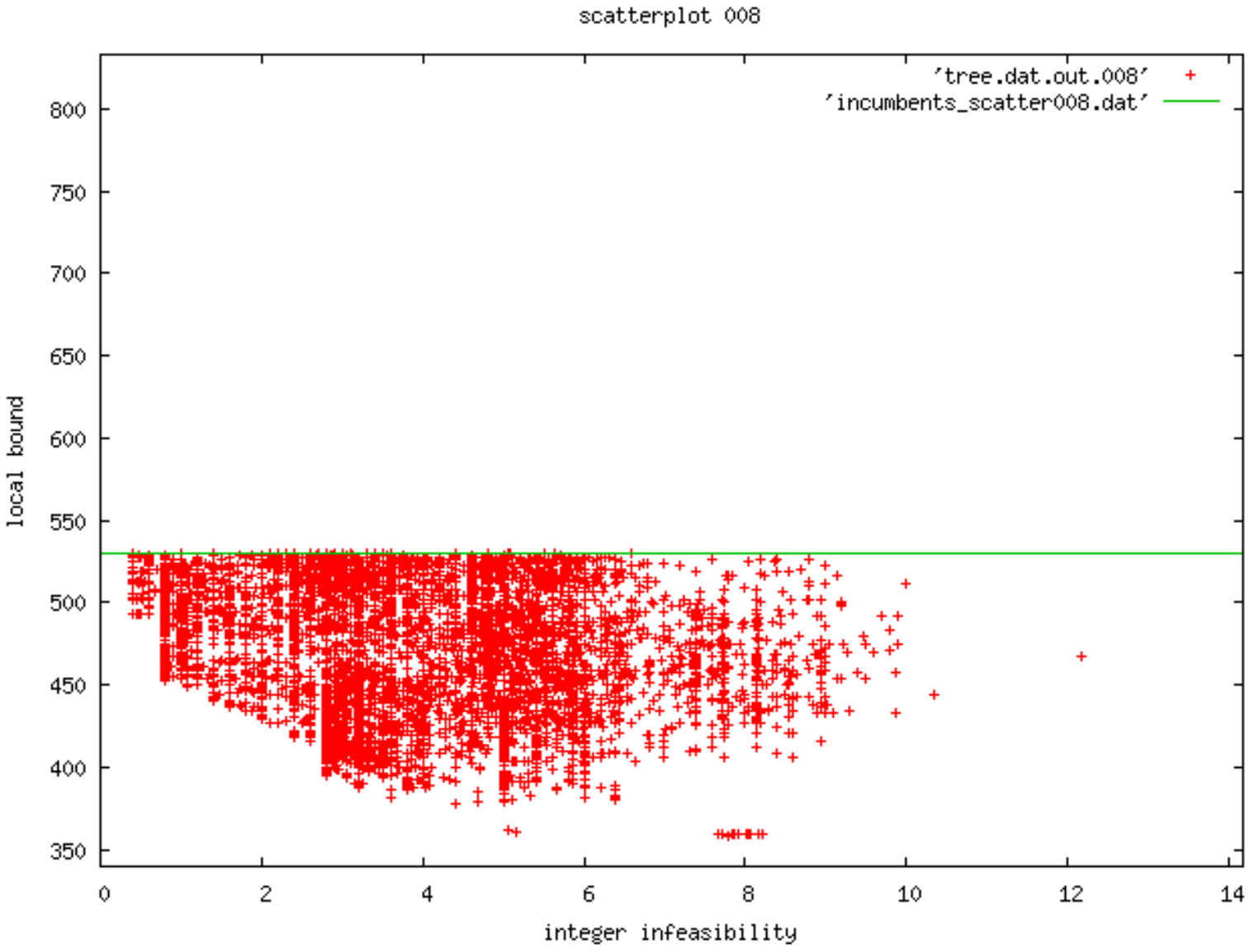
Example scatter plot series 1: swath (MIPLIB 2003)



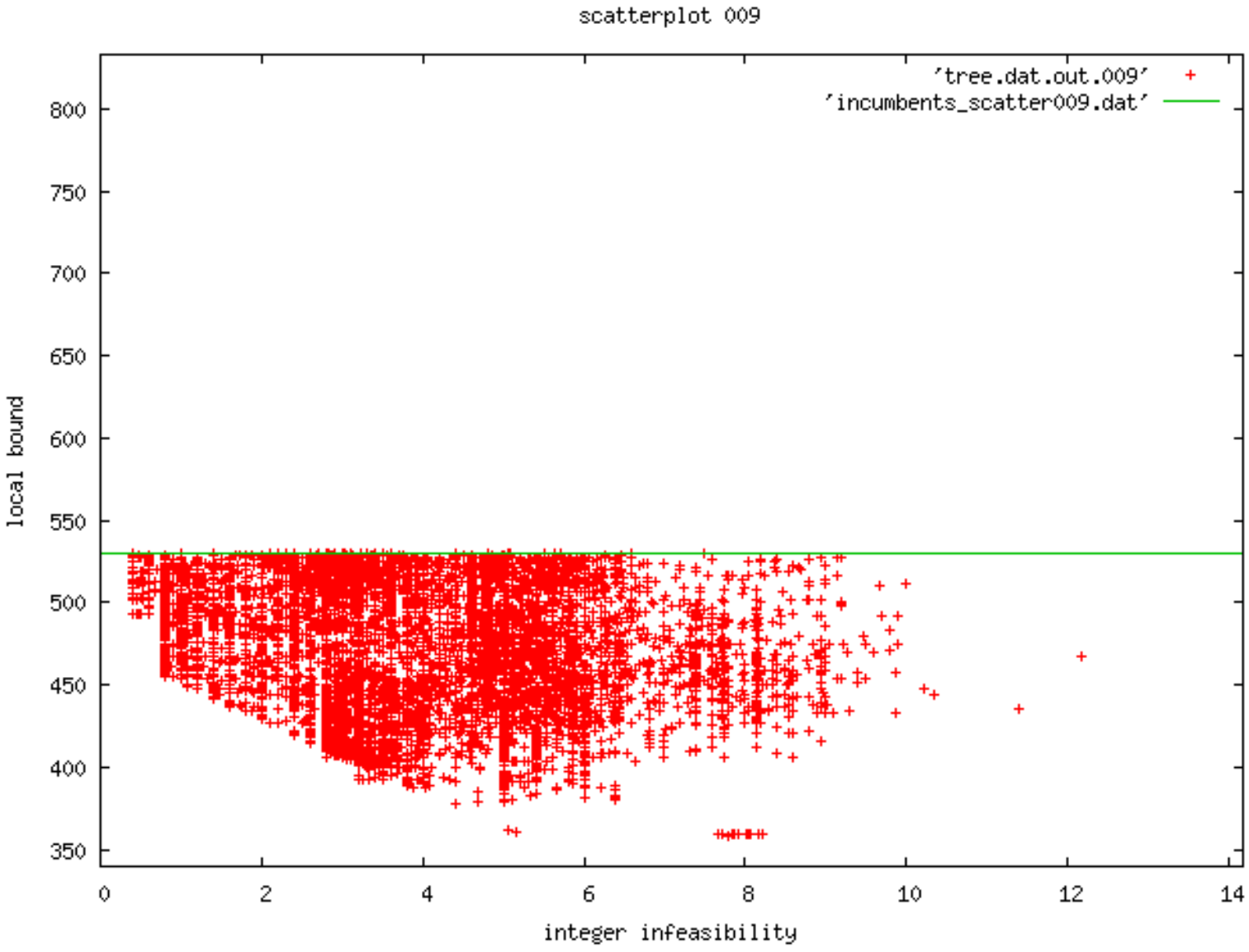
Example scatter plot series 1: swath (MIPLIB 2003)



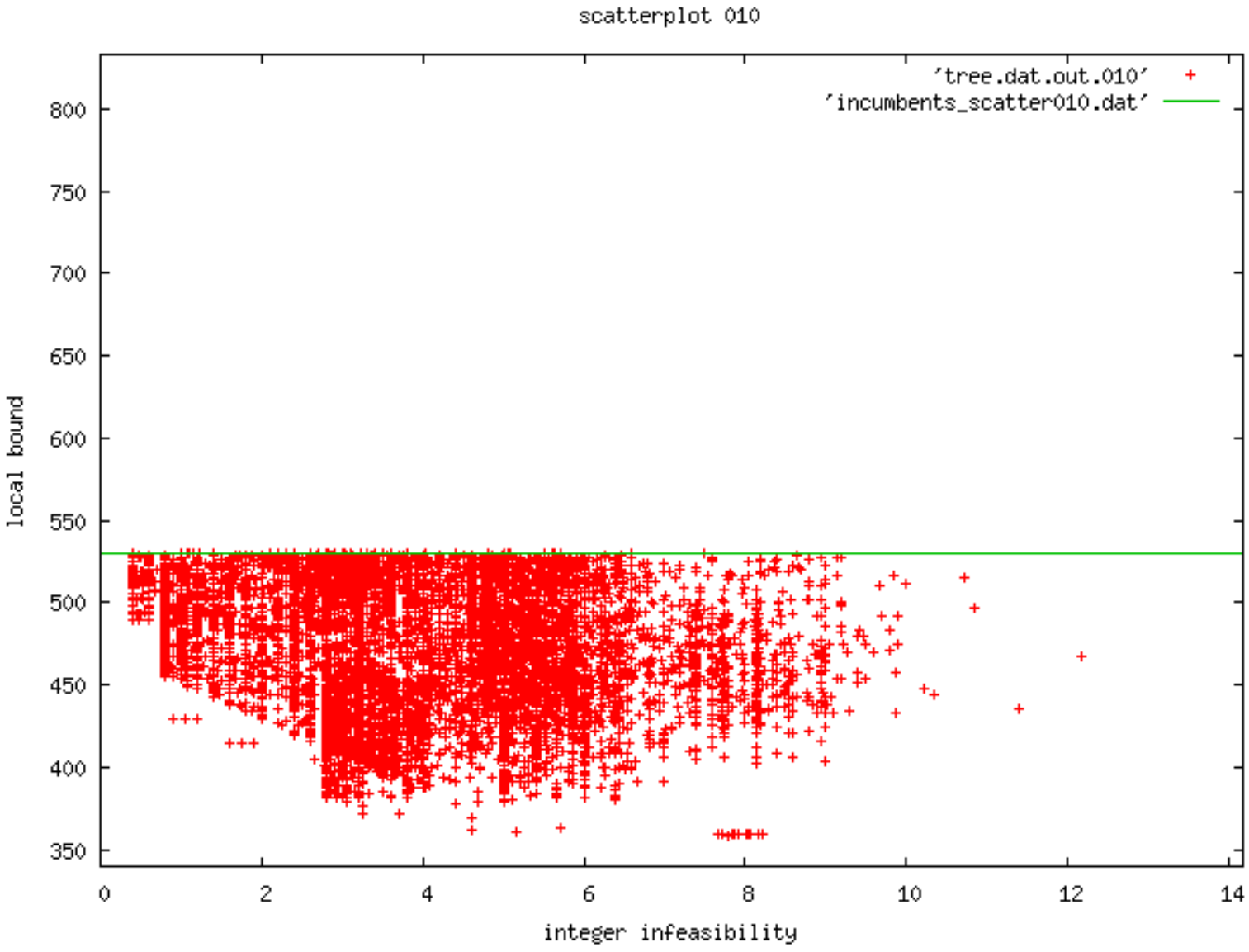
Example scatter plot series 1: swath (MIPLIB 2003)



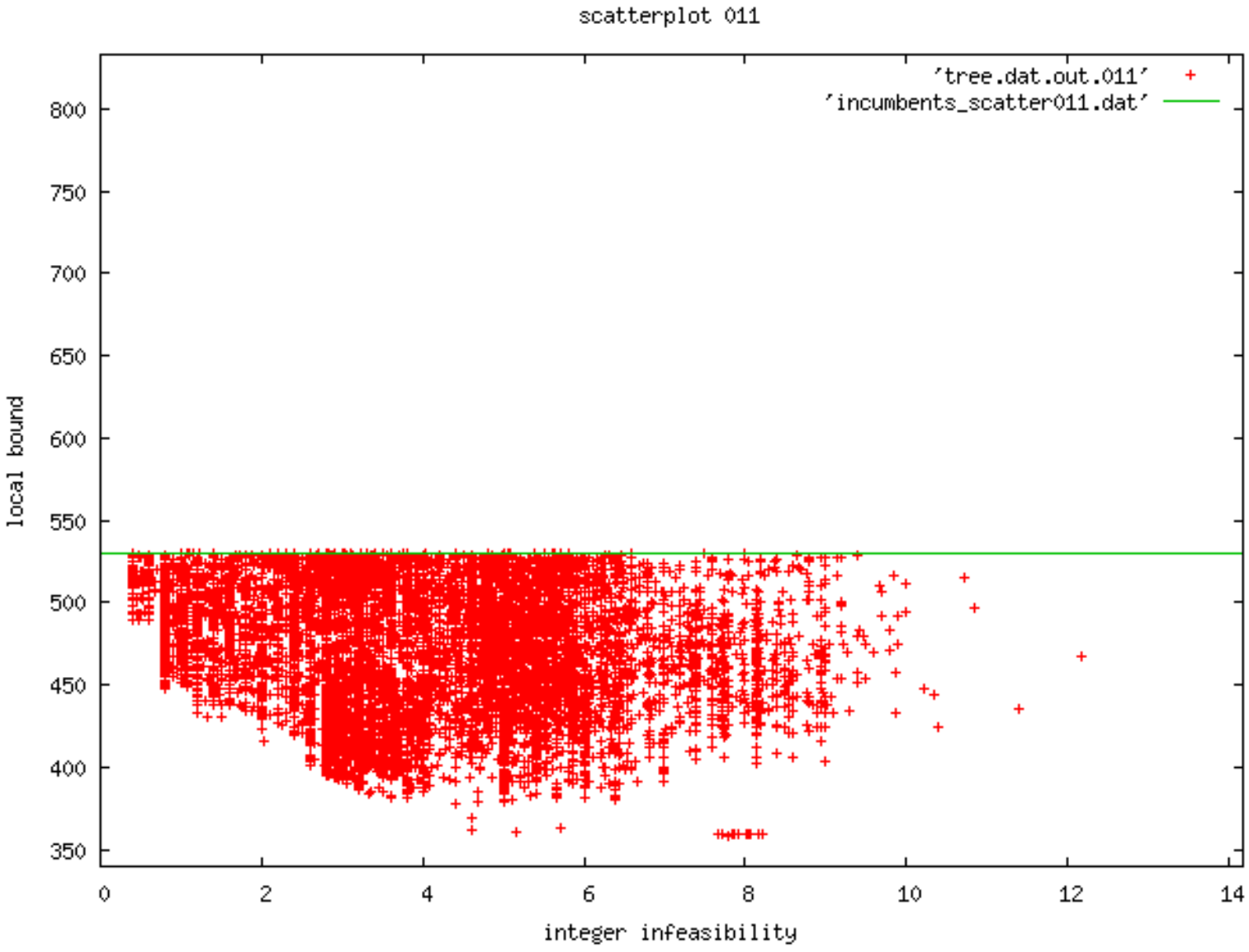
Example scatter plot series 1: swath (MIPLIB 2003)



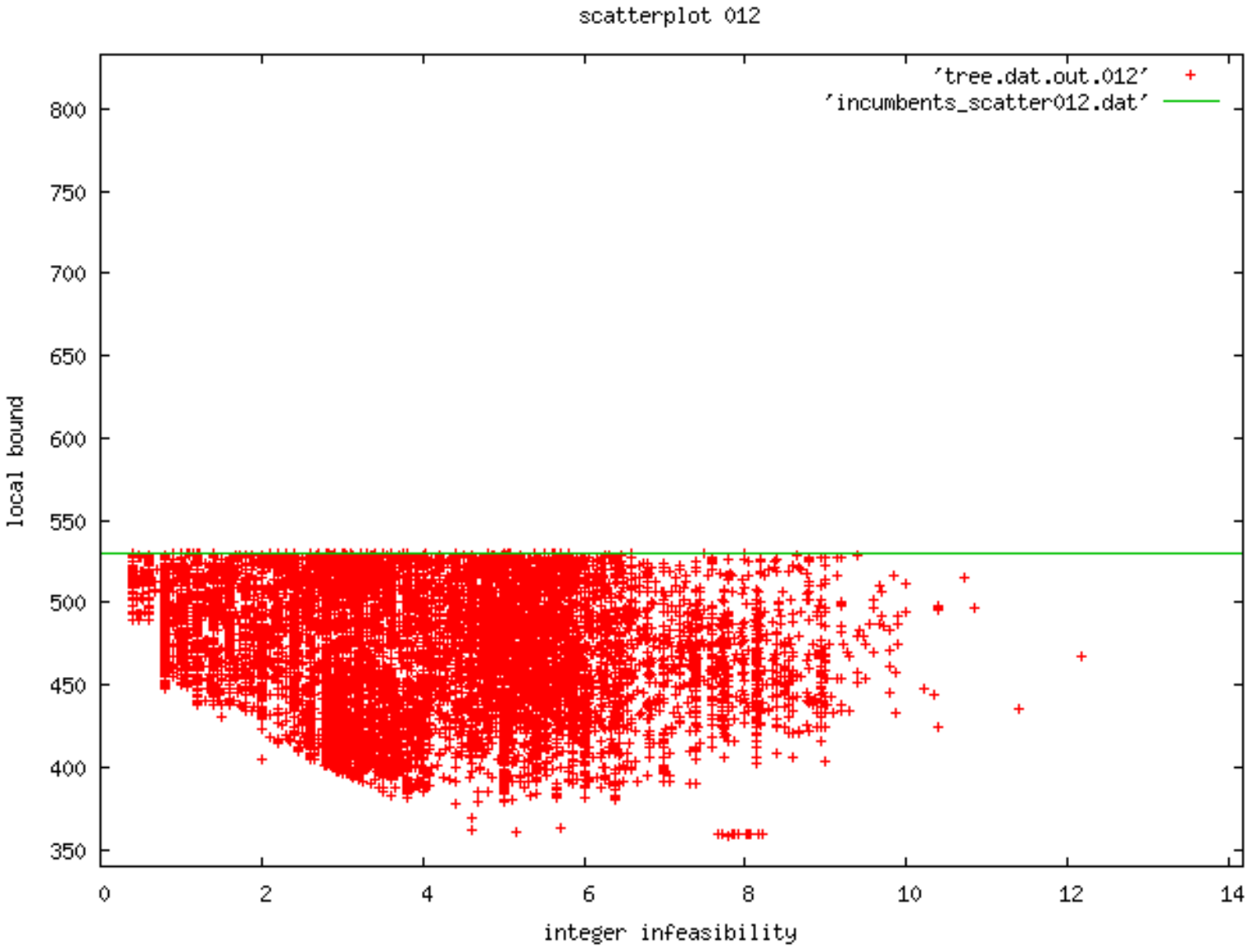
Example scatter plot series 1: swath (MIPLIB 2003)



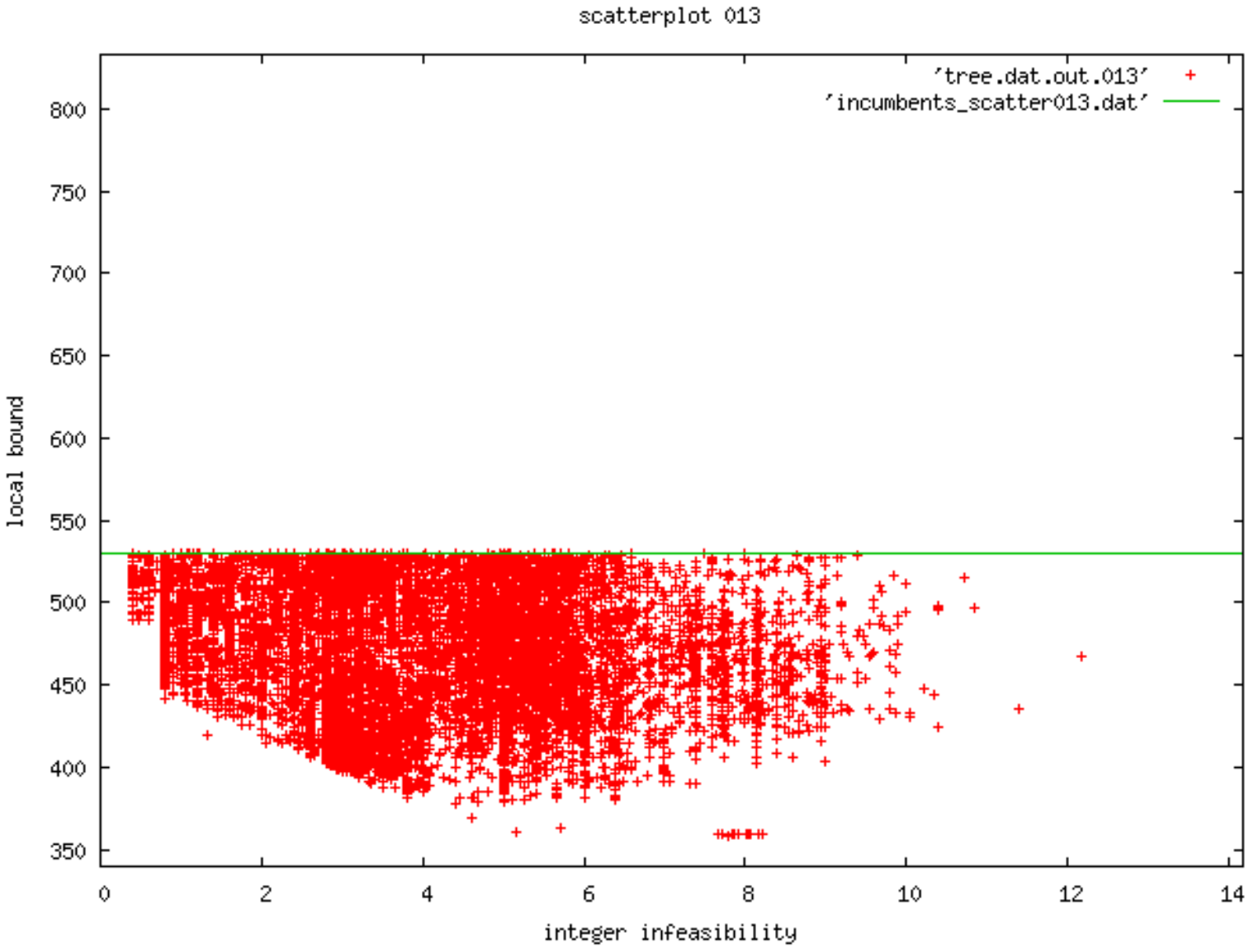
Example scatter plot series 1: swath (MIPLIB 2003)



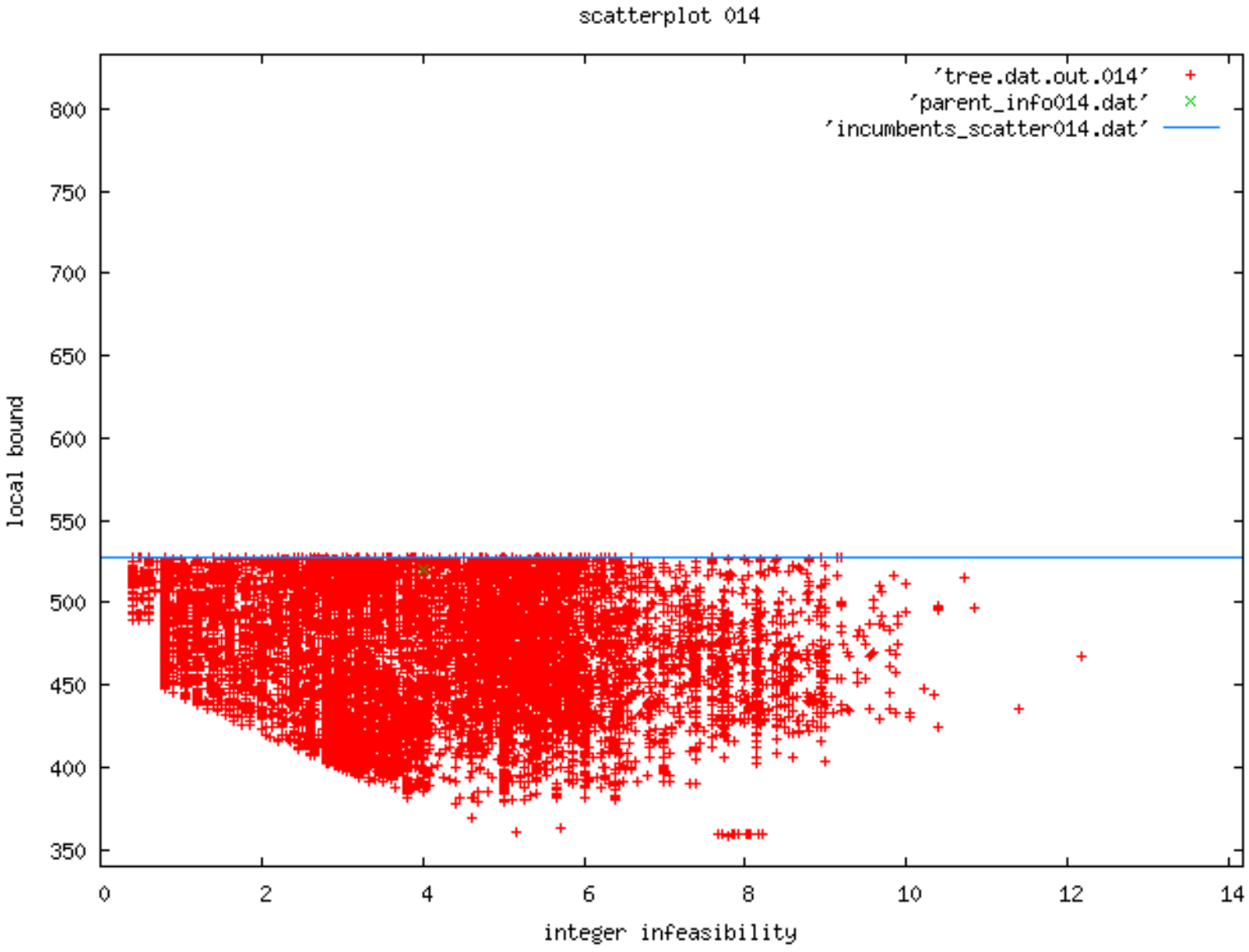
Example scatter plot series 1: swath (MIPLIB 2003)



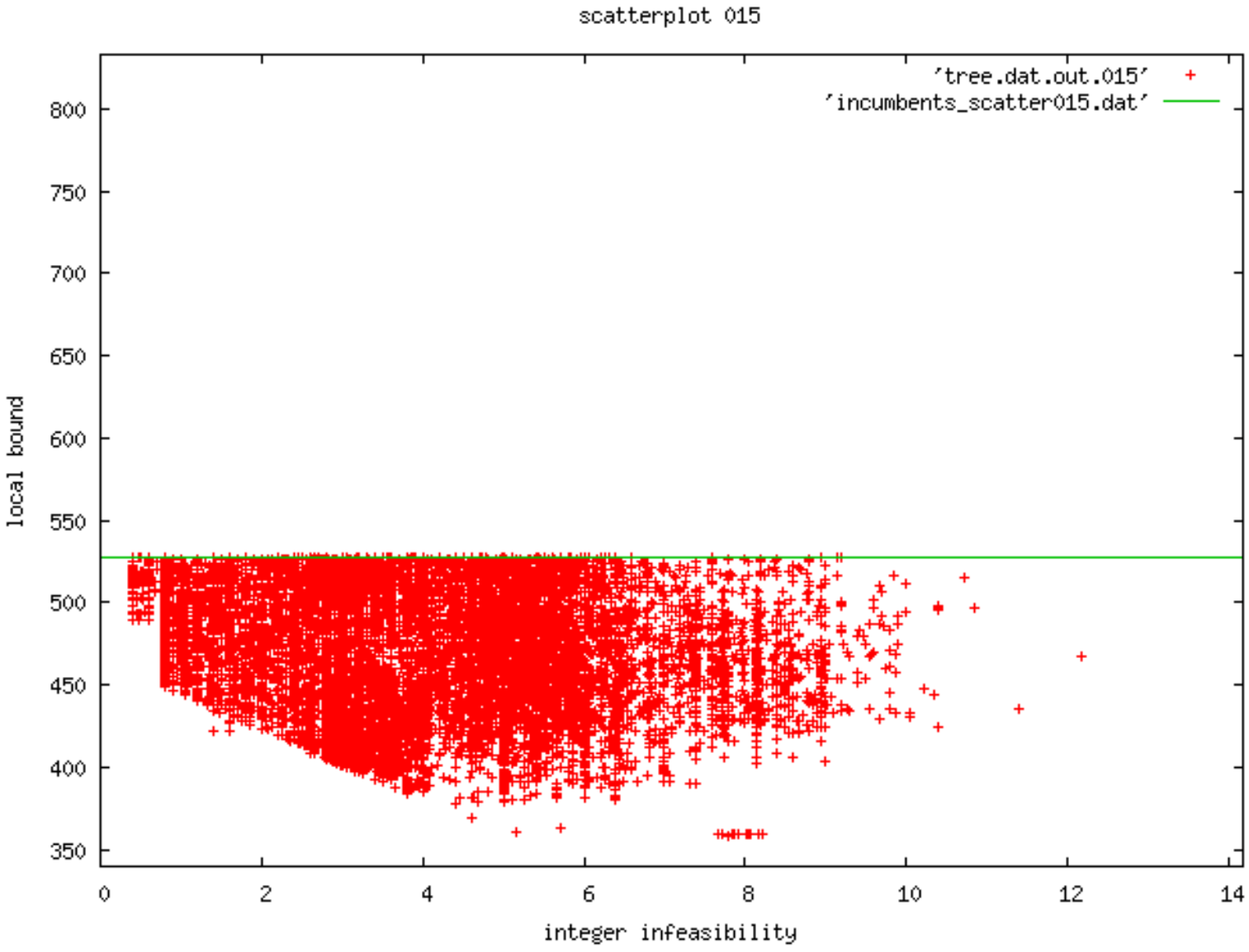
Example scatter plot series 1: swath (MIPLIB 2003)



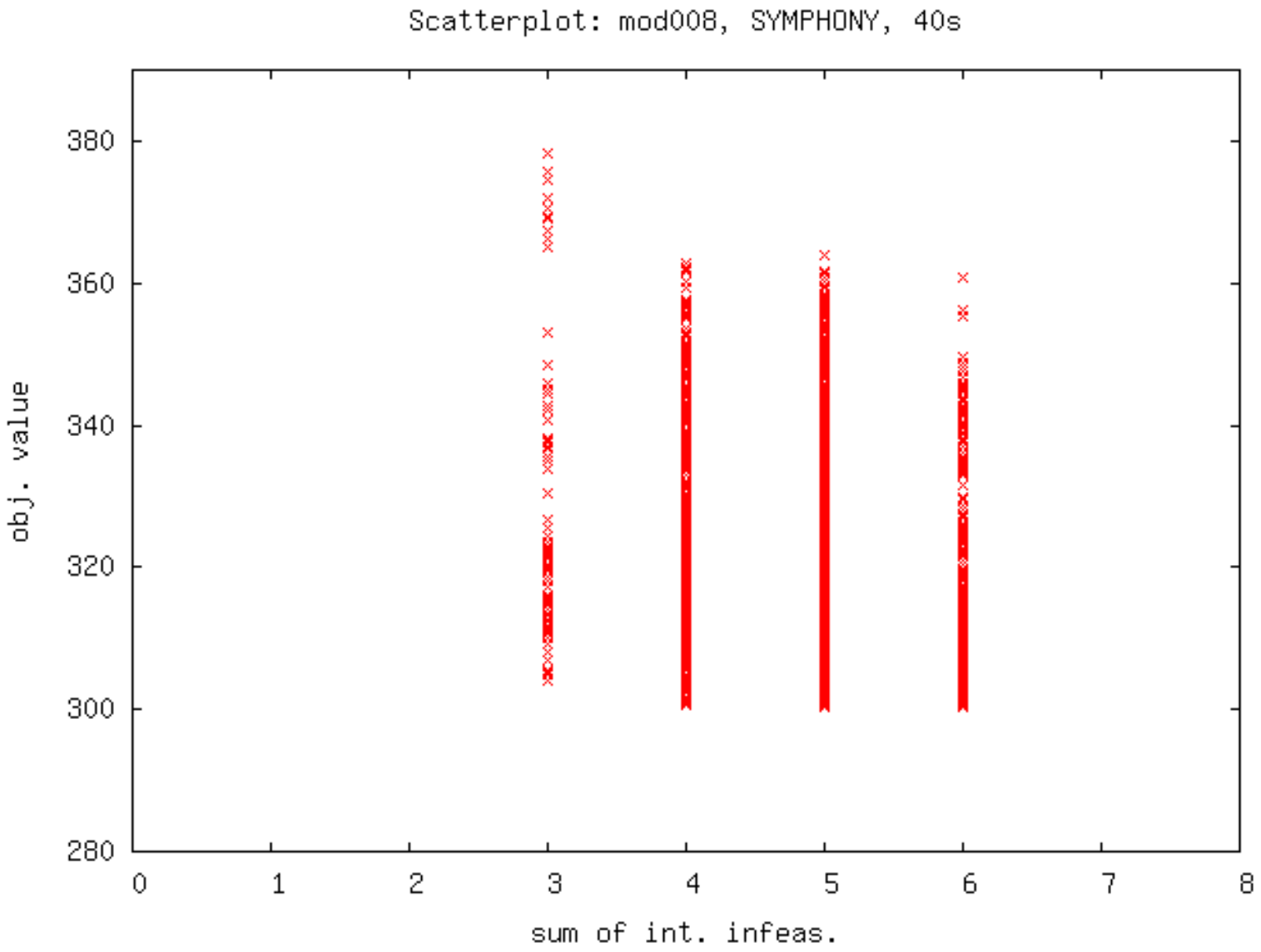
Example scatter plot series 1: swath (MIPLIB 2003)



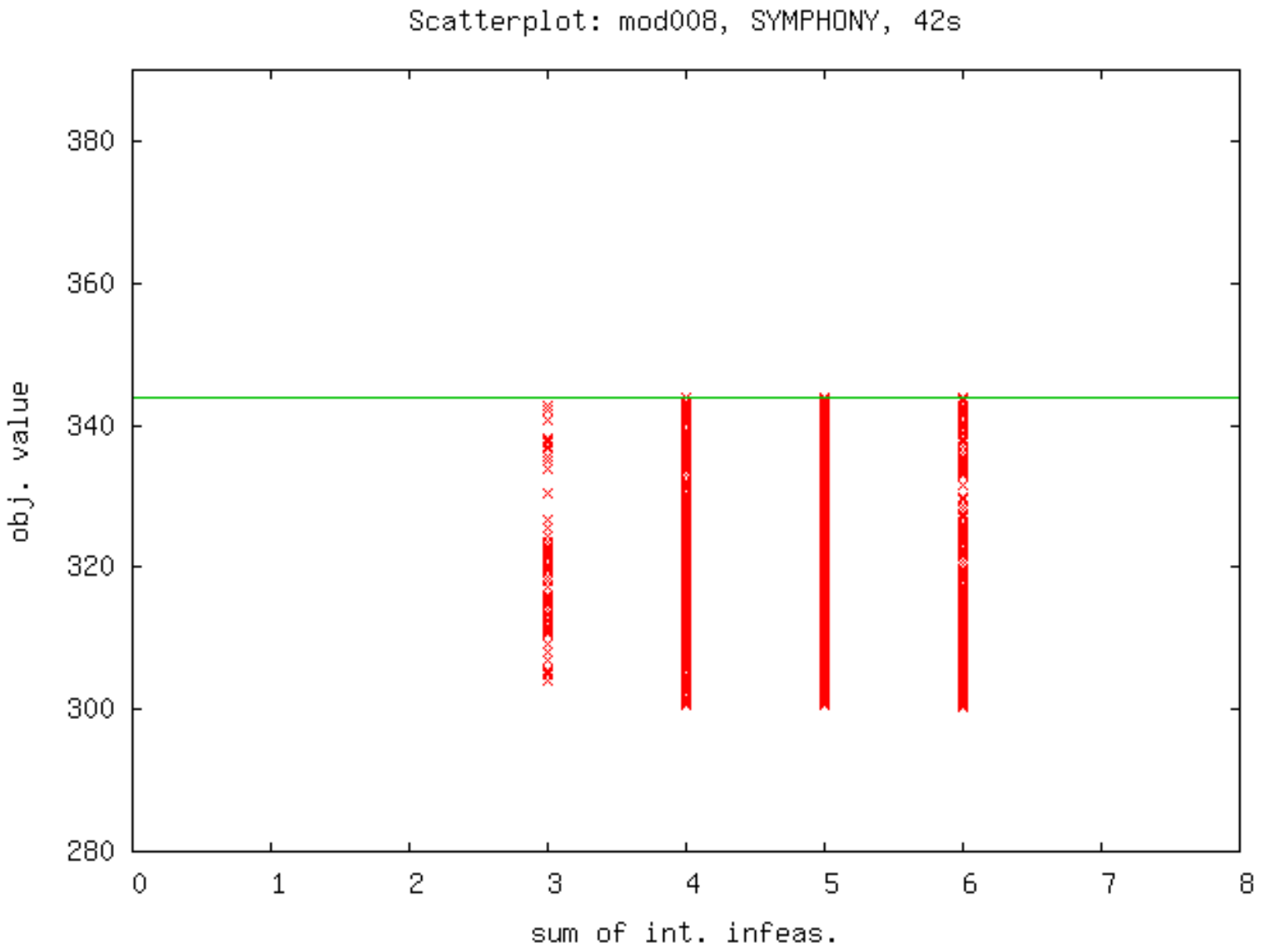
Example scatter plot series 1: swath (MIPLIB 2003)



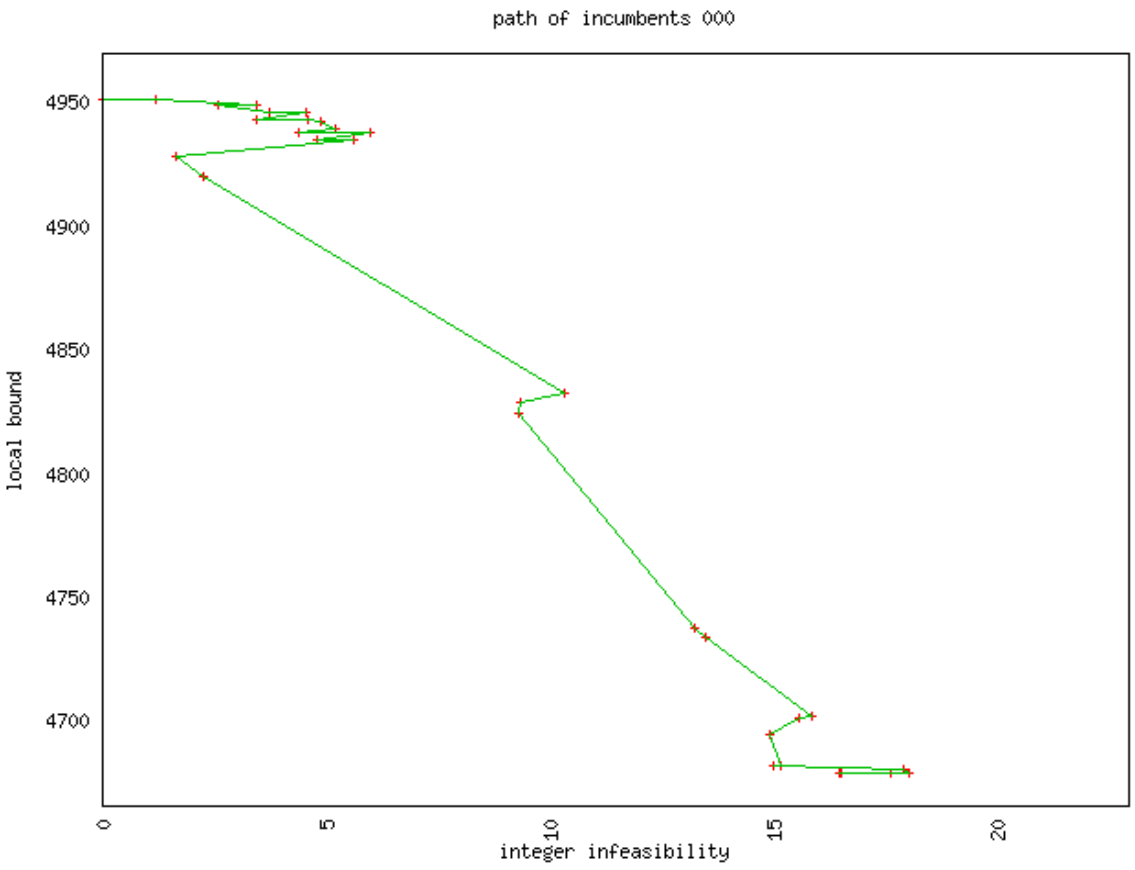
Patterns in integer infeasibility: SYMPHONY



Patterns in integer infeasibility: SYMPHONY

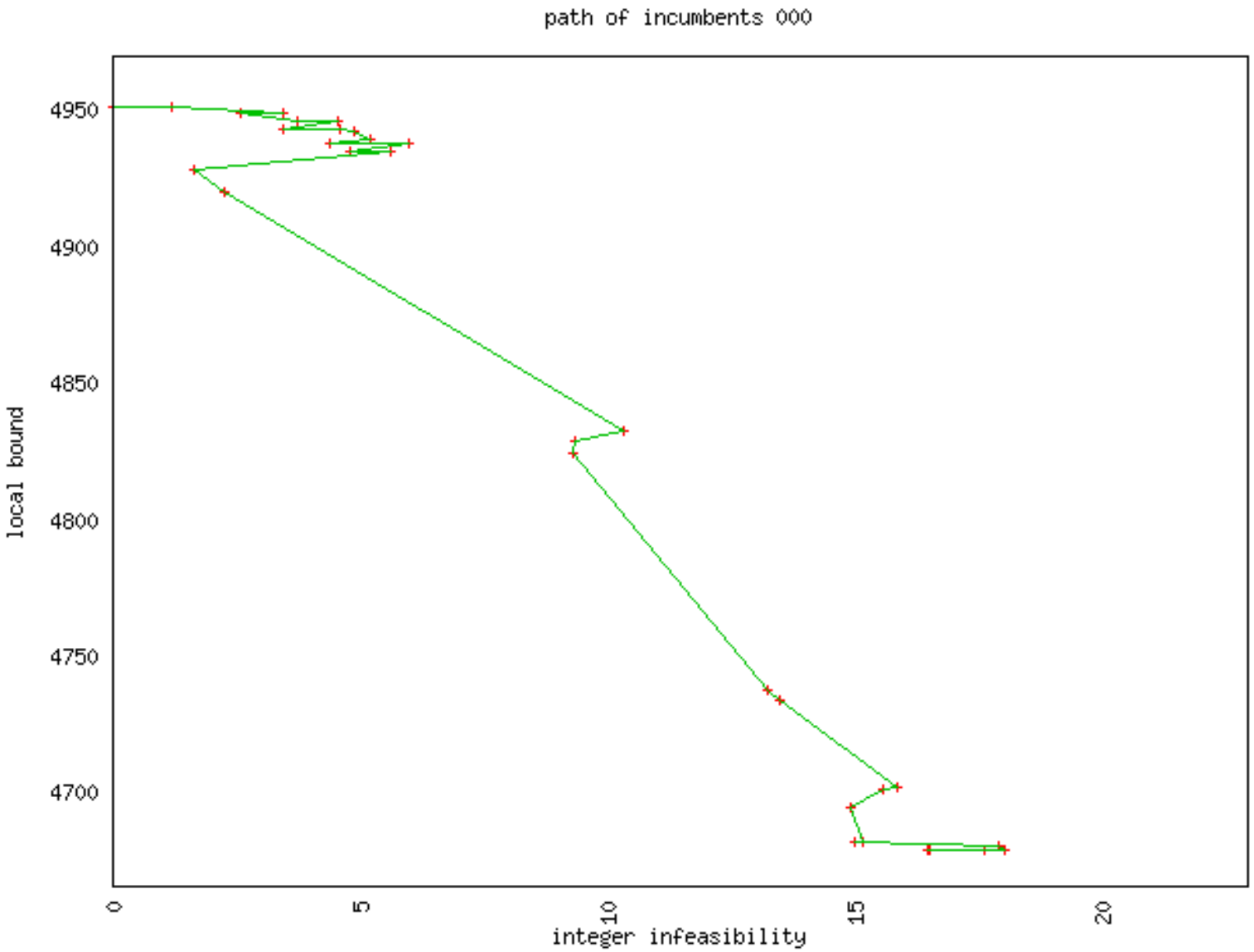


Visualization tools: Incumbent node history in scatter plot

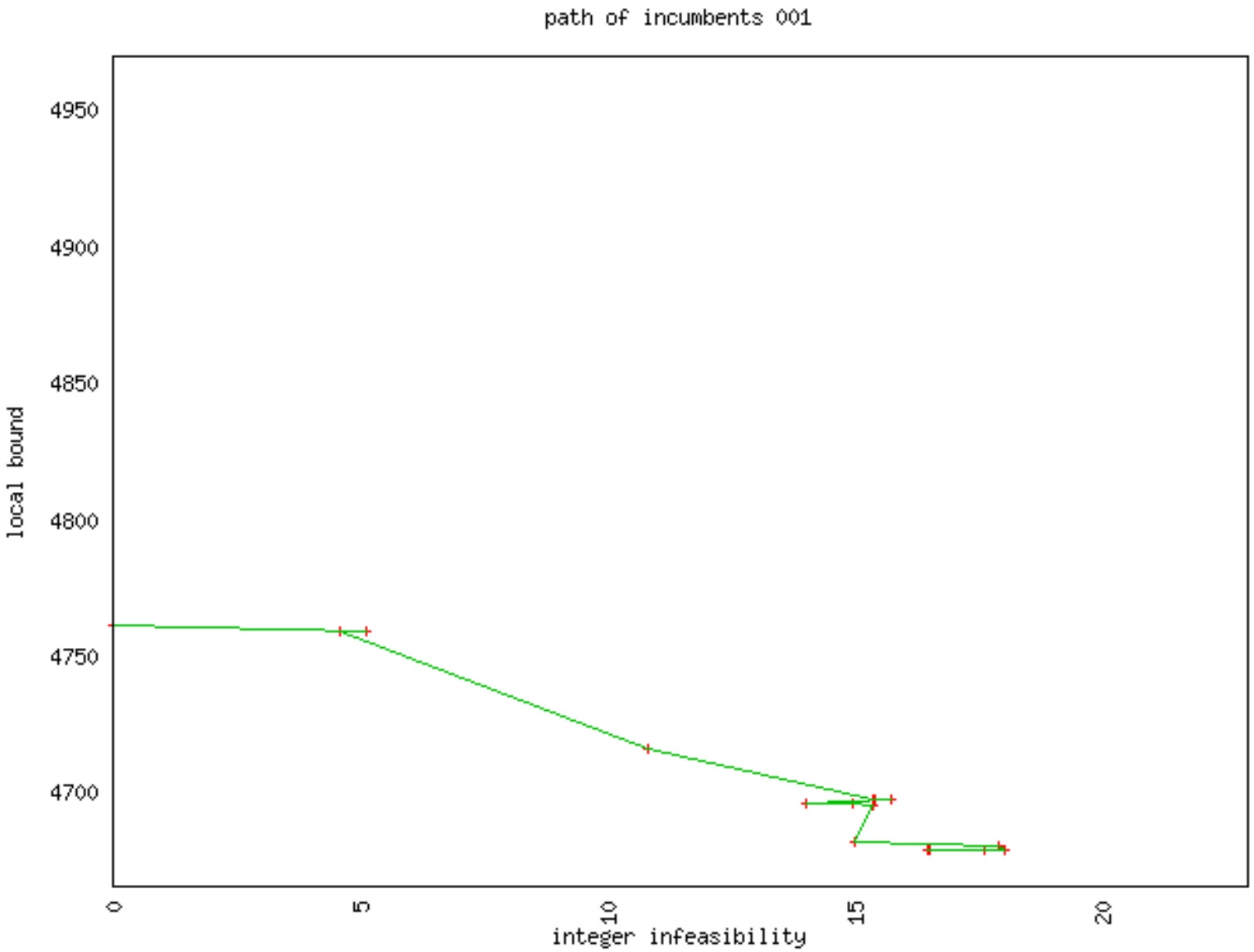


- Horizontal axis is the integer infeasibility
- Vertical axis is the LP bound
- Green line shows ancestors of the incumbent node

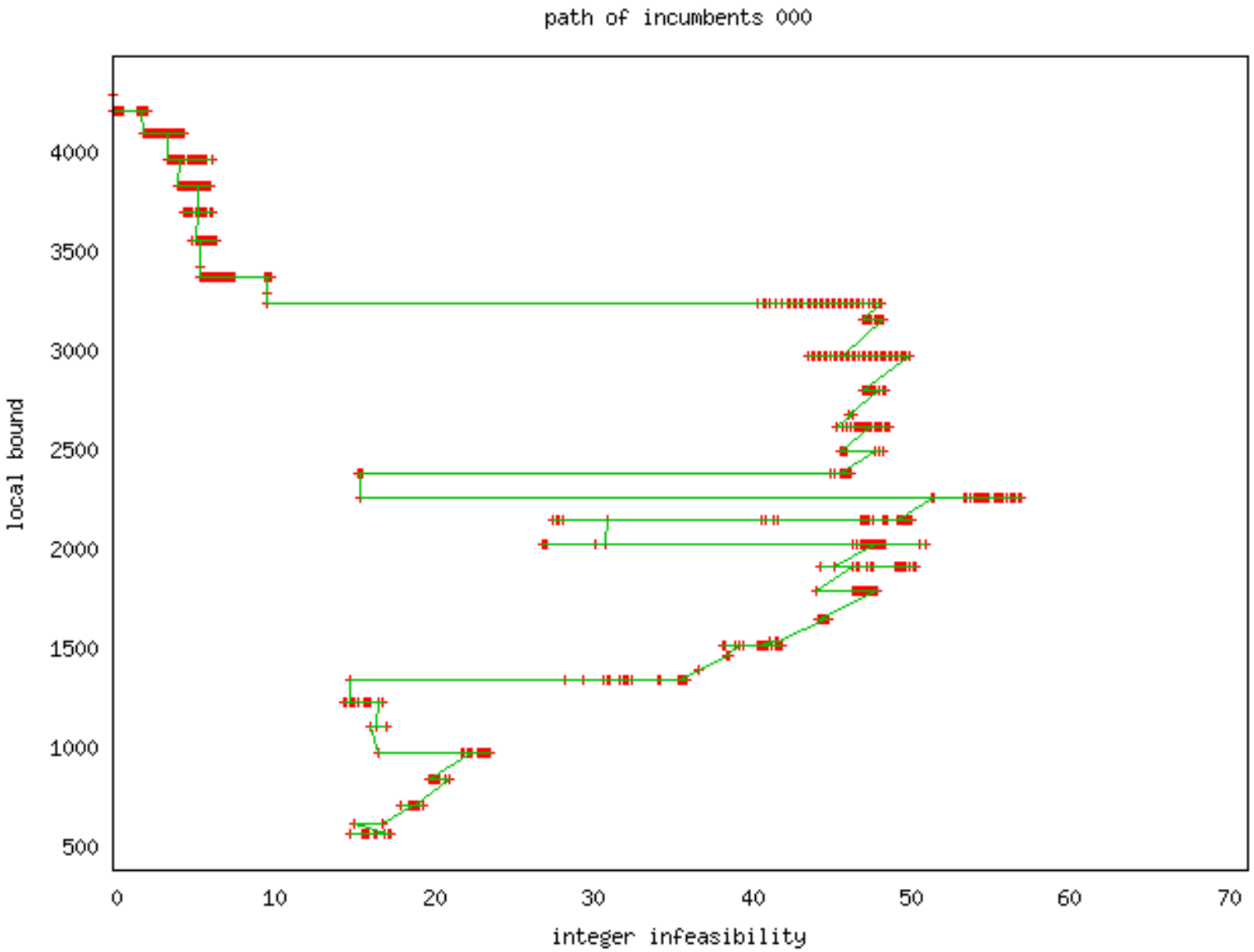
Example incumbent node history series 1: l152lav (MIPLIB 2003)



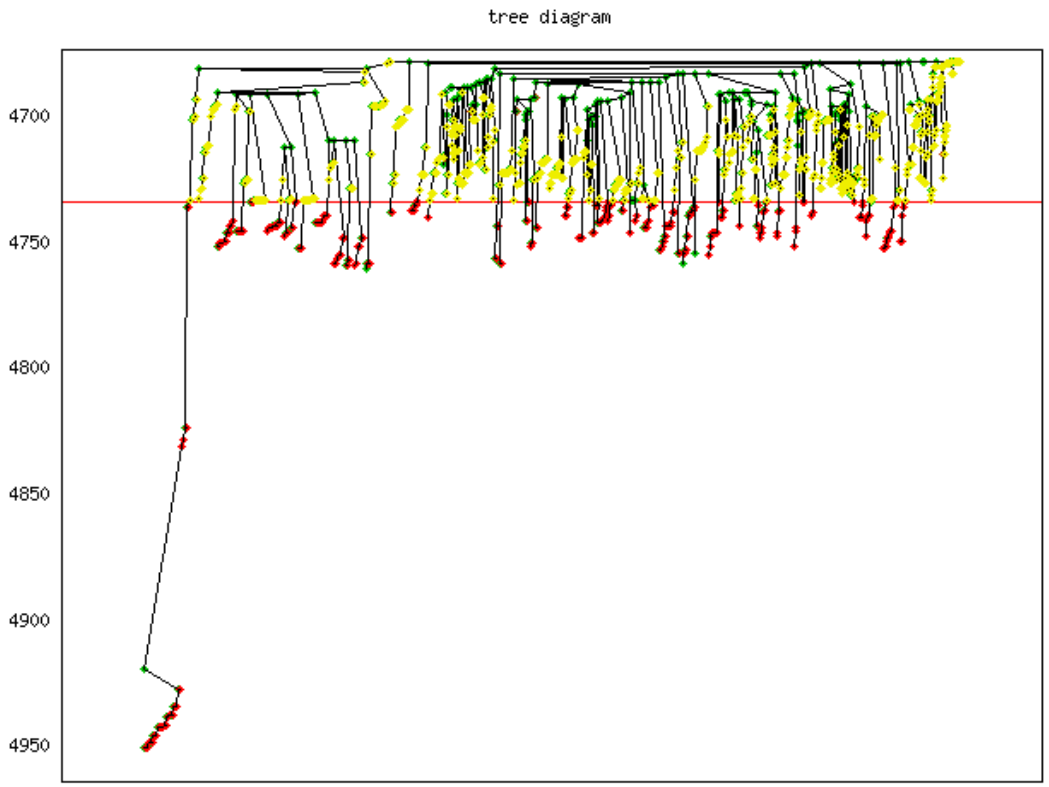
Example incumbent node history series 1: l152lav (MIPLIB 2003)



Example incumbent node history series 2: liu (MIPLIB 2003)

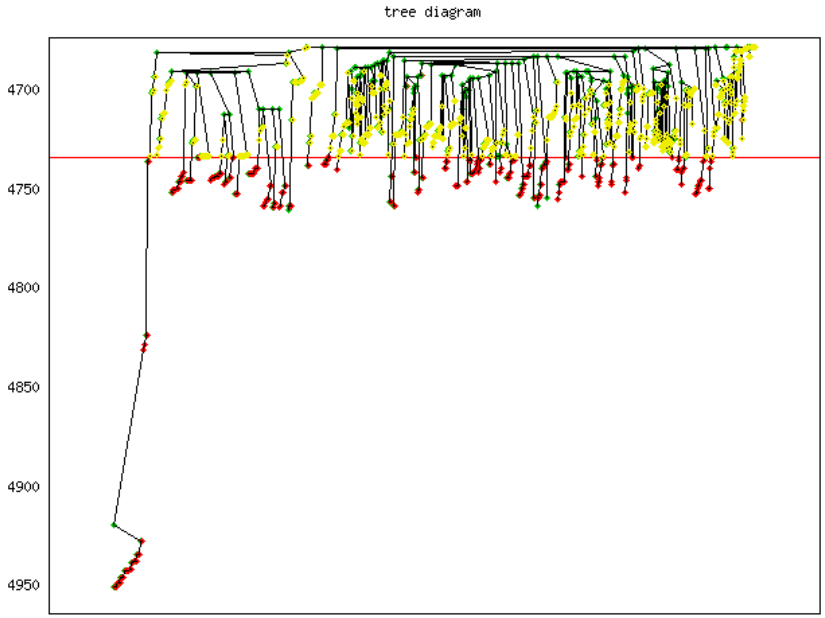


Visualization tools: B&B trees



- Vertical axis is the LP bound
- Nodes are horizontally positioned to make the pictures more readable
- Alternatively, horizontal positions may be fixed based on position in the tree

Visualization tools: B&B trees

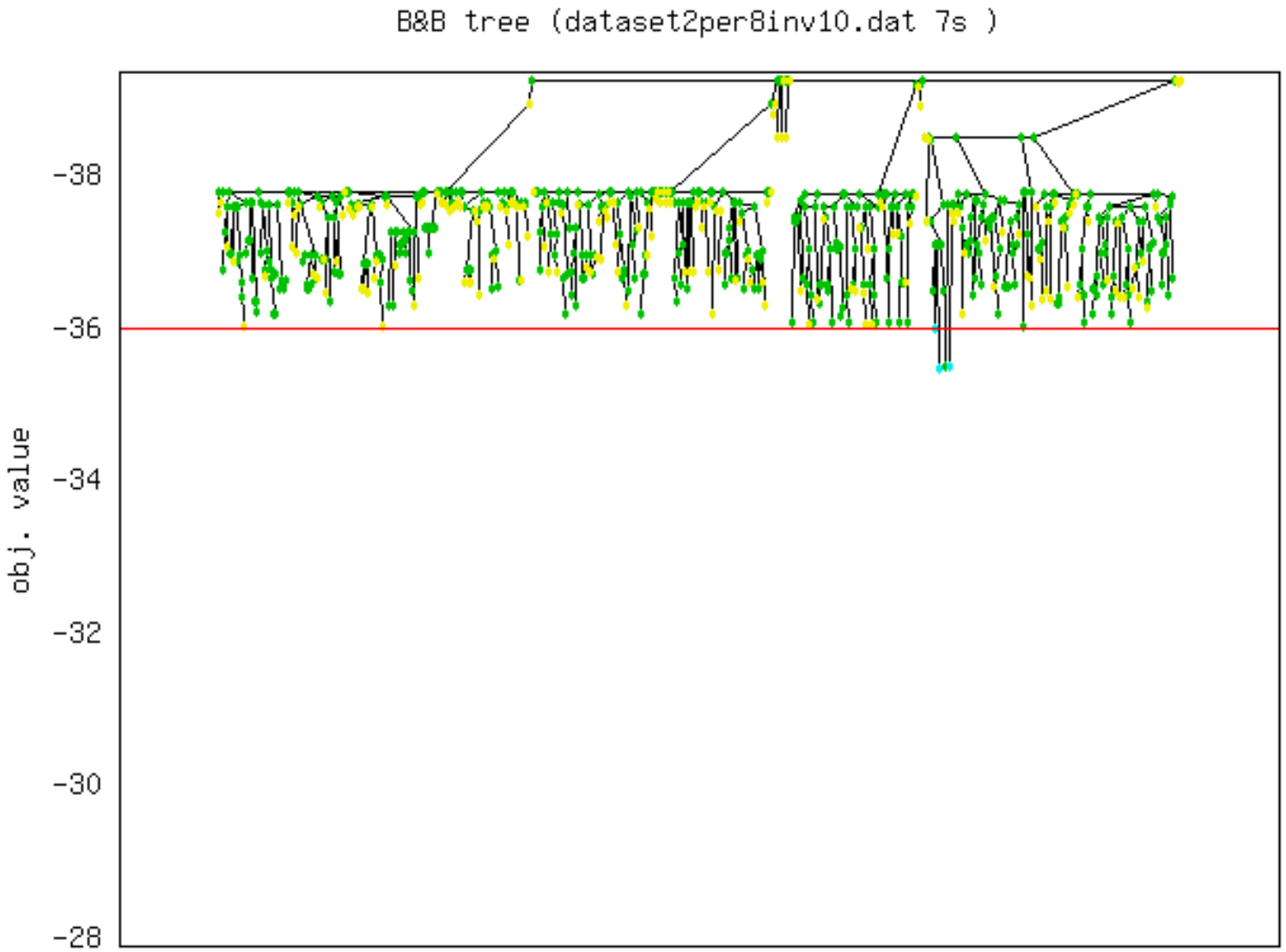


- Node color legend:
  - green: branched
  - yellow: candidate or pregnant
  - red: fathomed
  - blue: infeasible

Example B&B trees

Example B&B trees

Example B&B trees

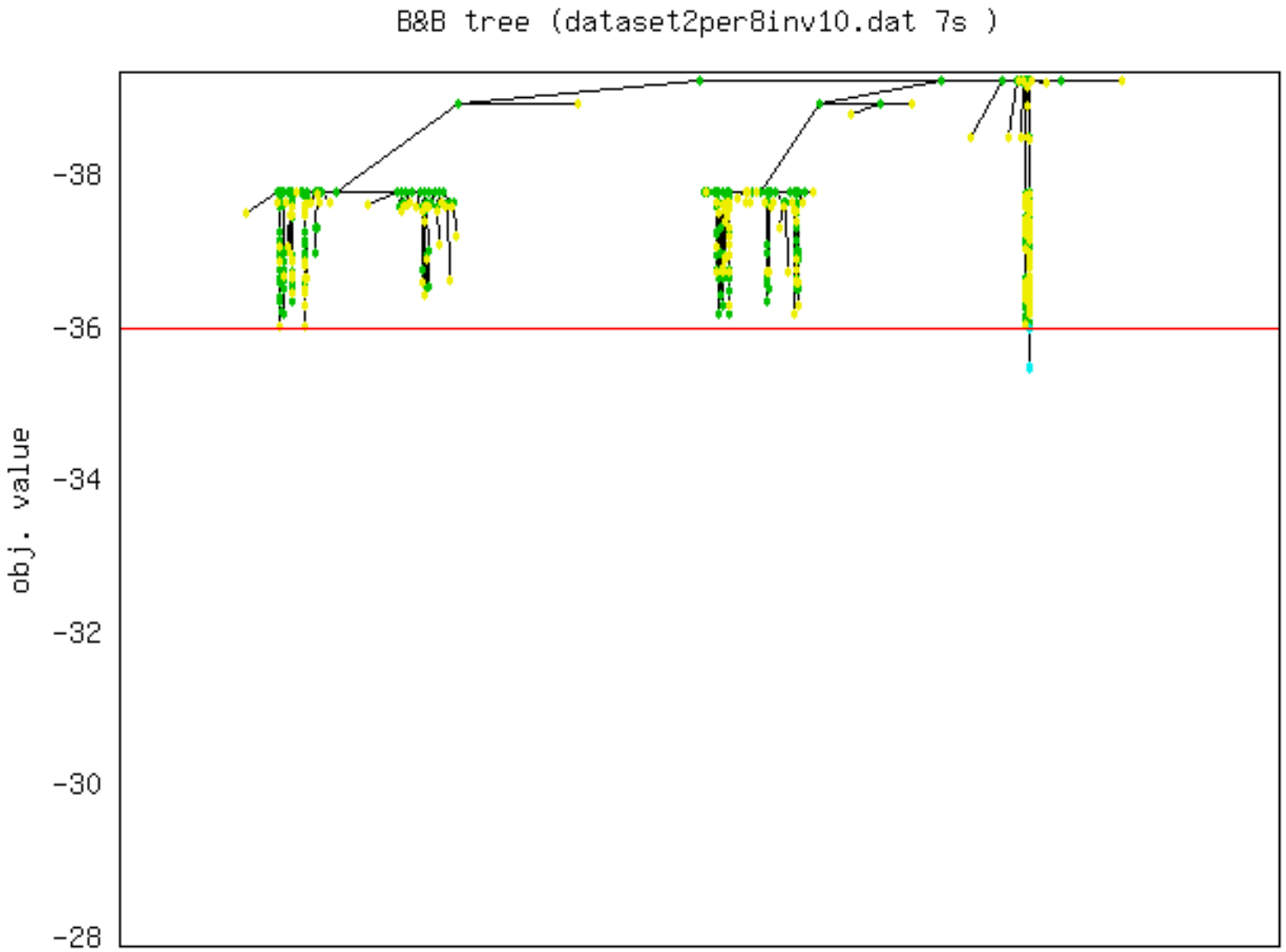


Example B&B trees

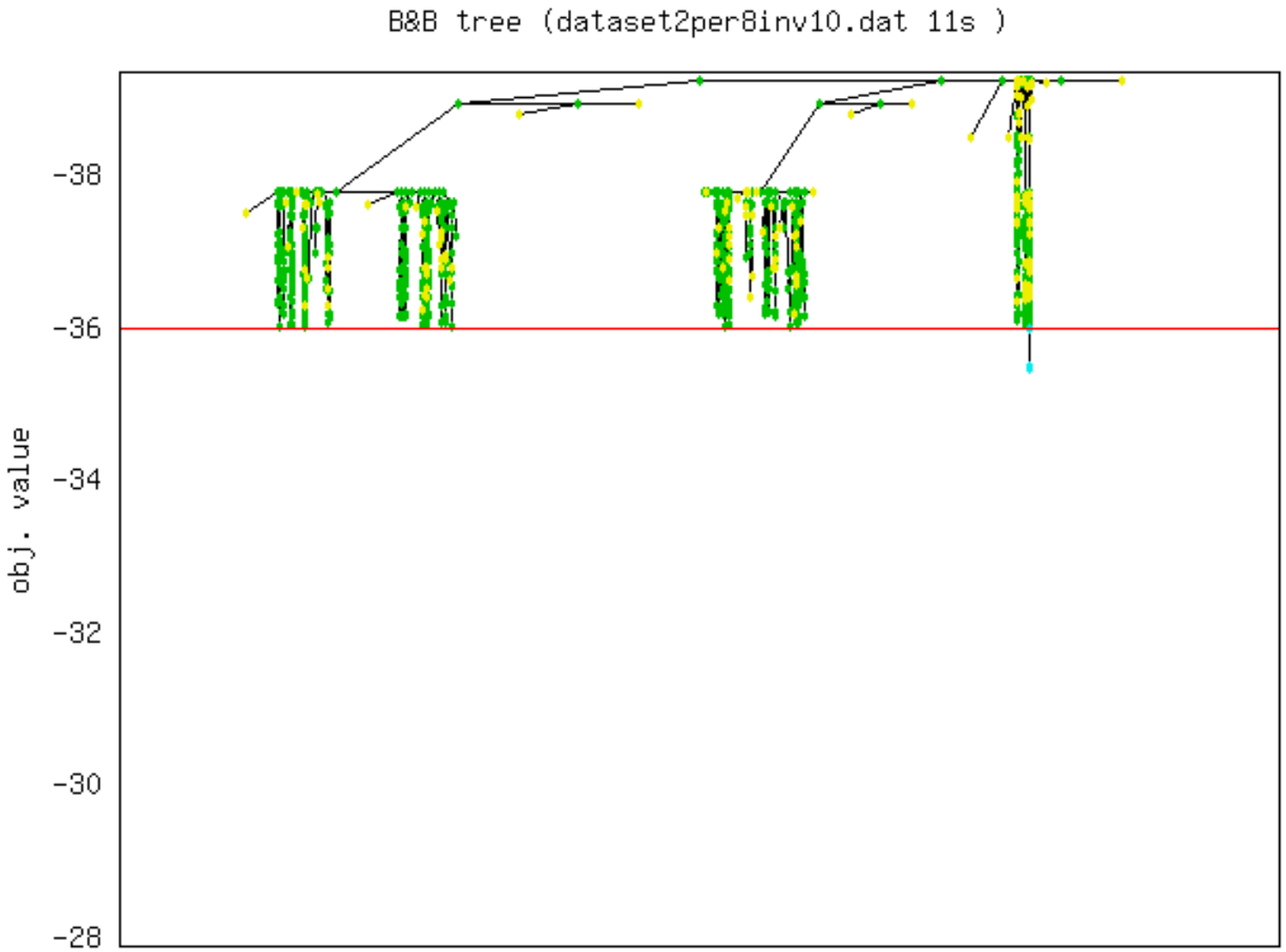
Example B&B trees

Example B&B trees

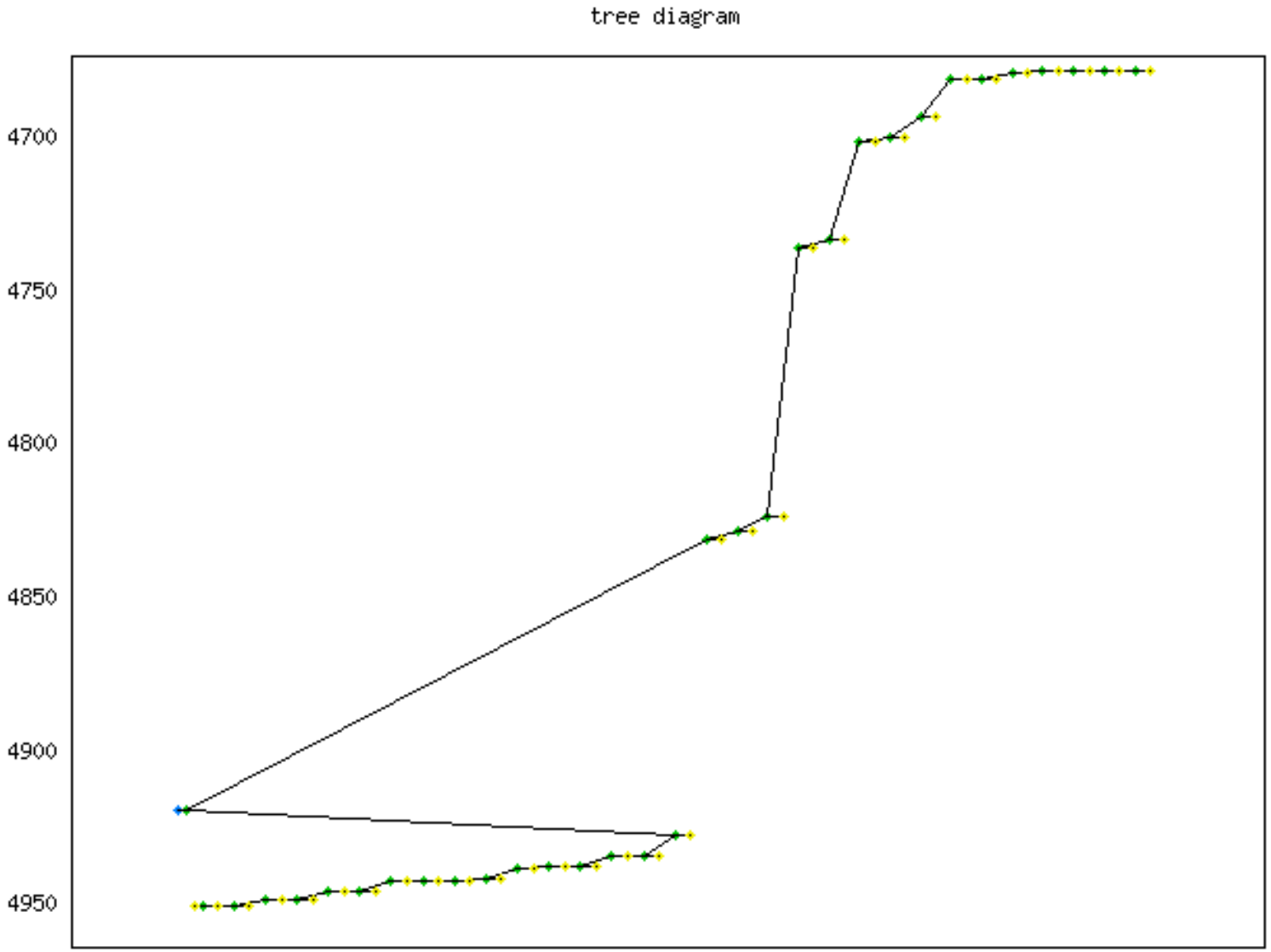
Example B&B trees



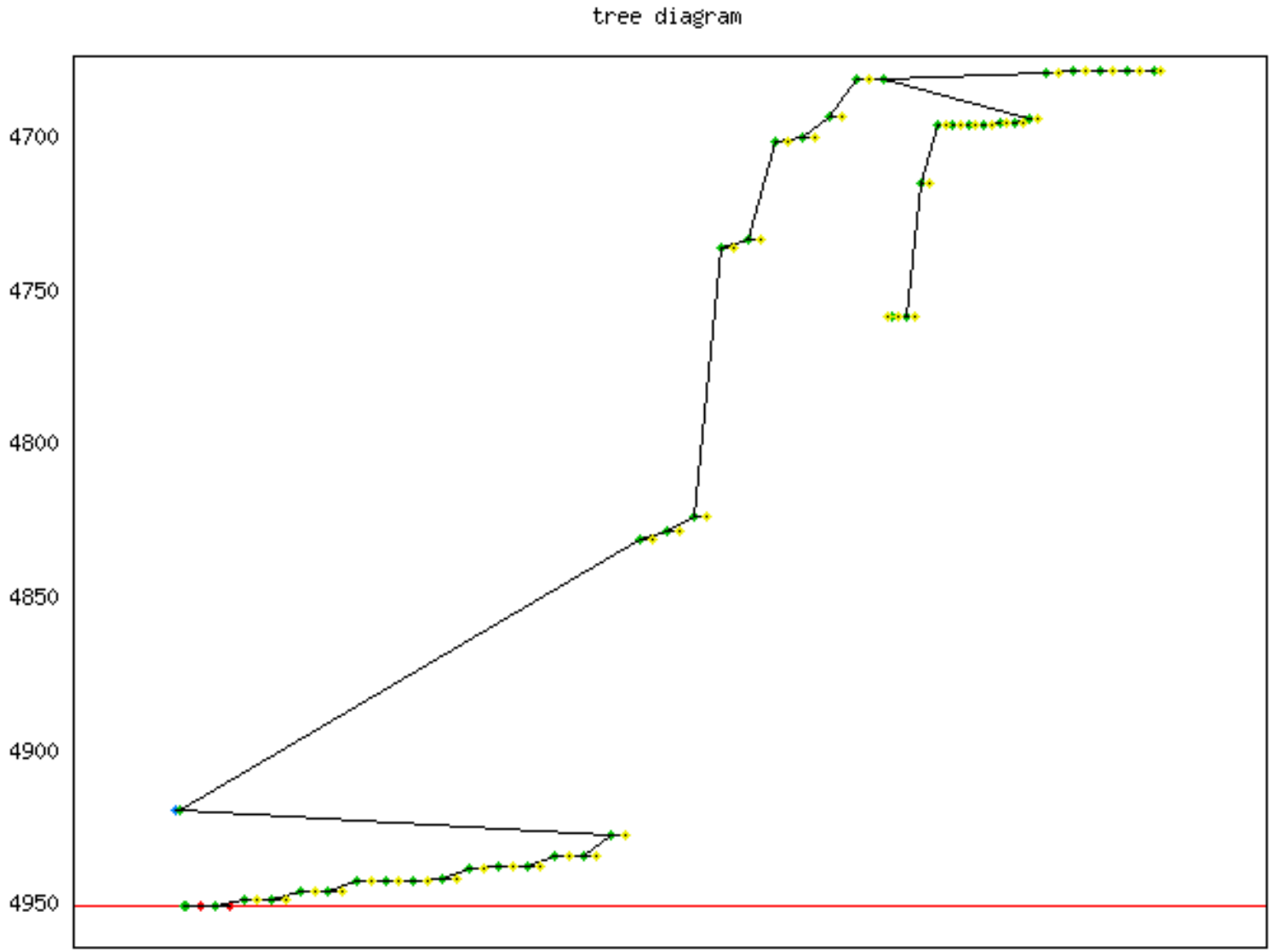
Example B&B trees



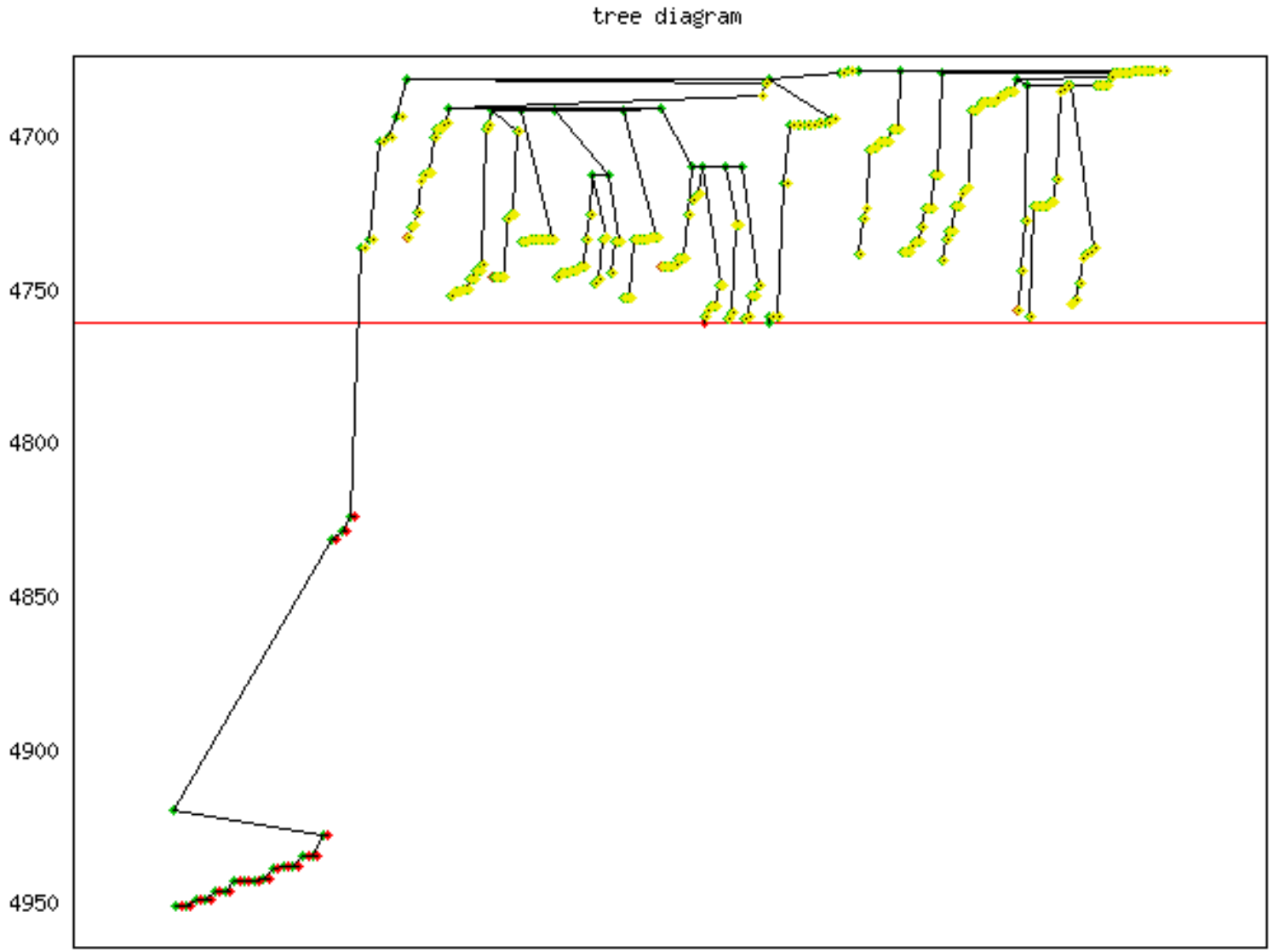
Example B&B tree series 1: l152lav (MIPLIB 2003)



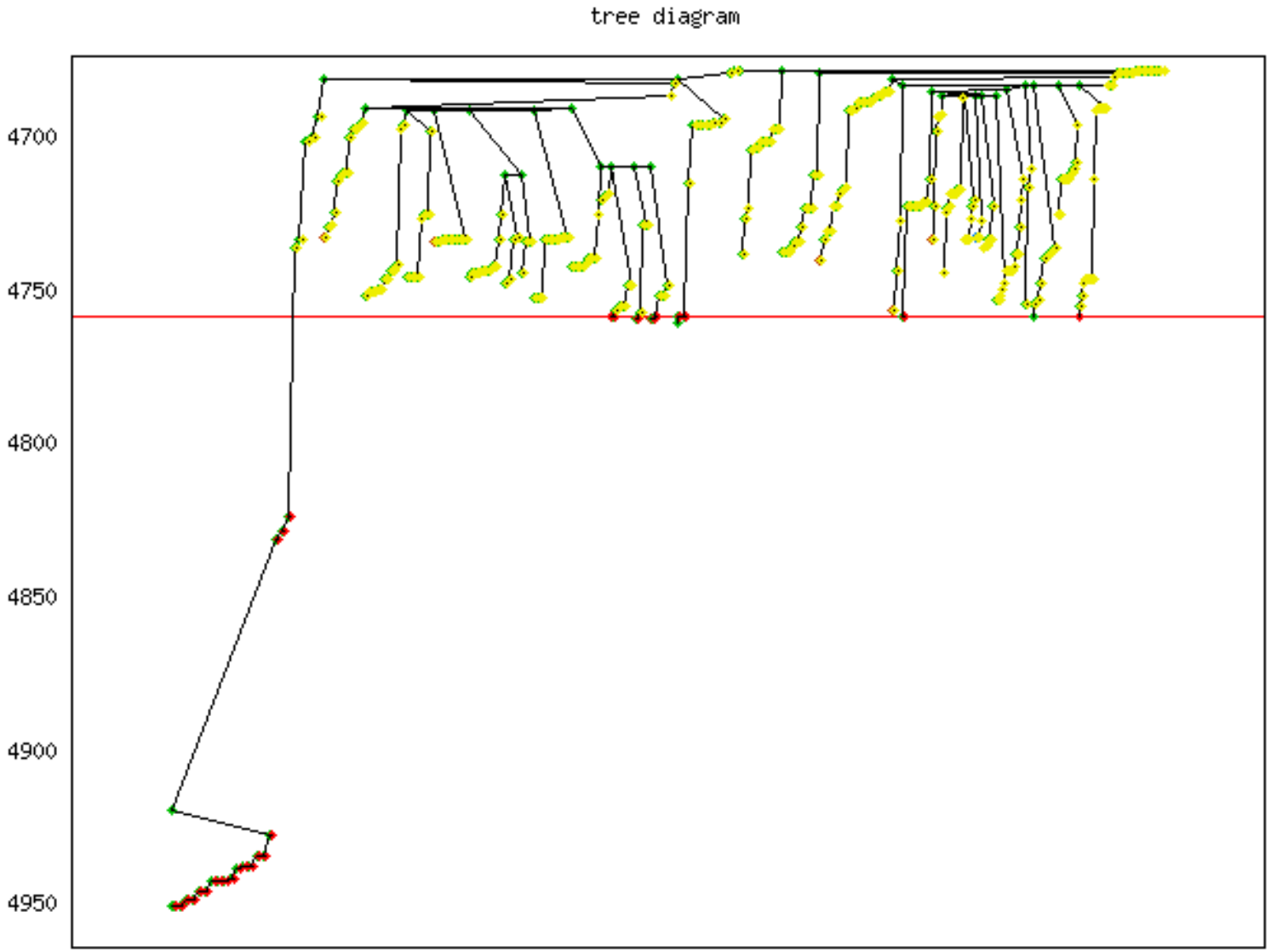
Example B&B tree series 1: l152lav (MIPLIB 2003)



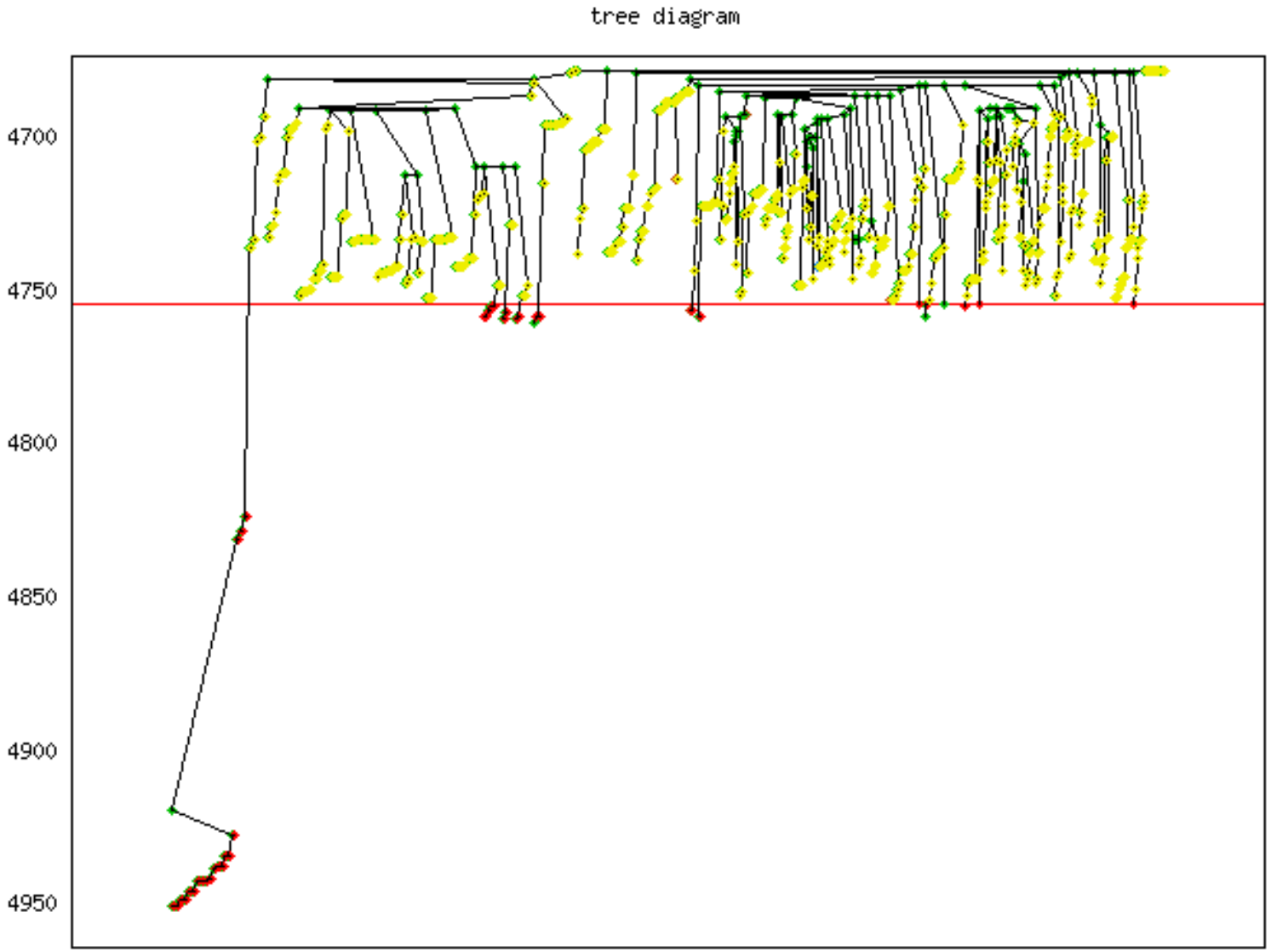
Example B&B tree series 1: l152lav (MIPLIB 2003)



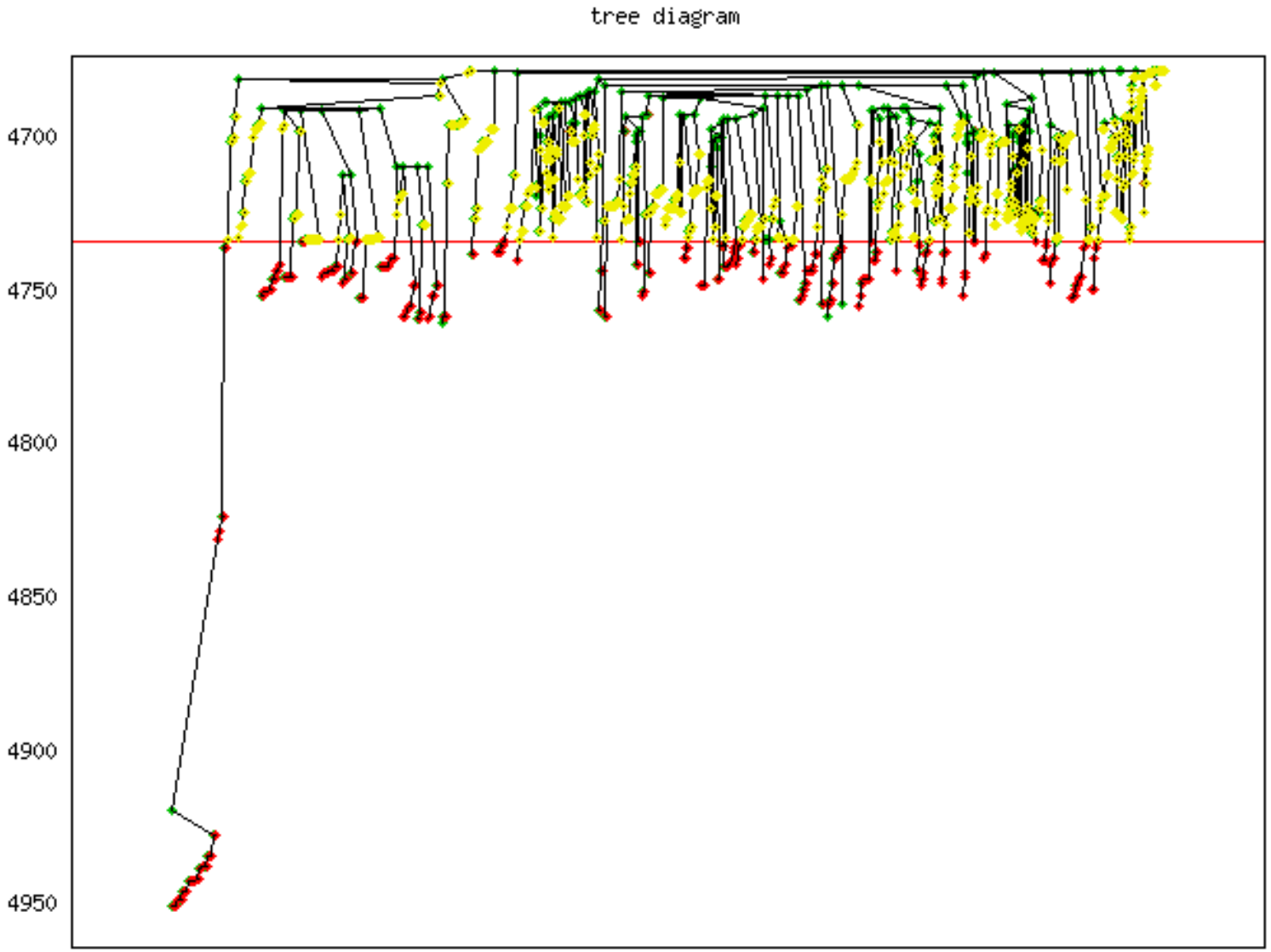
Example B&B tree series 1: l152lav (MIPLIB 2003)



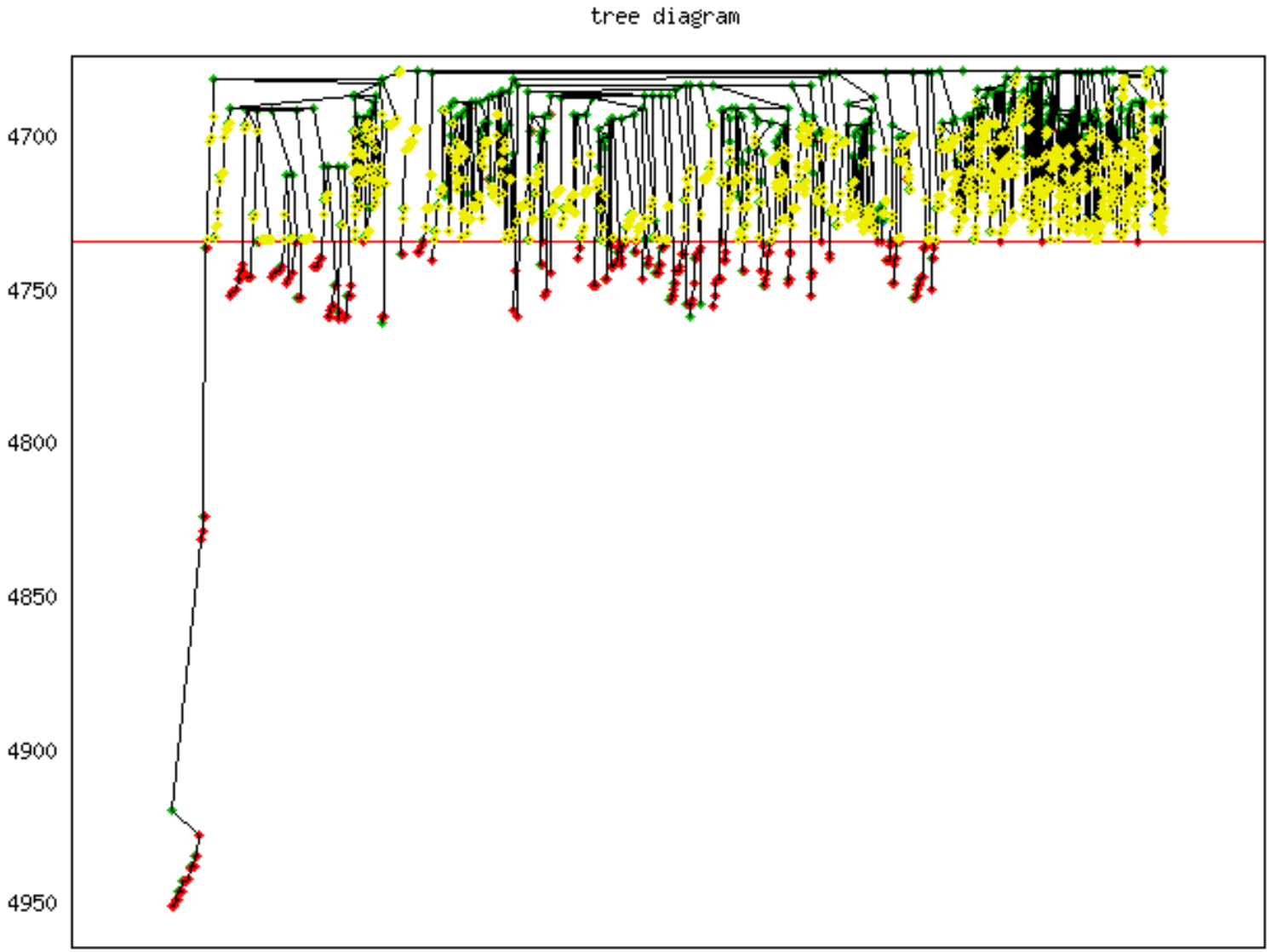
Example B&B tree series 1: l152lav (MIPLIB 2003)



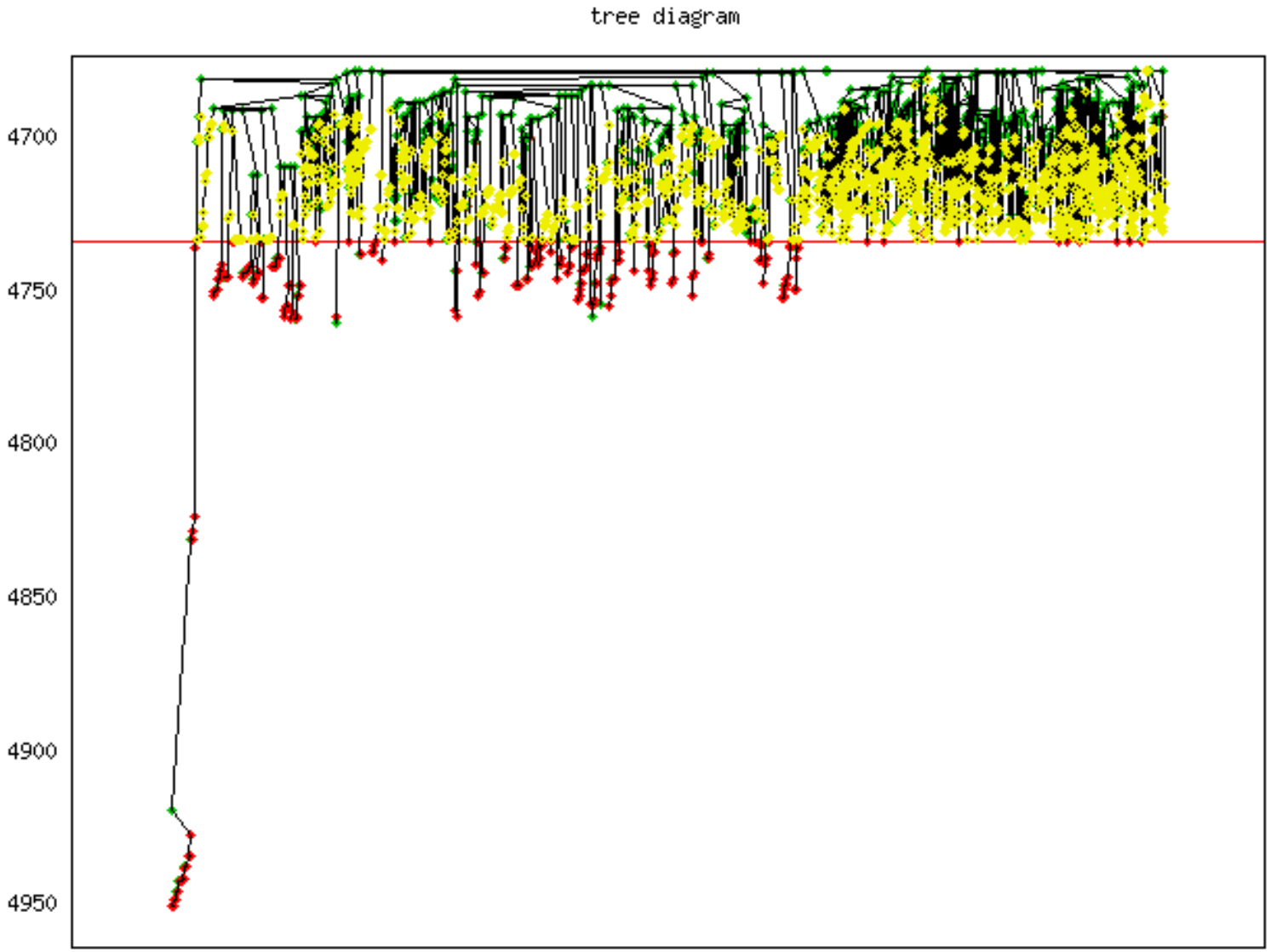
Example B&B tree series 1: l152lav (MIPLIB 2003)



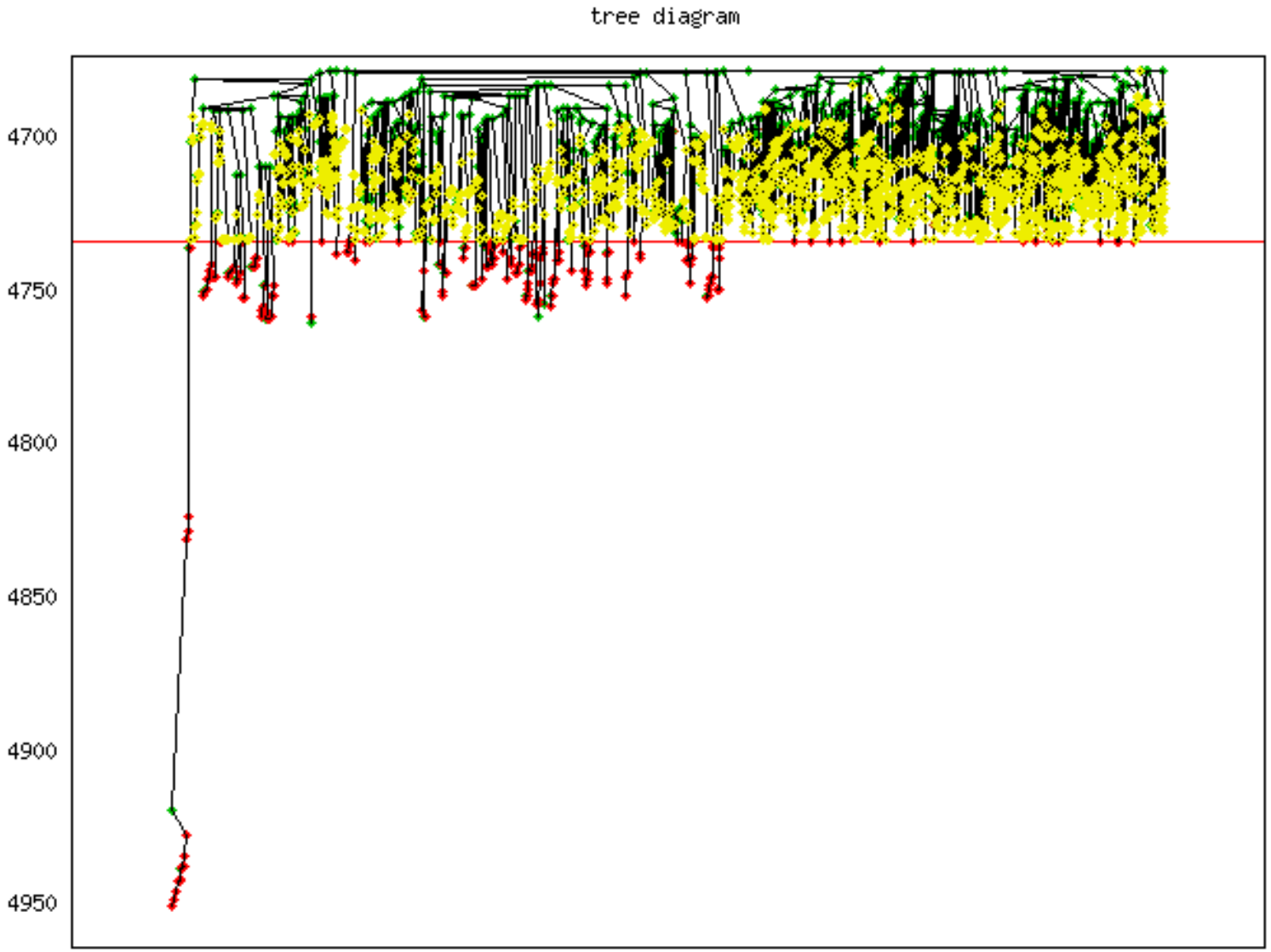
Example B&B tree series 1: l152lav (MIPLIB 2003)



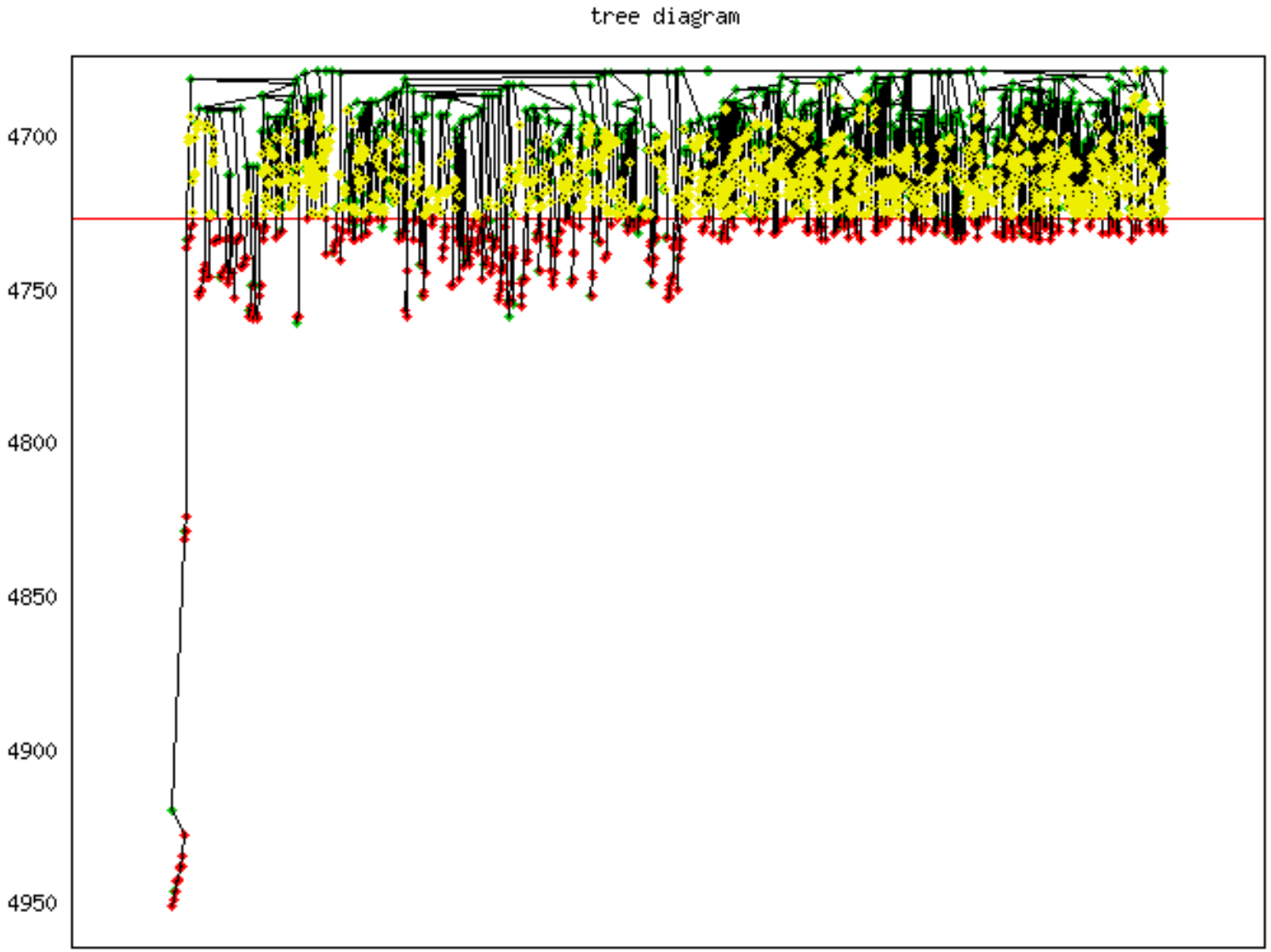
Example B&B tree series 1: l152lav (MIPLIB 2003)



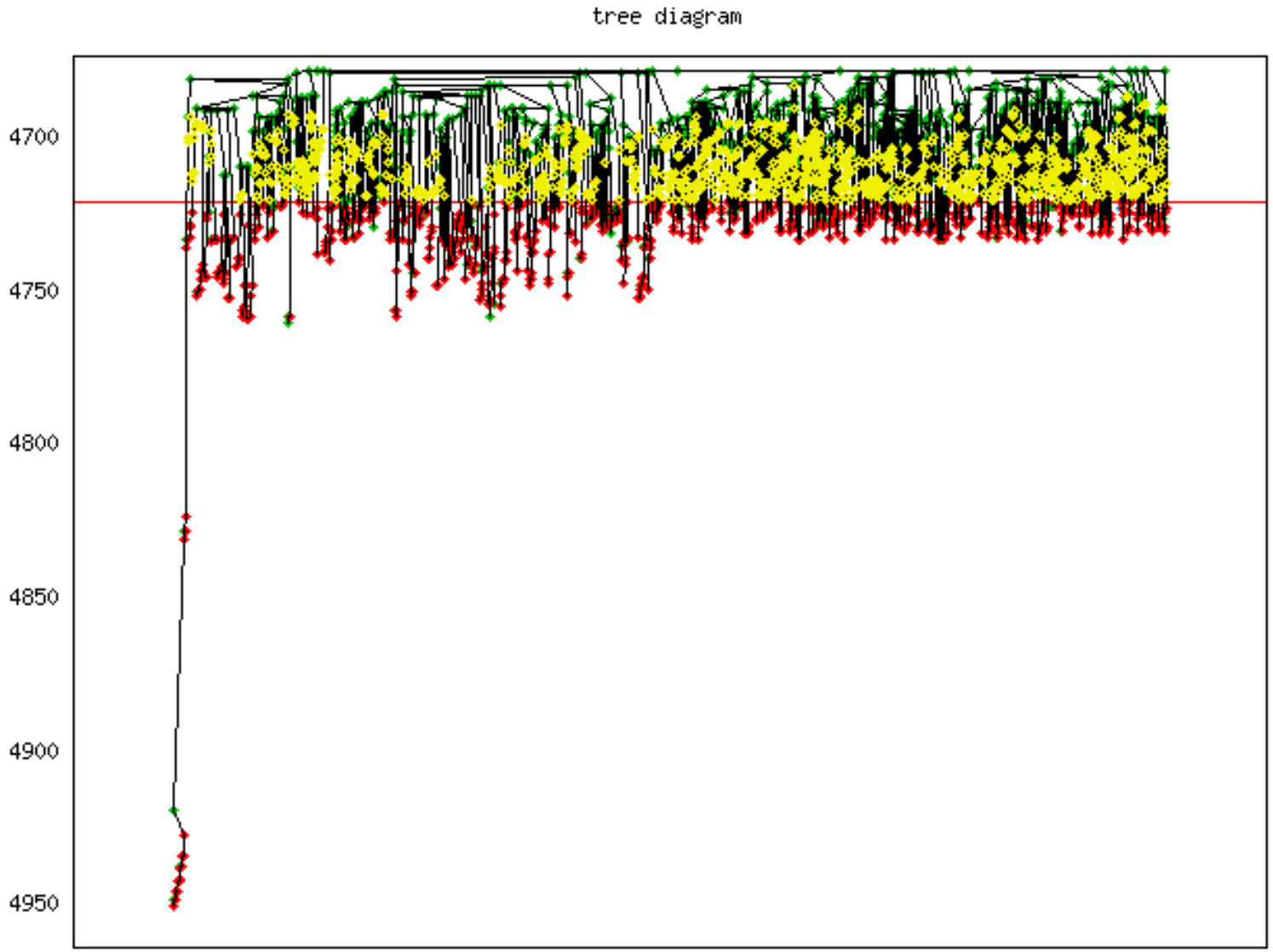
Example B&B tree series 1: l152lav (MIPLIB 2003)



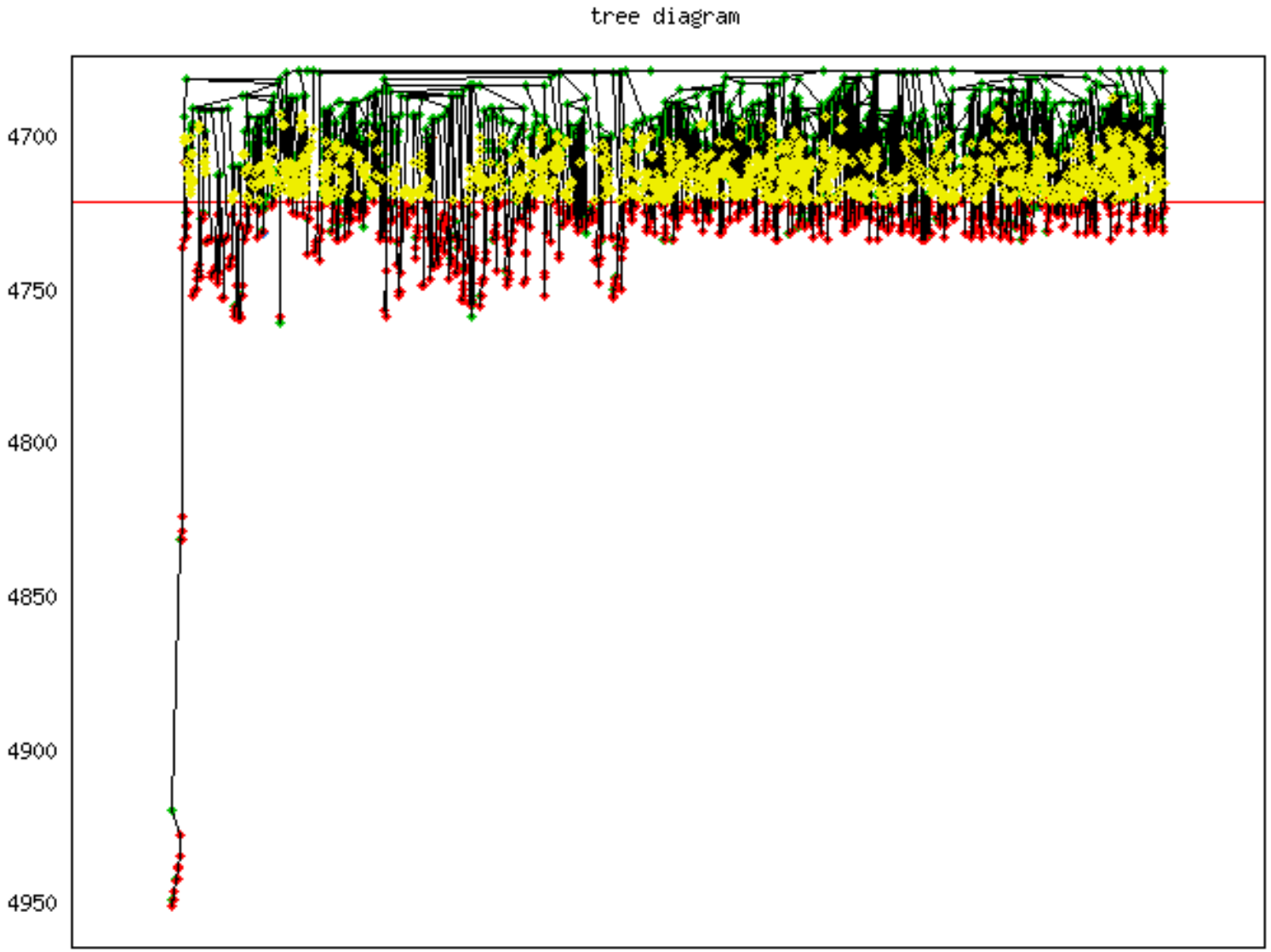
Example B&B tree series 1: l152lav (MIPLIB 2003)



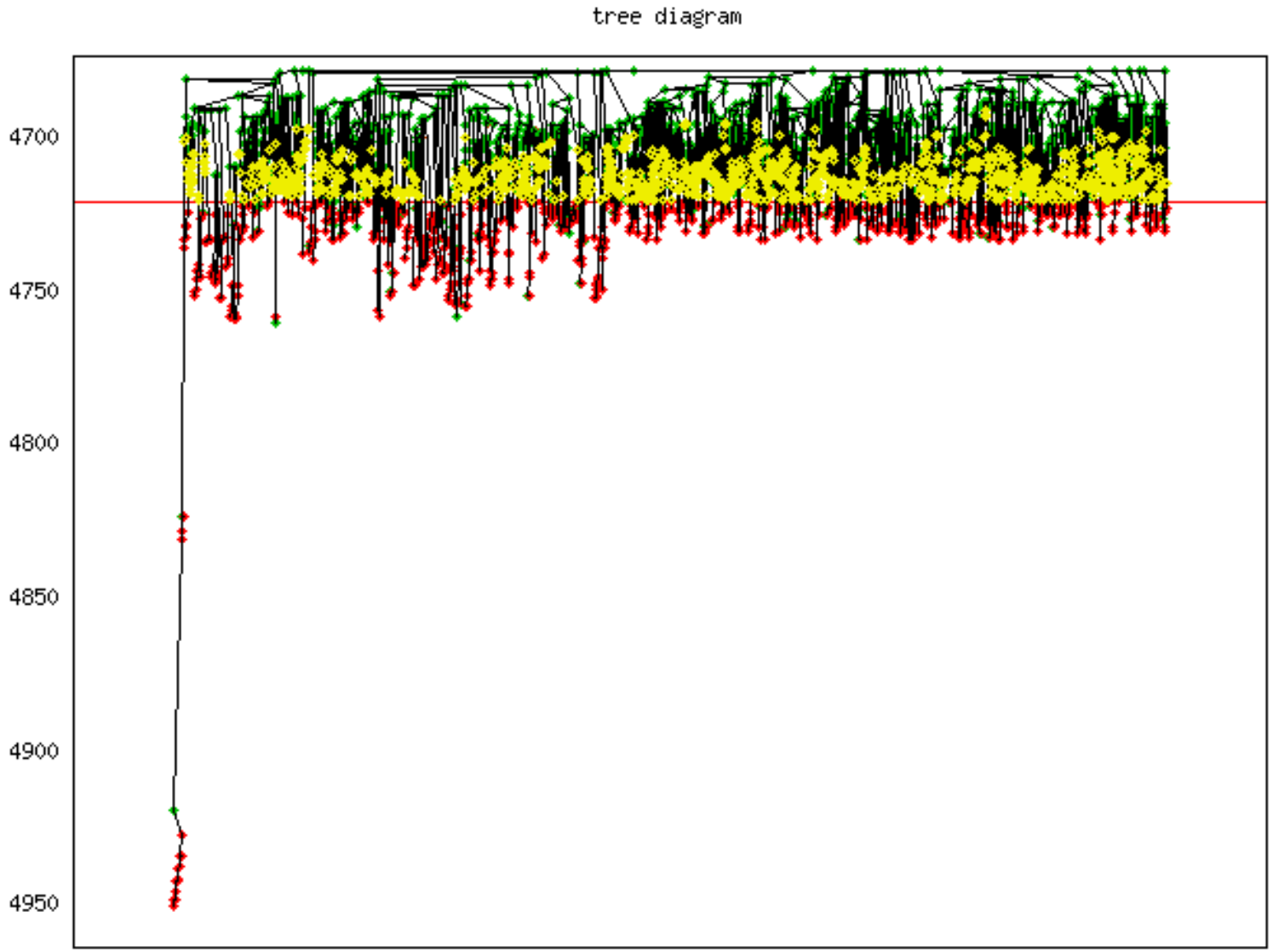
Example B&B tree series 1: l152lav (MIPLIB 2003)



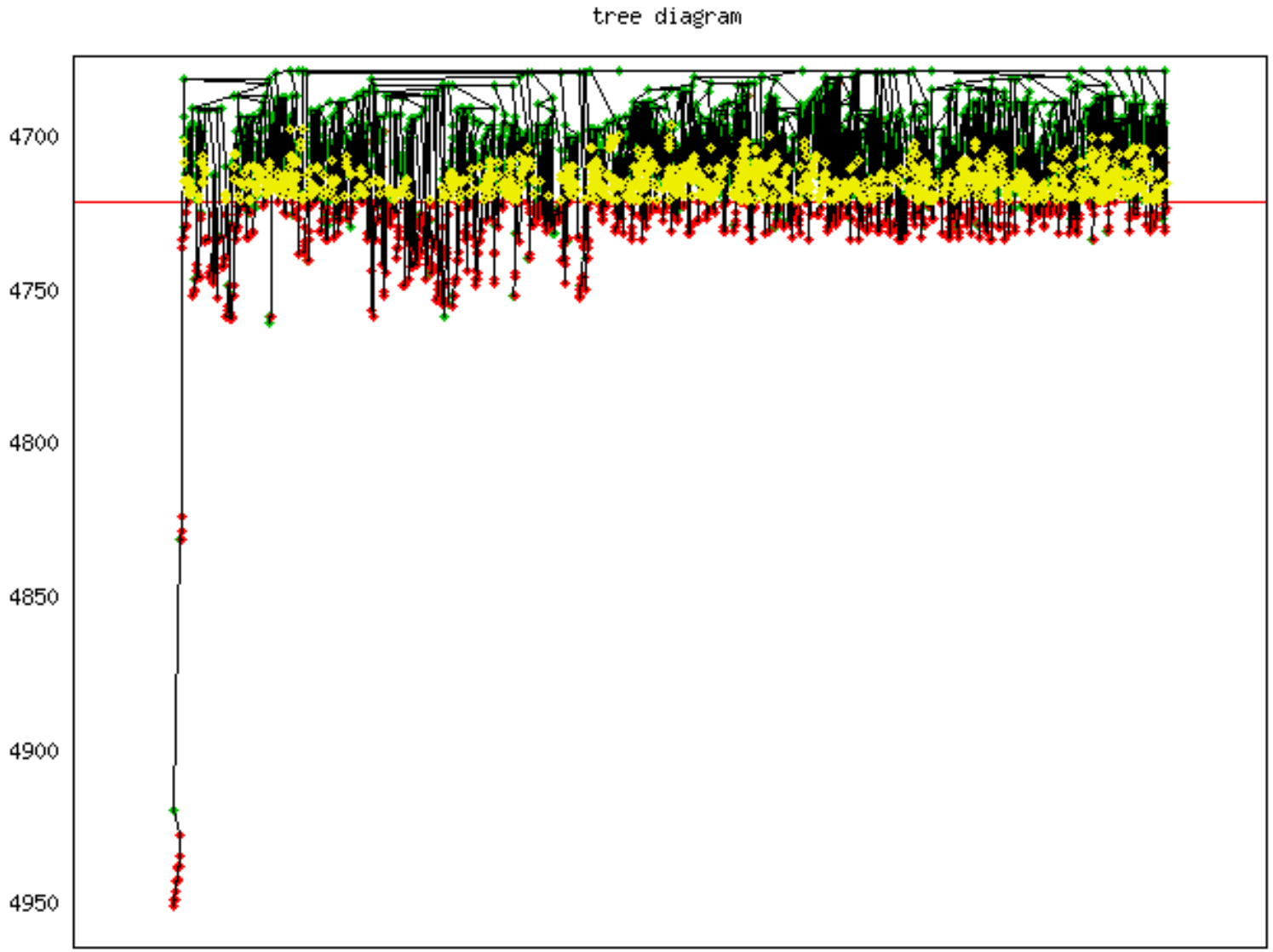
Example B&B tree series 1: l152lav (MIPLIB 2003)



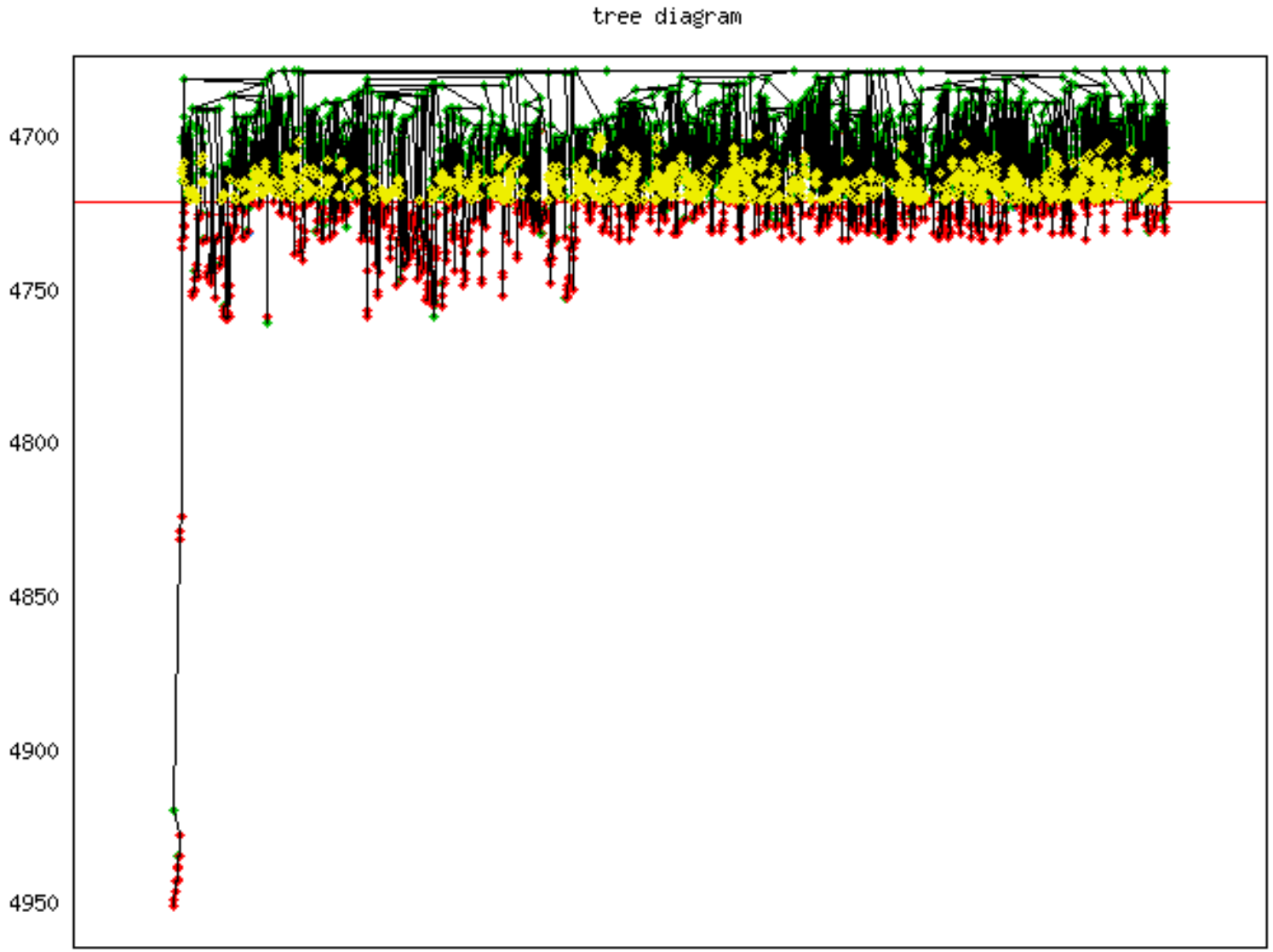
Example B&B tree series 1: l152lav (MIPLIB 2003)



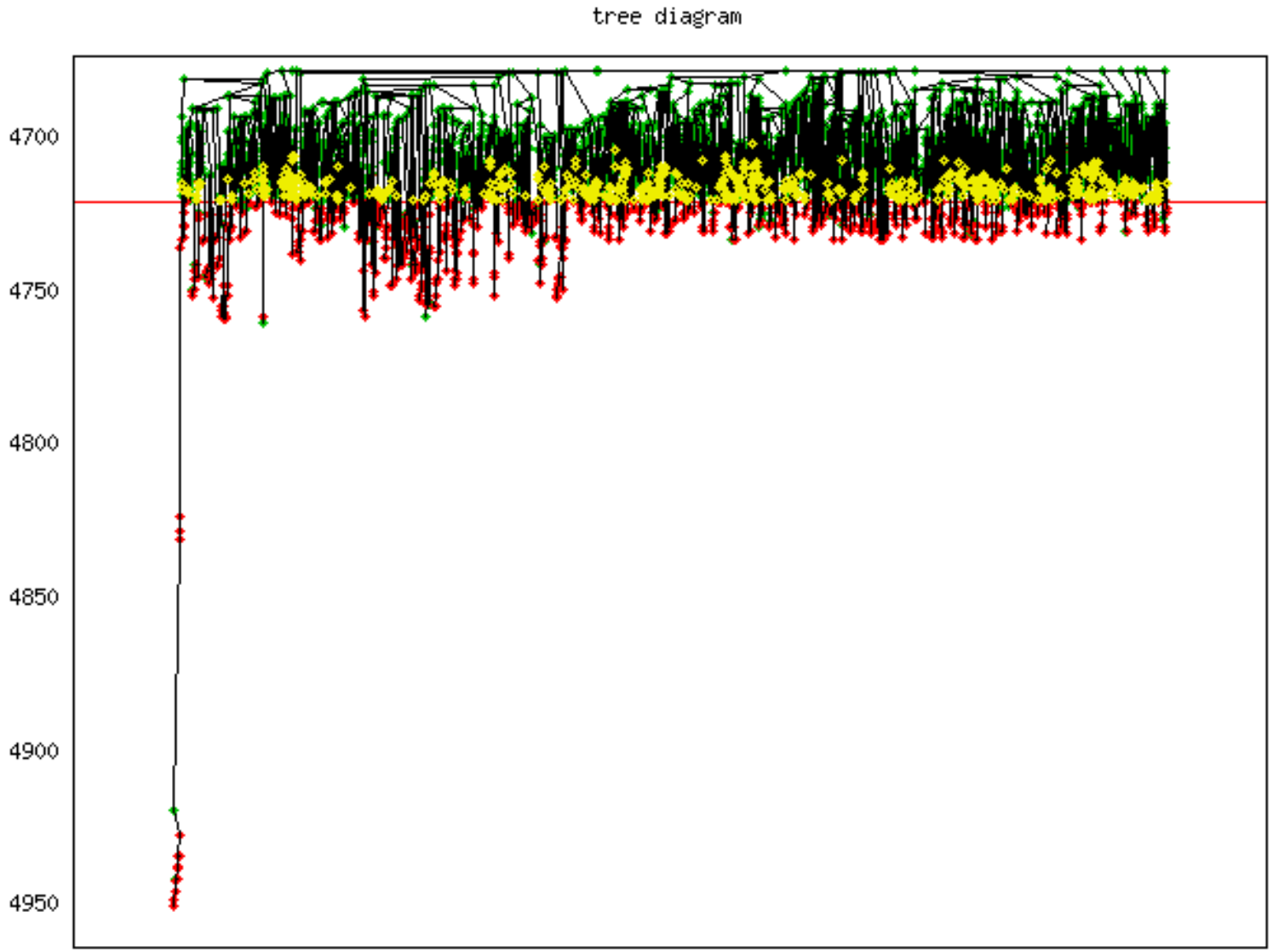
Example B&B tree series 1: l152lav (MIPLIB 2003)



Example B&B tree series 1: l152lav (MIPLIB 2003)



Example B&B tree series 1: l152lav (MIPLIB 2003)



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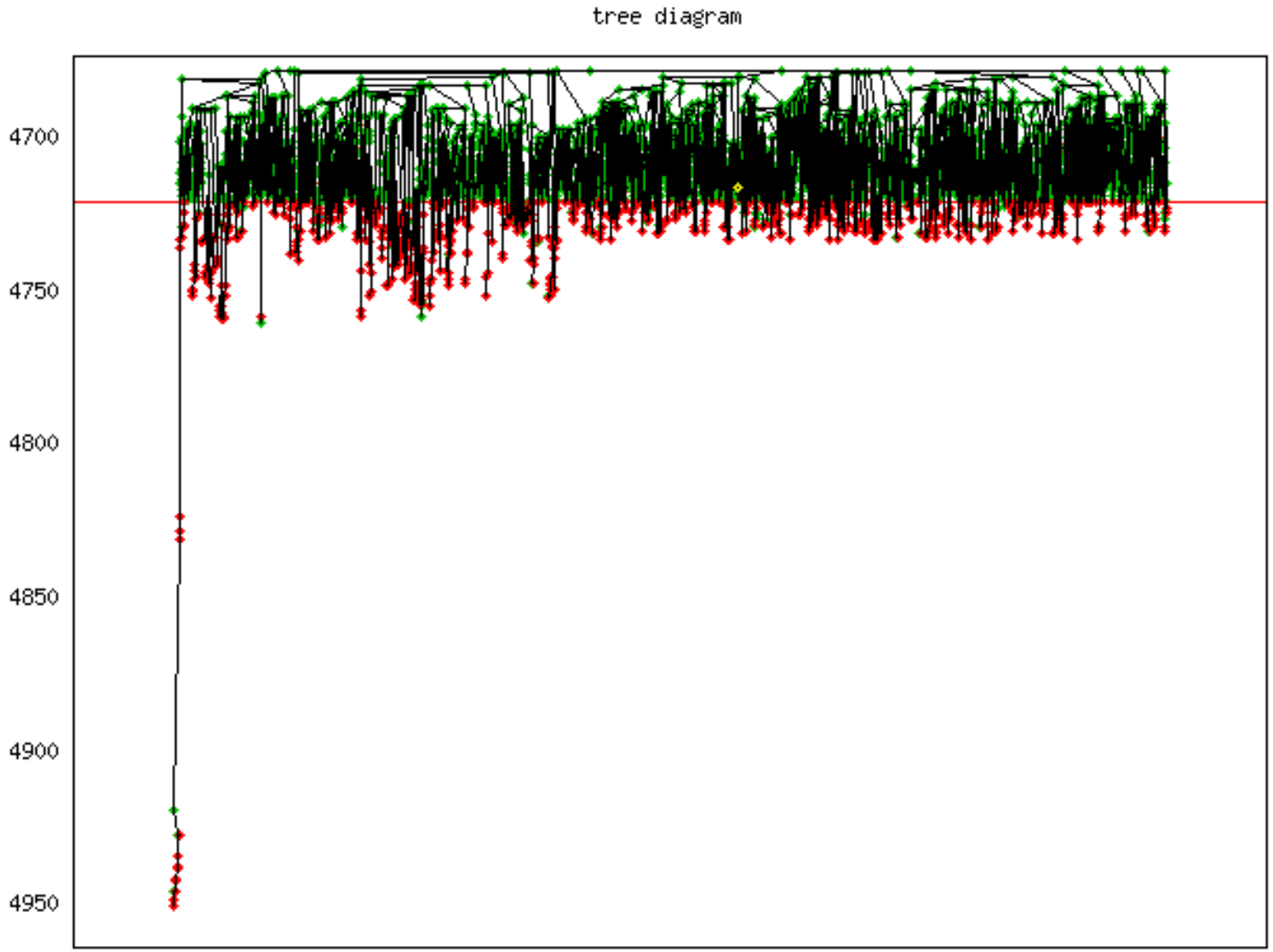
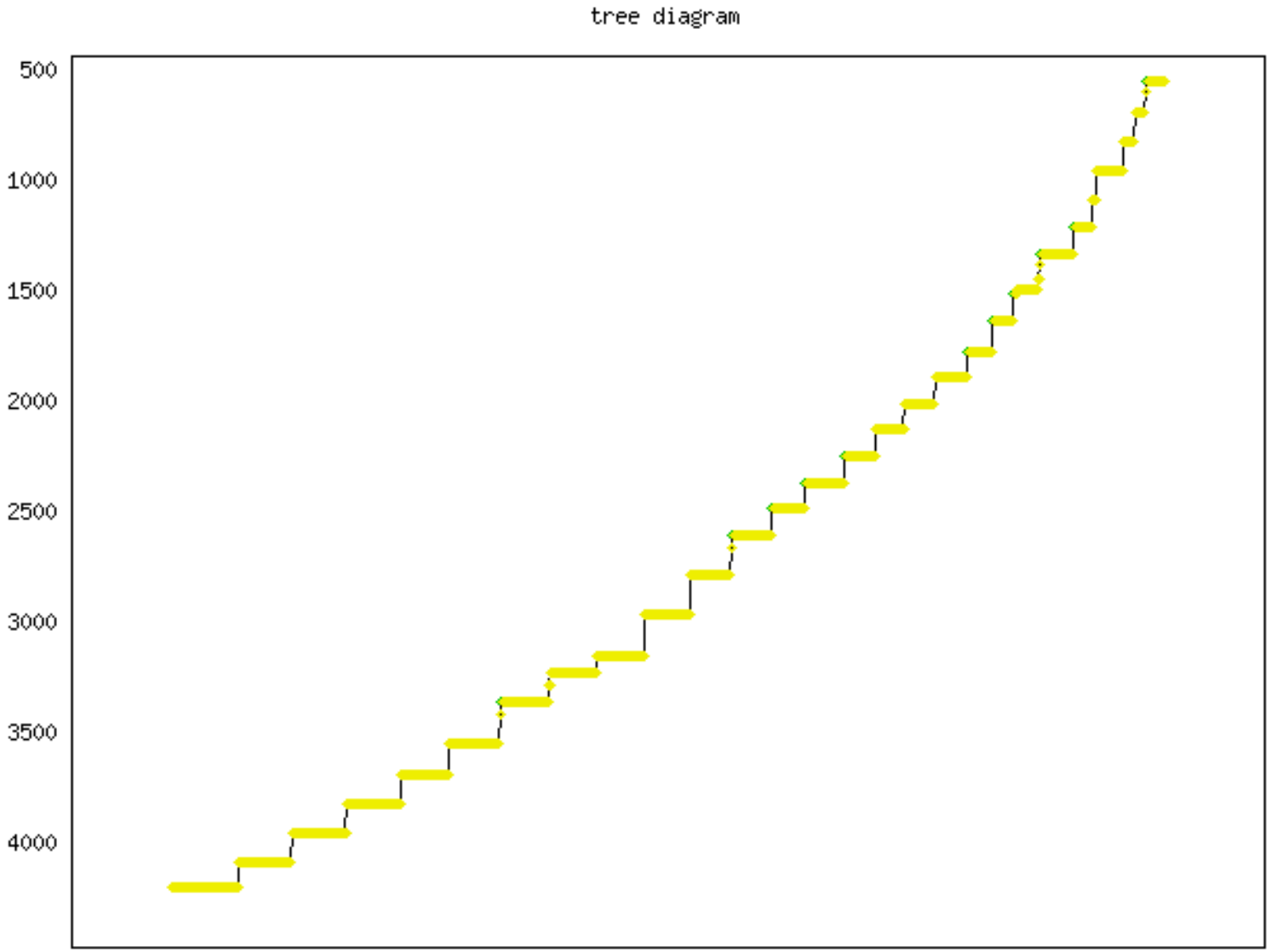


Example B&B tree series 2: liu (MIPLIB 2003)



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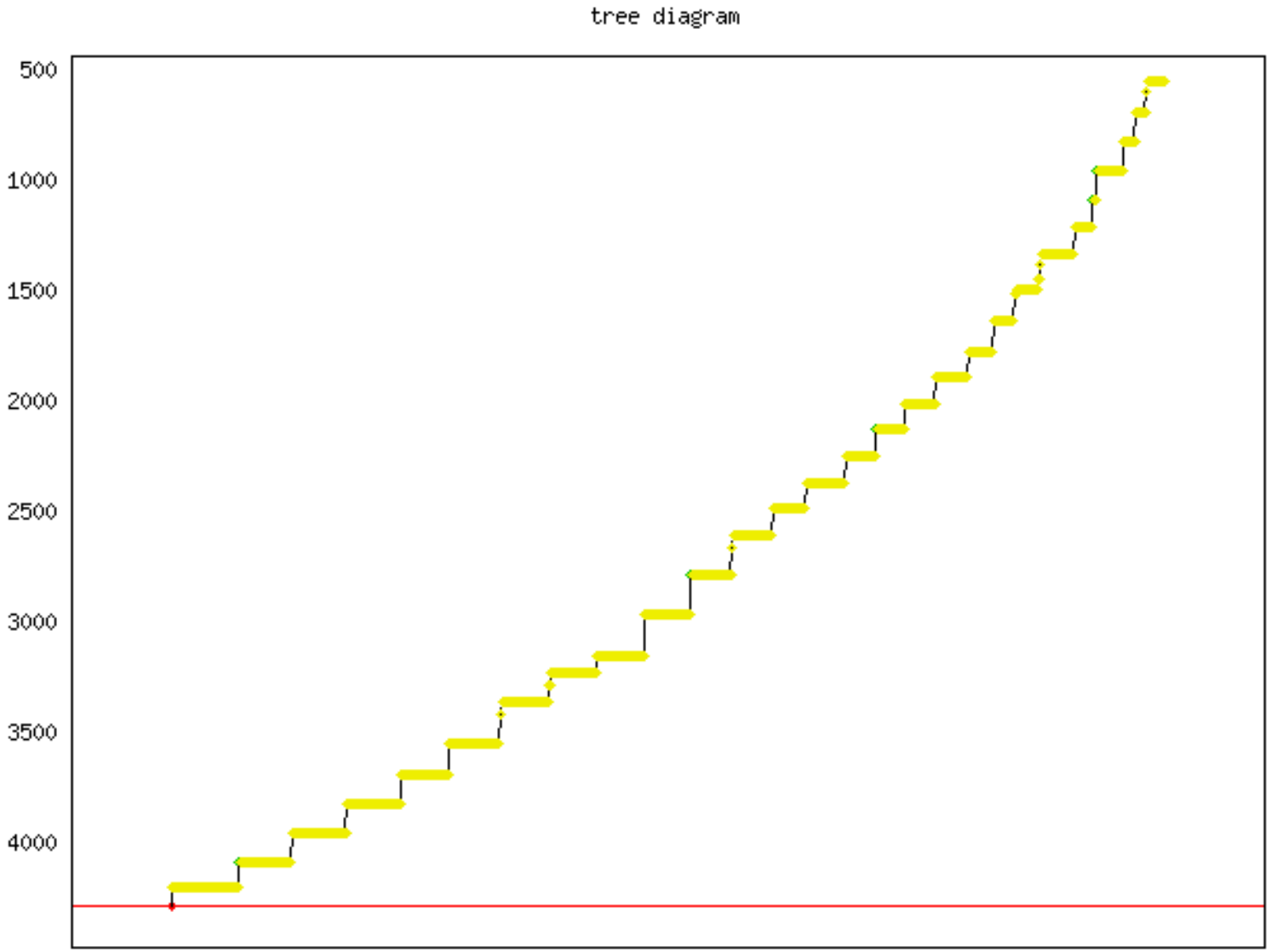

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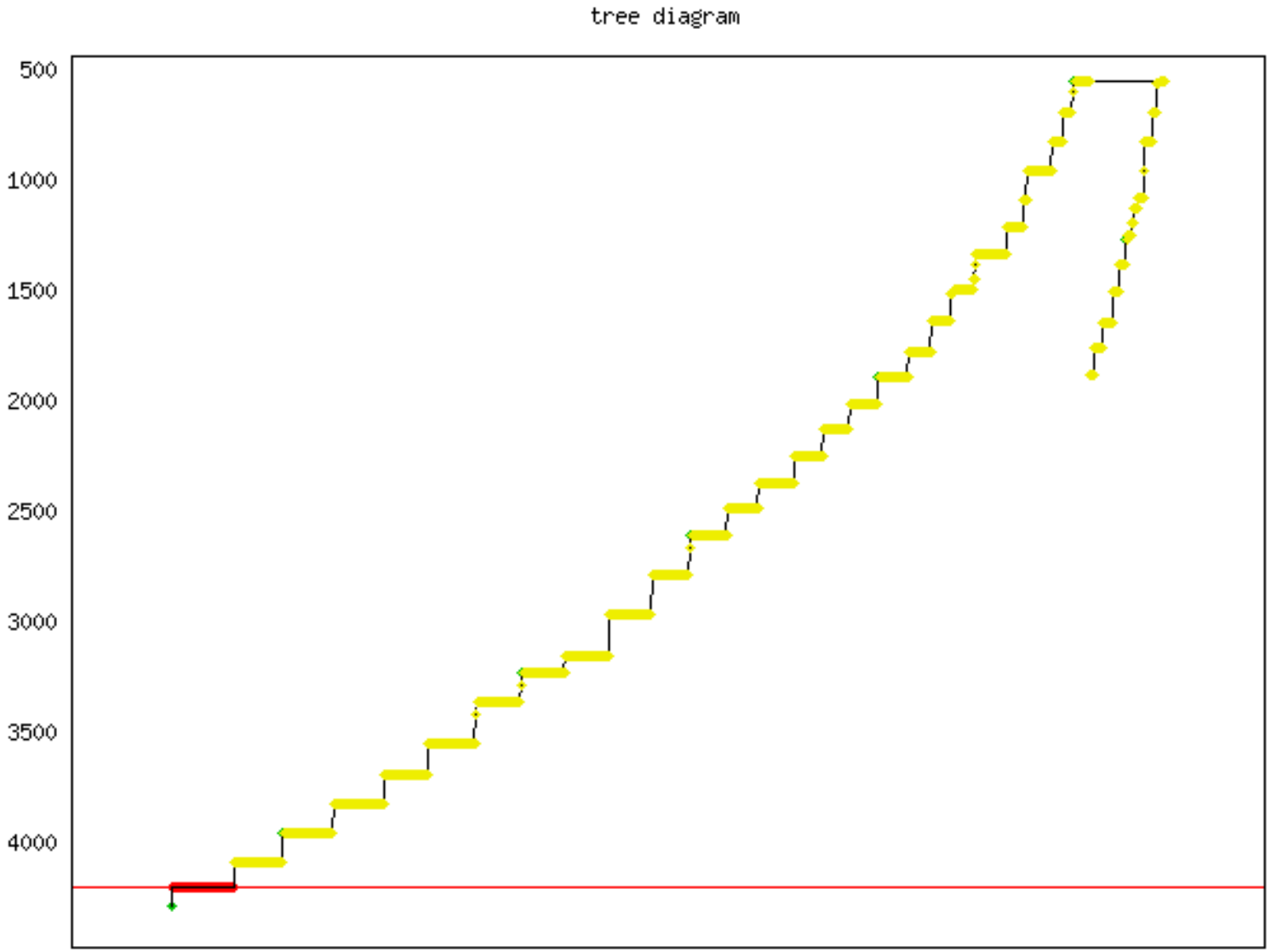

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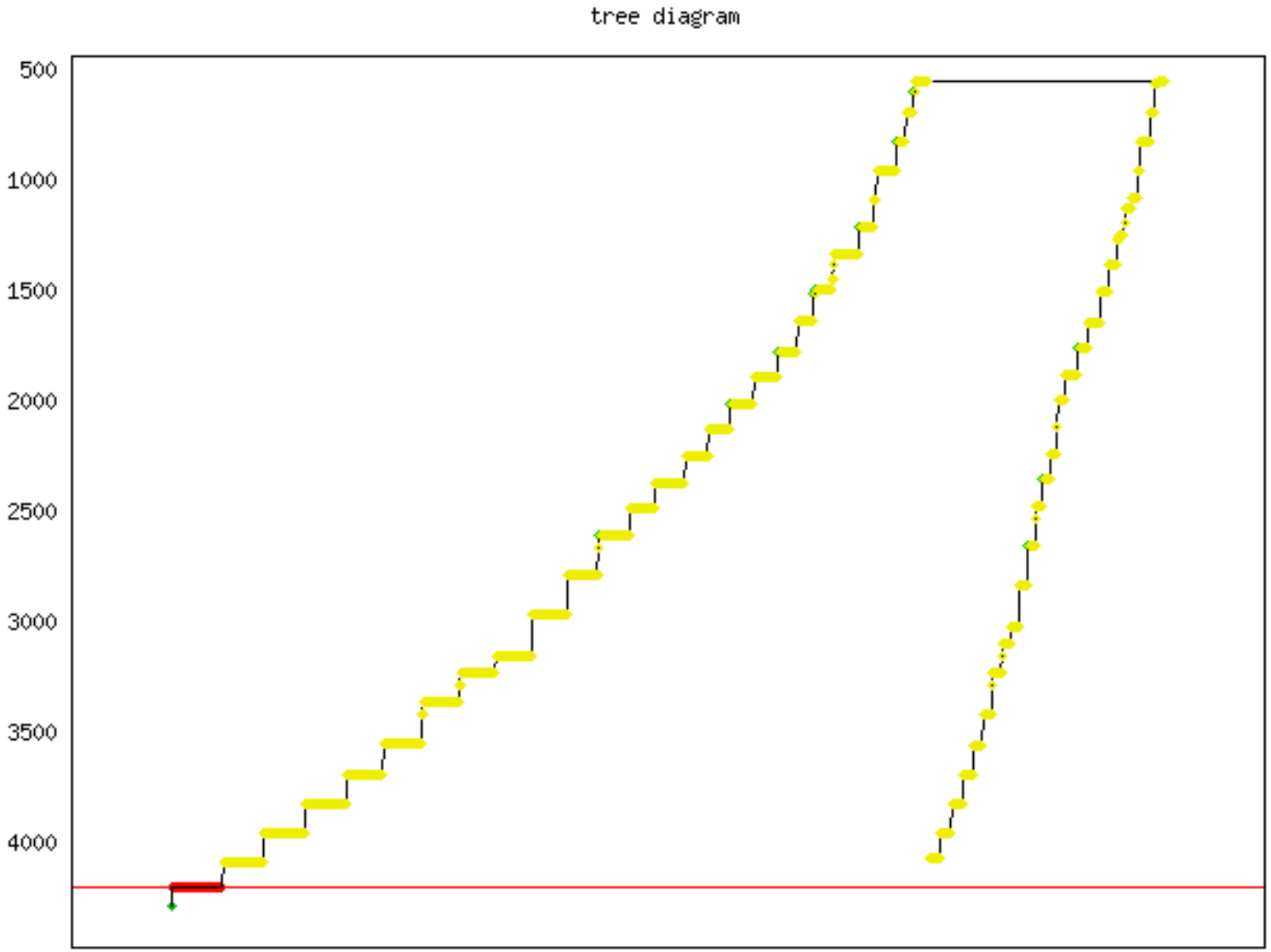

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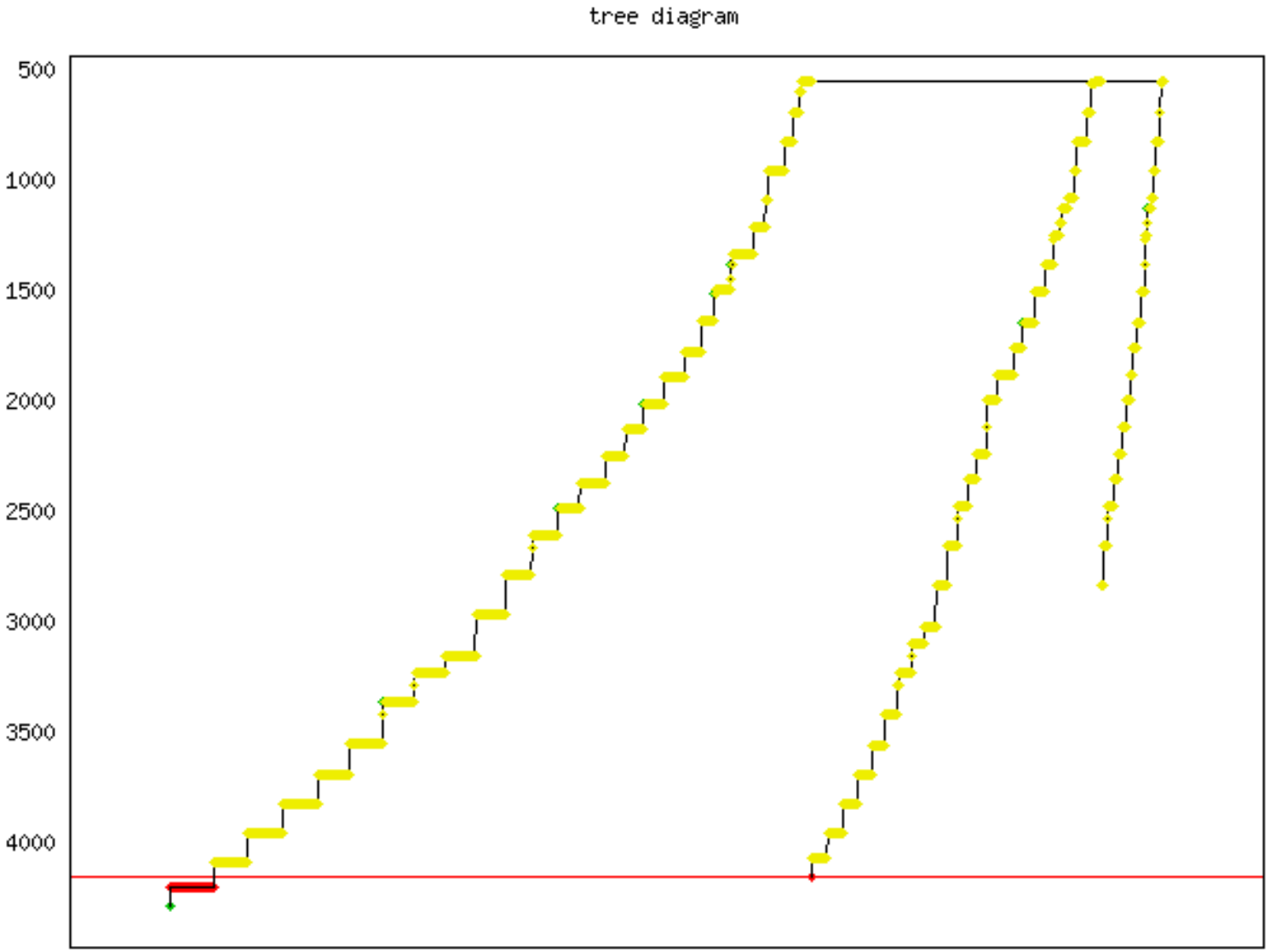

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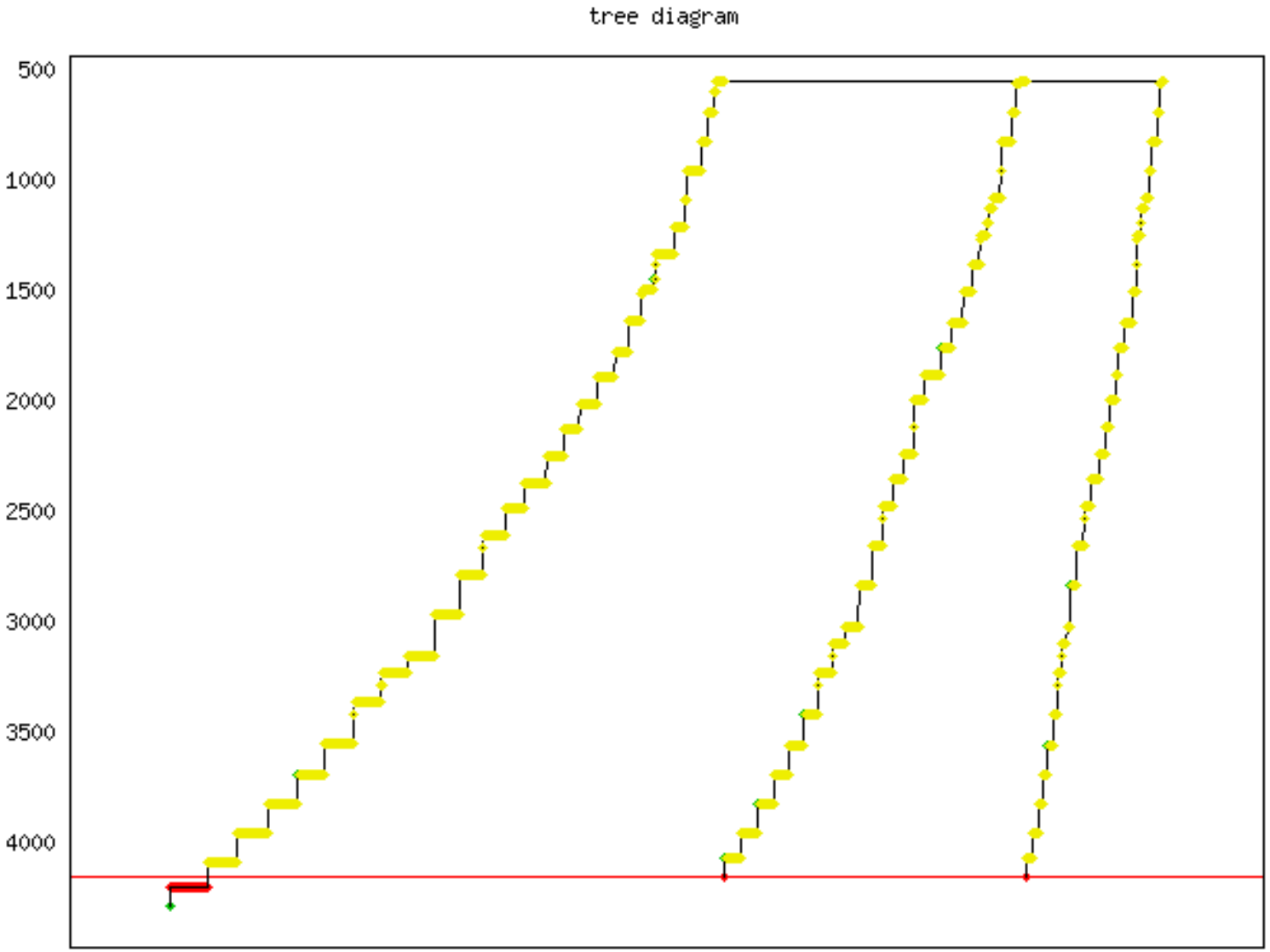

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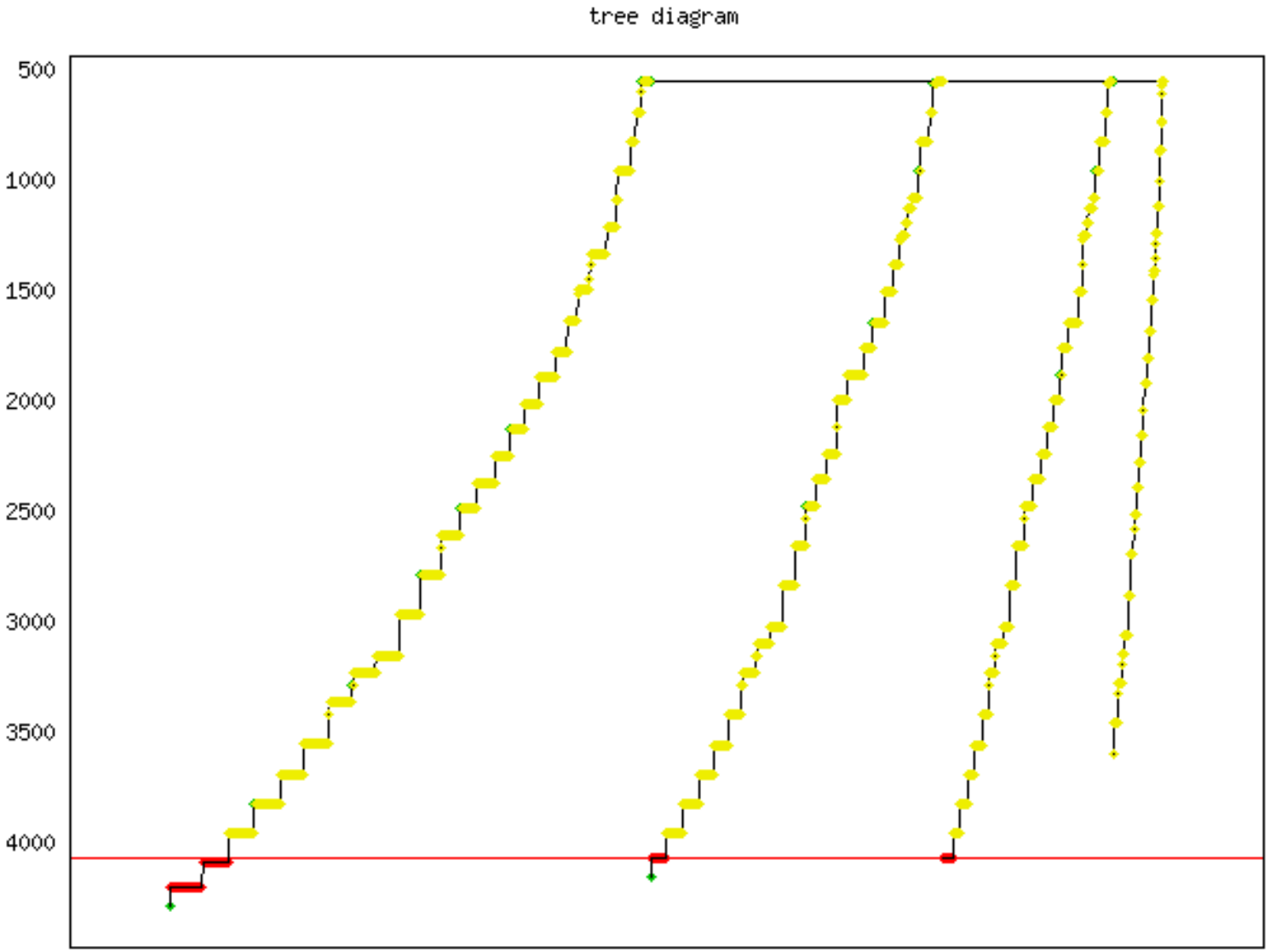

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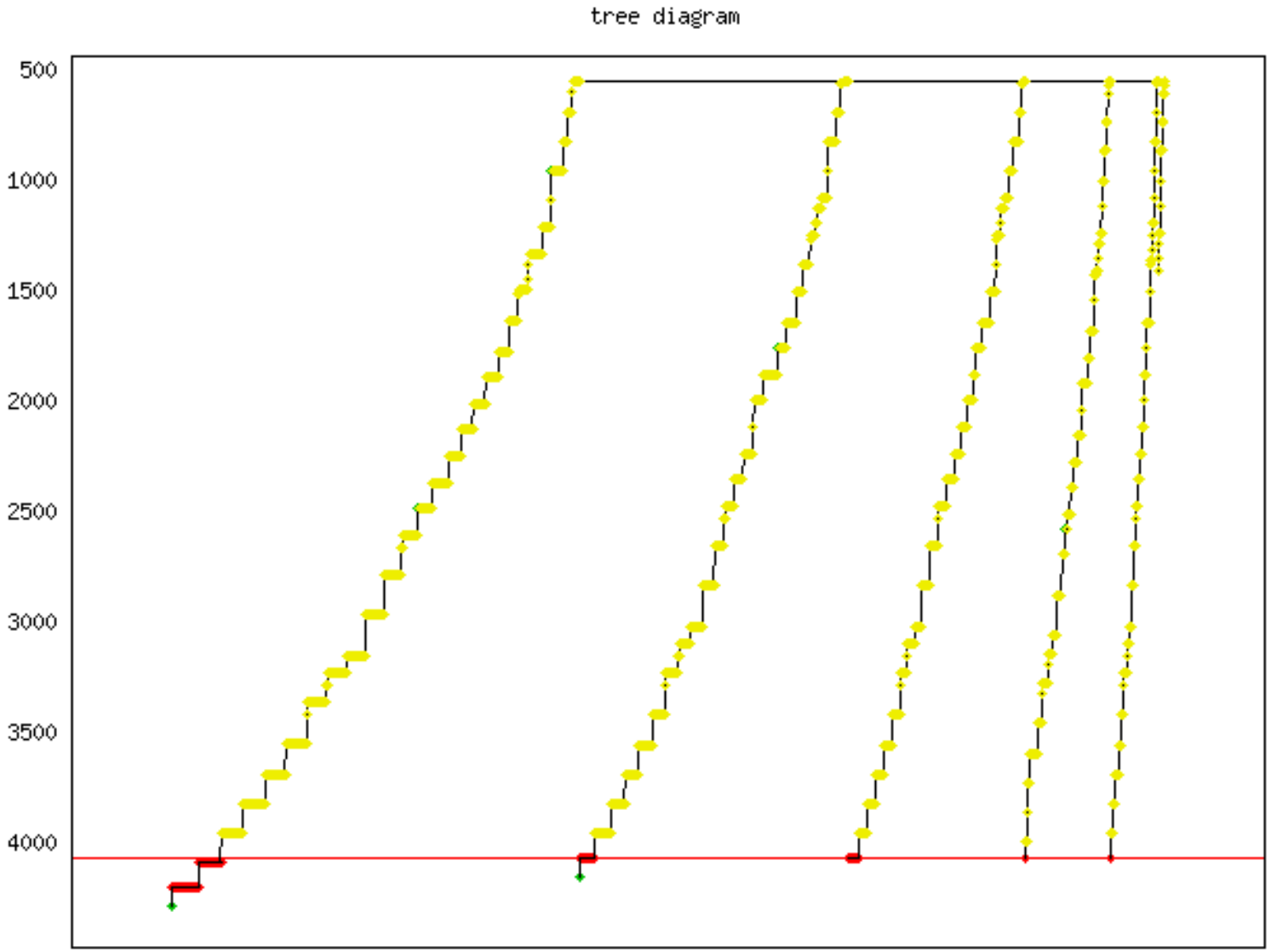

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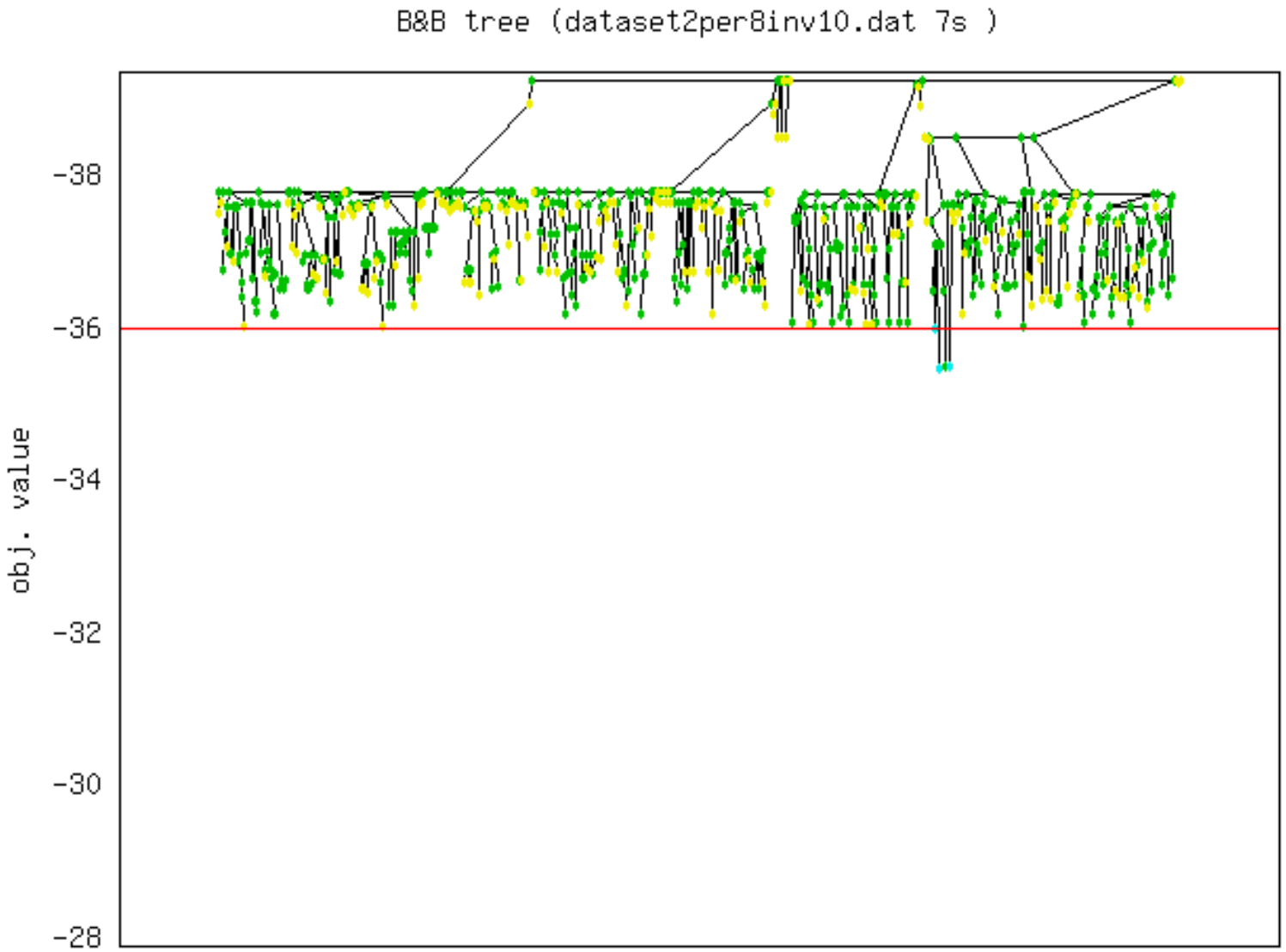

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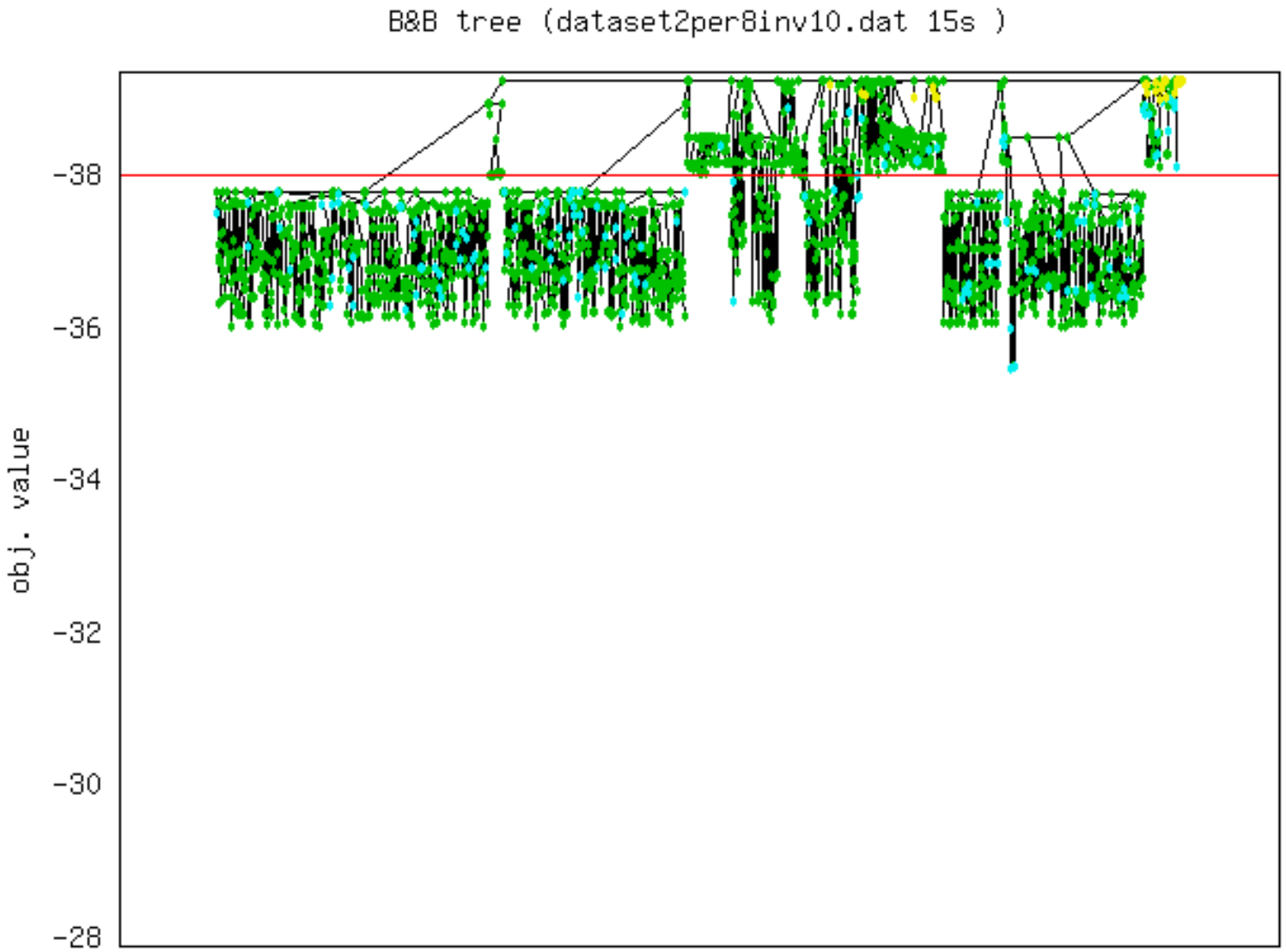

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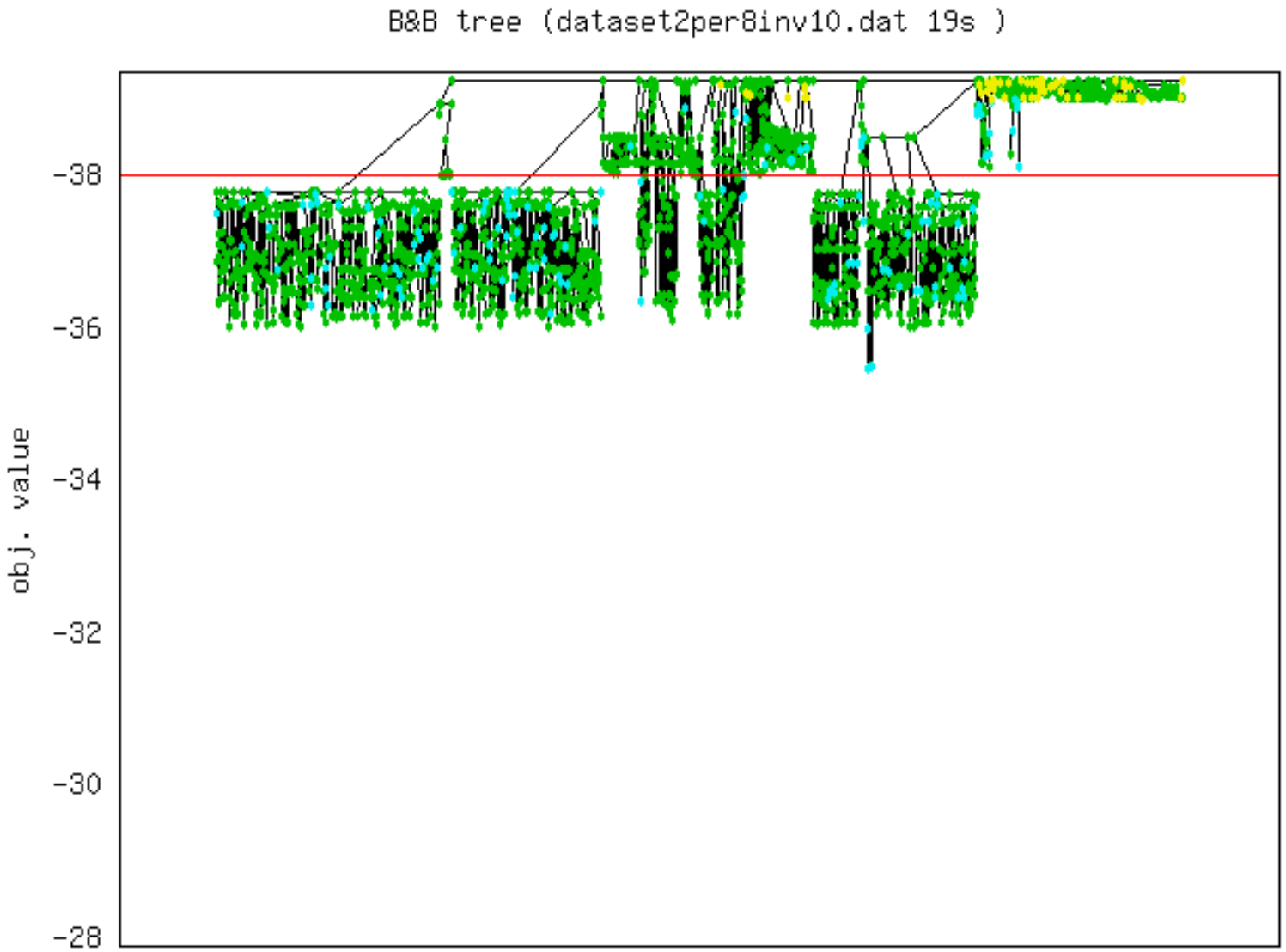

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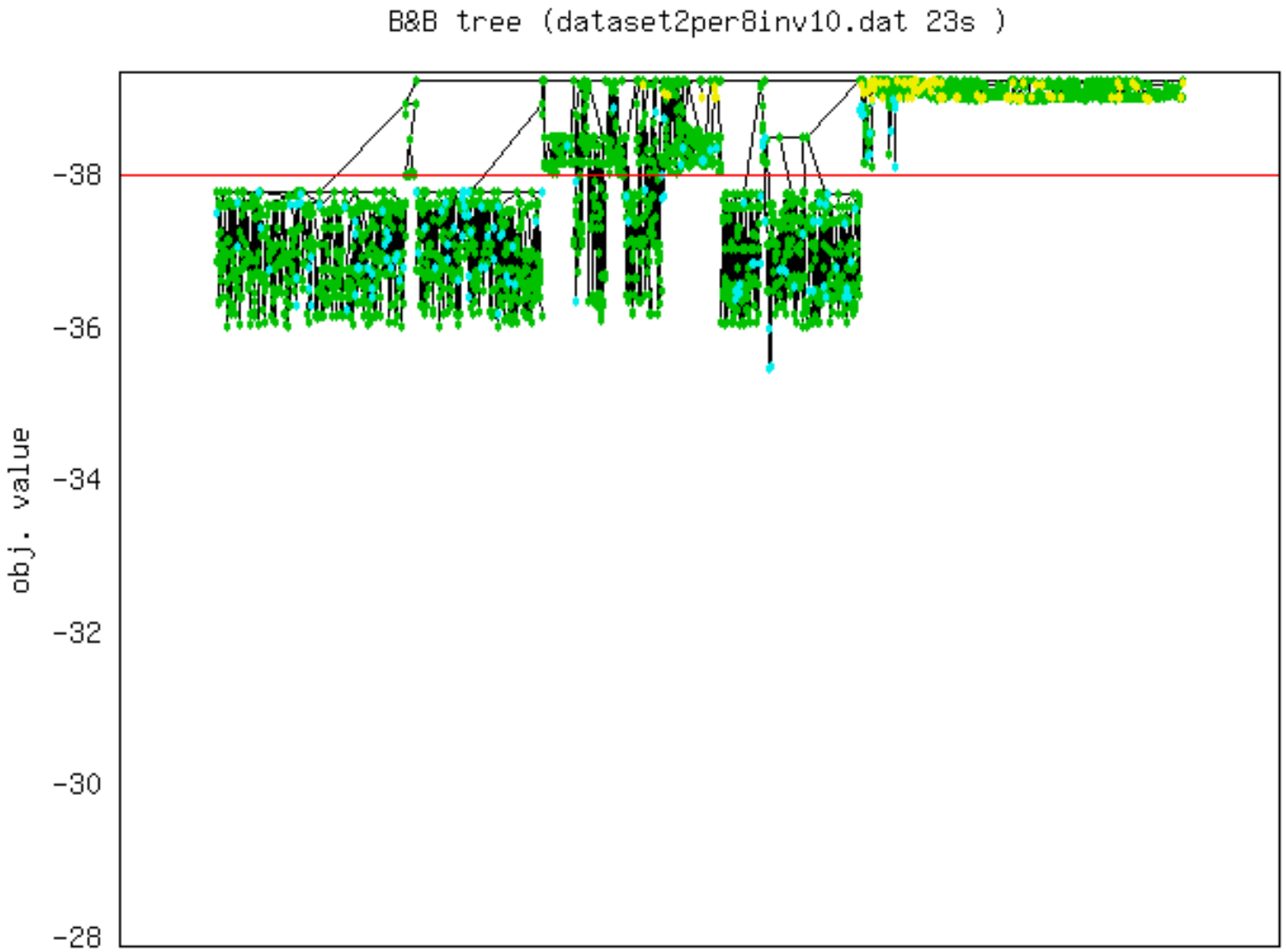

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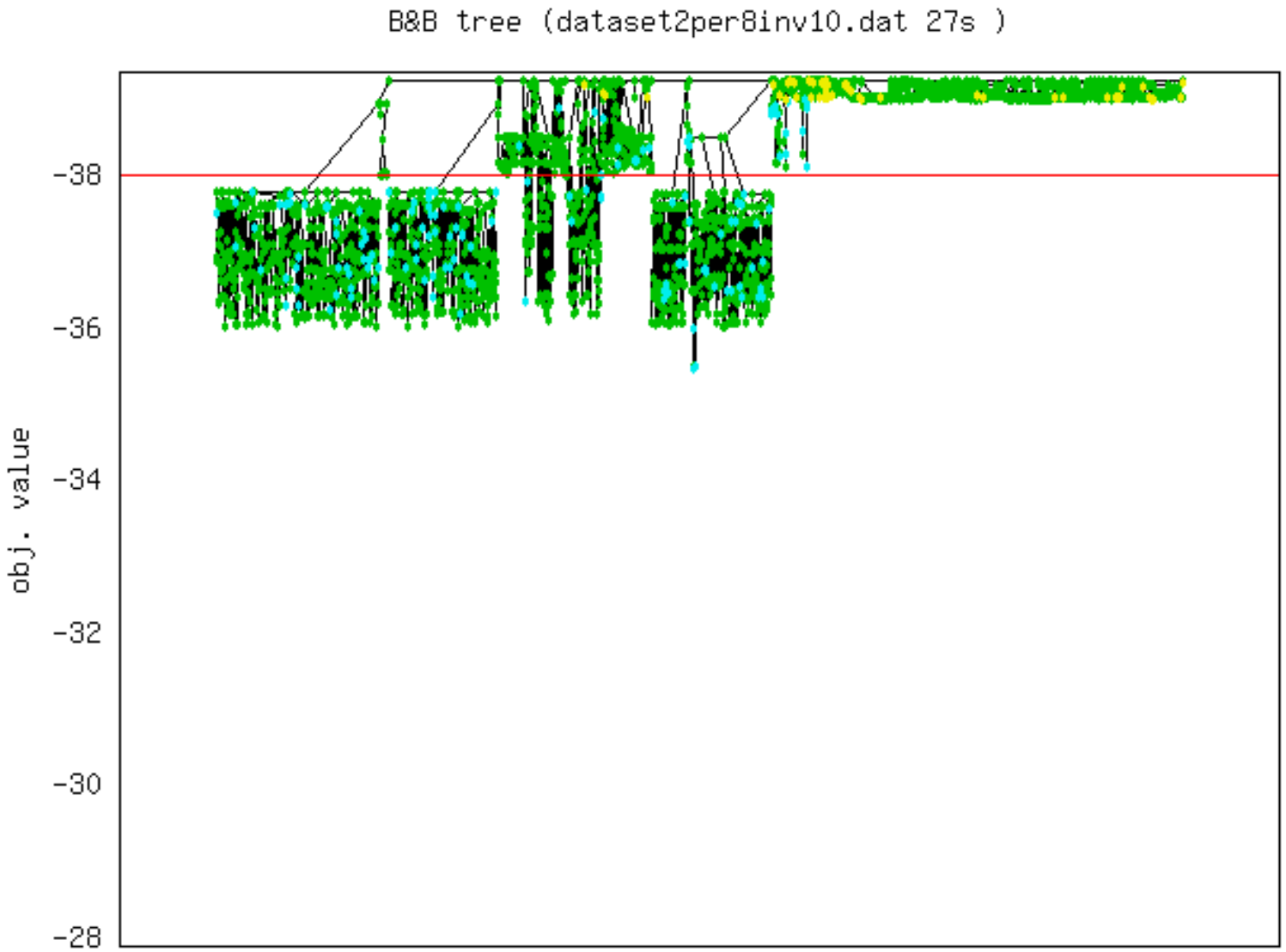

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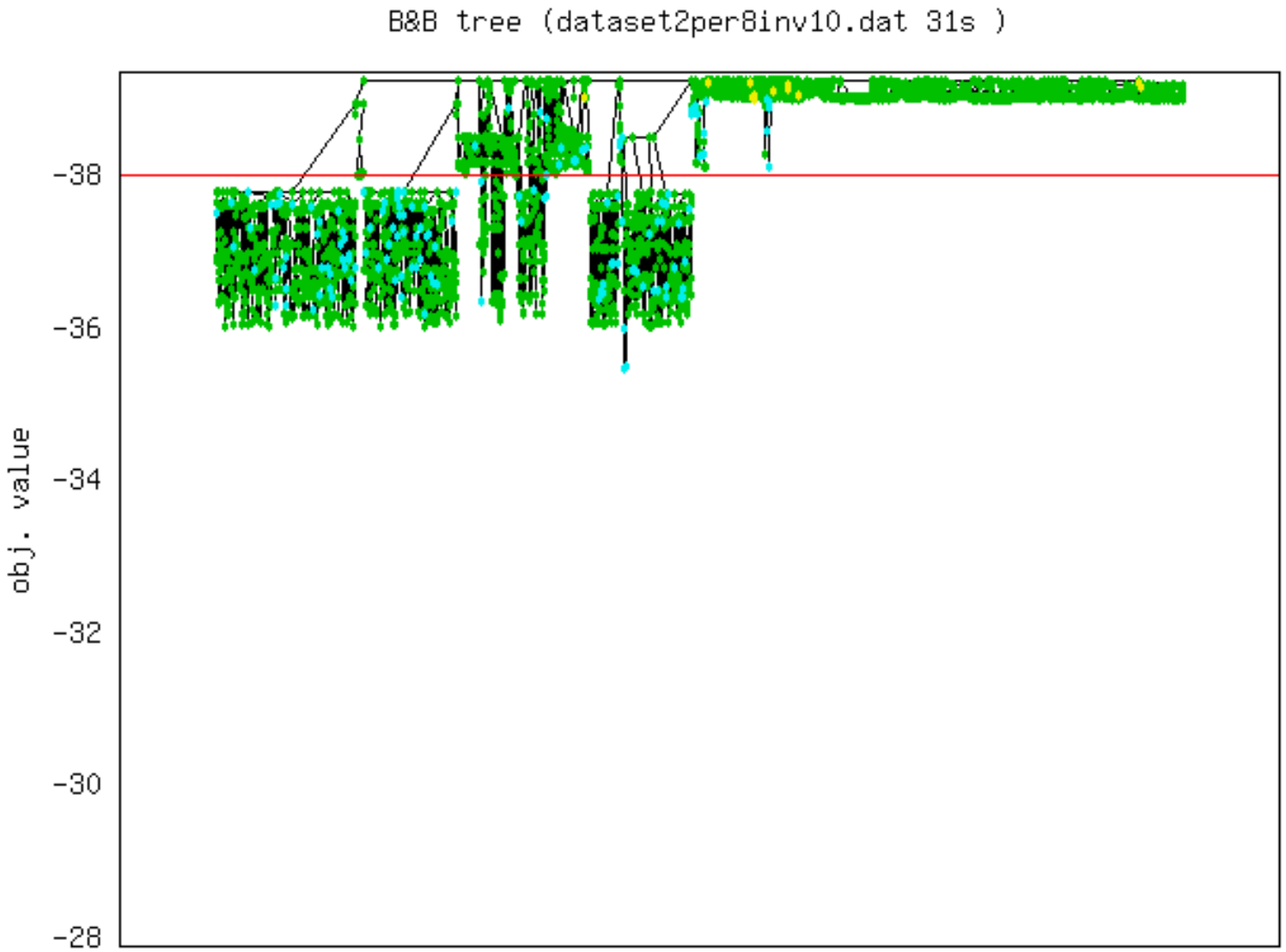

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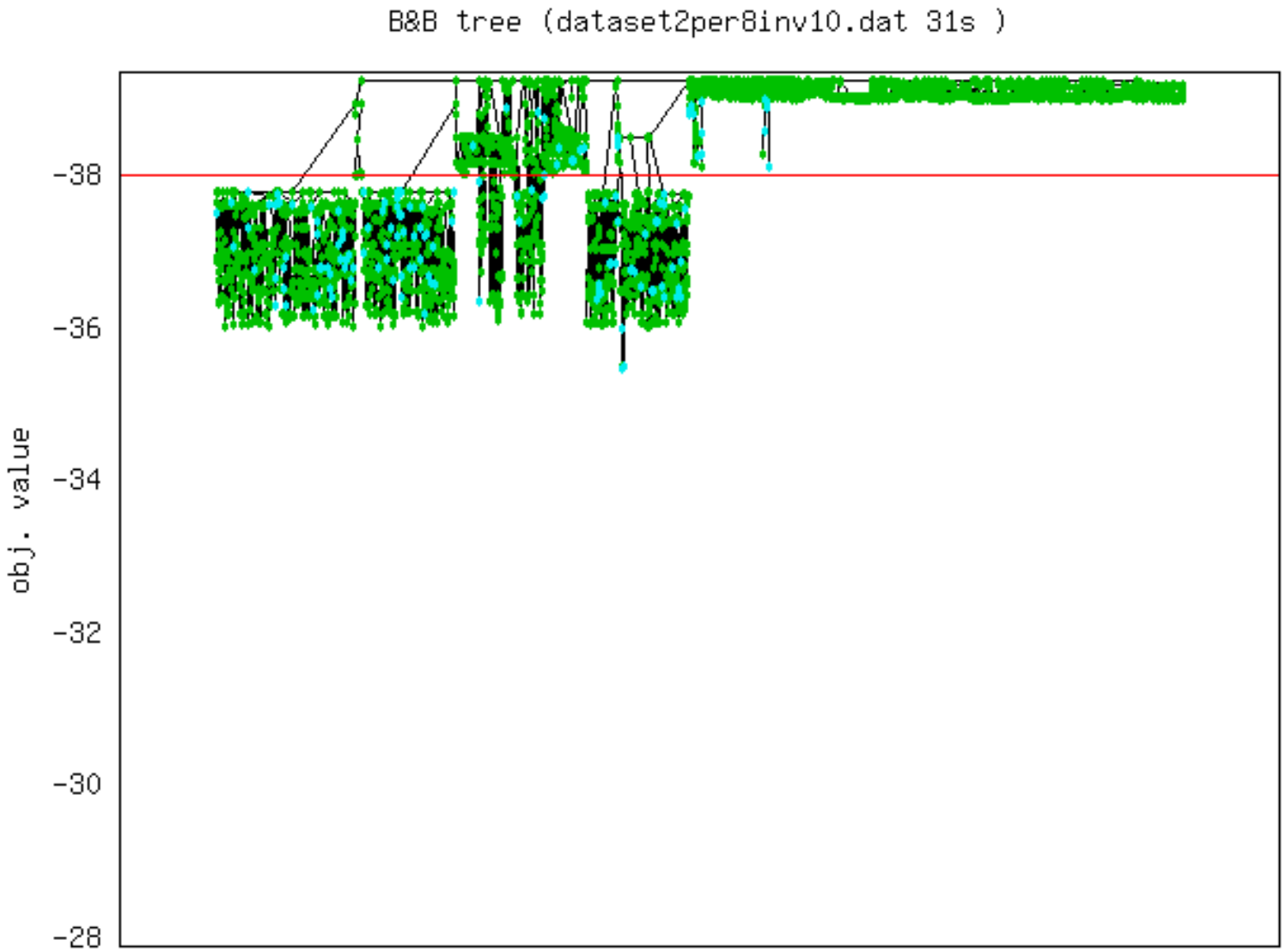

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