Cross-Layer Network Design Optimization

Mixed Integer Nonlinear Programming Models with Multiple Objectives

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Mobile Ad Hoc Wireless Networks

We consider the design of *mobile ad hoc wireless networks* (MANETs)

- autonomous mobile nodes
- dynamic wireless links and network topology
- self-organizing

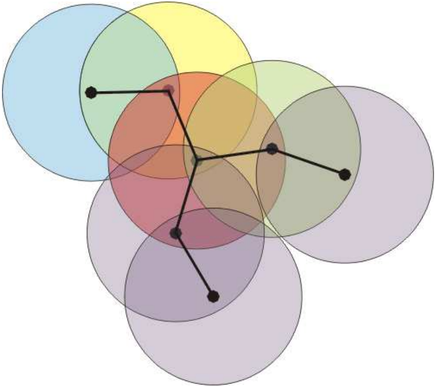


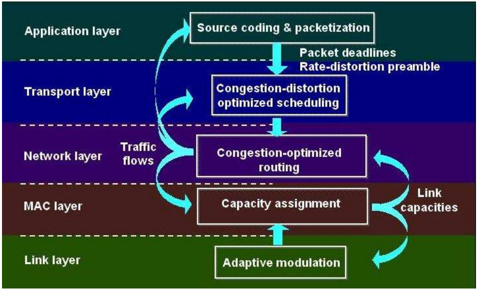
Figure: A representation of a MANET. The circles represent the transmission area for nodes.

MANET Structure

- We study MANETs whose nodes are equipped with *cognitive transmitters*.
- Allows dynamic adjustment of transmission power and constellation size

MANETs consist of five layers:

- Application Layer
- Transport Layer
- Network Layer
- Medium Access Control (MAC) Layer
- Physical/Link Layer



We present an optimization model for the bottom three layers.

Cross Layer Network Design

The objectives of the network design are:

- Minimization of utilized transmission power in the physical layer
- Maximization of total throughput

Traditionally, this design problem is modeled as a standard MINLP

- Maximization of capacity
- Optimization is done across all three layers (Fridman et al., 2008)
- Provides a better solution than layer-by-layer optimization (typically done in wired network design)
- Does not optimize with respect to transmission power

We consider this problem as a MINLP, with *multiple* objectives.

Physical Layer

This layer represents the actual physical nodes (i.e. foot soldiers) and controls

- data transmission and
- constellation size.

Let

$$
\begin{aligned}
N &:= \text{the set of mobile nodes} \\
p_i^t &:= \text{the power of node } i \in N \text{ at time } t \\
m_i^t &:= \text{the constellation size of node } i \text{ at time } t \\
\tau^t &:= \text{the set of nodes transmitting at time } t
\end{aligned}
$$

Both constellation size and power must lie in a finite discrete set. Denote these sets $\mathcal{M}$ and $\mathcal{P}$, respectively.

Physical Layer

The *Signal to Interference and Noise Ratio* (SINR) for node j, when listening to node i, is given by

$$SINR_j^t = \frac{p_i^t d_{ij}^{-\alpha}}{\sum_{k \in \tau^t \setminus \{i\}} p_j^t d_{kj}^{-\alpha} + \sigma^2}, \qquad (1)$$

where

$$\begin{aligned} d_{ij} &:= \text{the distance between node } i \text{ and node } j \\ \alpha > 2 &:= \text{the path-loss constant} \\ \sigma^2 &:= \text{the additive Gaussian noise to which the channel is subject} \end{aligned}$$

We define the *Bit Error Rate* (BER) for each receiver:

$$BER_j^t = 2Q\left(\sqrt{2SINR_j^t}\sin\frac{\pi}{m_i^t}\right). \qquad (2)$$

where, Q is the Q *Function*, $Q(Z) = P\{Z > z\}$ for $Z \sim N(0,1)$. The maximum allowable BER on any link is given by β.

Transmission Power Versus Constellation Size

Cognitive radios can change both transmission power and constellation size.

- Increasing Constellation Size m_i . . .
  - Increases the capacity of links emanating from node i
  - Increases the chance of symbol decoding error (BER)
  - Has no effect on the neighboring nodes (other than link capacity)
- Increasing Transmission Power p_i . . .
  - Increases the ratio of ratio of signal to noise (SINR) for transmitter i
  - Decreases the SINR for all other receivers

Thus, these two functions can be used in a complementary manner to improve network performance.

MAC Layer

This layer determines the optimal scheduling for data transmission.

- We assume a slotted protocol.
- At any time, a node can be transmitting, receiving, or idle.

Let

$$S := \text{the set of time slots}$$
$$s_i := \text{the maximum number of time slots in which node } i \text{ can transmit}$$

A schedule is given by $|S|$ different $N \times N$ matrices, $B^1, \ldots, B^s$, where

$$B^t_{ij} = \begin{cases} 1 & \text{if node } i \in N \text{ transmits to node } j \in N \text{ in slot } t \in S \\ 0 & \text{otherwise.} \end{cases}$$

and we require that if $B^t_{ij} = 1$, then $BER^t_j \leq \beta$.

MAC Layer

For each transmitter, we define the nominal capacity:

$$c_i = \sum_{t \in S} \log_2 m_i^t.$$

But, node i can only transmit in s_i time slots, so the *effective capacity* of link (i, j) is

$$\tilde{c}_{ij} = \frac{s_i}{|S|} \sum_{t \in S} \log_2 m_i^t. \tag{3}$$

Each schedule yields a *capacity graph* $G = (V, E)$, where

$$V = \{i \in N \mid \tilde{c}_{ij} > 0, \text{for some } j \in N \setminus \{i\}\}$$
$$E = \{(i, j) \in V \times V \mid B_{ij}^t = 1, \text{for some } t \in S\}$$

and edge (i, j) is assigned capacity $\tilde{c}_{ij}$.

Note, B_{ij}^t is not a variable in this formulation (see e.g. Fridman et al. (2008)).

Network Layer

Responsible for managing the transmission of data packets between a specified source and destination.

- Several commodities flow across the network layer
- each has a different source and sink

Let

$$\begin{aligned} K &:= \text{the set of commodities} \\ \sigma_k \in N &:= \text{the sink node for commodity } k \\ \delta_k \in N &:= \text{the sink node for commodity } k \\ x_e^k \geq 0 &:= \text{the flow of commodity } k \in K \text{ over edge } e \in E \end{aligned}$$

The sum of the flow over all edges which terminate at the sink node for commodity k,

$$\sum_{i \in N} x_{i,t_k}^k = f_k,$$

yields the total *throughput* f_k of the commodity.

Multicommodity Maximum Throughput Problem

Fridman et al. (2008) describe a MINLP to determine the maximum commodity flow:

$$
\begin{aligned}
\max \quad & \sum_{k=1}^{K} \sum_{i \in N} x^k_{i,\delta_k} \\
\text{subject to} \quad & \sum_{k \in K} x^k_{ij} \leq \frac{s_i}{|S|} \sum_{t \in S} \log_2 m^t_i, \quad \forall i,j \in N \\
& \sum_{j \in N} x^k_{ji} = \sum_{j \in N} x^k_{ij}, \quad \forall i \in N \setminus \{\sigma_k, \delta_k\}, k \in K \\
& B^t_{ij} \cdot \mathrm{BER}^t_j \leq \beta, \quad \forall i,j \in N, t \in S \\
& p \in \mathcal{P}, m \in \mathcal{M}, x \geq 0,
\end{aligned}
\tag{MMTP}
$$

Dualing Objectives

There are several other performance measures of importance

- The total network capacity

$$\sum_{i,j \in N} \tilde{c}_{ij} \tag{4}$$

- The total transmission power

$$\sum_{t \in S} \sum_{i \in N} p_i^t \tag{5}$$

- Sum of the node constellation sizes

$$\sum_{t \in S} \sum_{i \in N} m_i^t \tag{6}$$

In the remainder of this talk, we demonstrate how to incorporate objective (5) into the optimization model.

Biobjective Integer Programming Model

Biobjective programming provides one way to do this.

Biobjective Programming

- Generalization of mathematical programming with multiple, possibly conflicting, objectives
- Enables us to study the tradeoffs between two conflicting objectives by a single DM

The general biobjective program (BP) is given by:

Biobjective Program

$$\text{vmax}_{x \in X}[f_1(x), f_2(x)]. \tag{BP}$$

Biobjective Integer Programming Model

Applying this framework to our design problem yields the *biobjective design problem* (BODP):

$$
\begin{aligned}
\text{vmax} \quad & \left[\sum_{k=1}^{K} \sum_{i \in N} x^k_{i,\delta_k}, \sum_{t \in S} \sum_{i \in N} p^t_i \right] \\
\text{subject to} \quad & \sum_{k \in K} x^k_{ij} \le \frac{s_i}{|S|} \sum_{t \in S} \log_2 m^t_i, \quad \forall i, j \in N \\
& \sum_{j \in N} x^k_{ji} = \sum_{j \in N} x^k_{ij}, \quad \forall i \in N \setminus \{\sigma_k, \delta_k\}, k \in K, \\
& B^t_{ij} \cdot \text{BER}^t_j \le \beta, \quad \forall i, j \in N, t \in S \\
& p \in \mathcal{P}, m \in \mathcal{M}, x \ge 0,
\end{aligned}
\tag{BODP}
$$

for fixed schedule B.

Solution Methods for (BODP)

We are looking for *efficient* solutions to (BP)

Definition

A feasible solution $\hat{x} \in X$ is *efficient* if there is no other $x \in X$ such that

$$\begin{aligned} f_i(x) &\geq f_i(\hat{x}), \text{ for } i = 1, 2 \text{ and} \\ f_i(x) &> f_i(\hat{x}) \text{ for either } i = 1 \text{ or } i = 2. \end{aligned}$$

We can convert (BP) into a single-objective problem with a nonnegative linear combination of the objective functions (Geoffrion, 1968):

$$\max_{x \in X} \quad \omega f_1(x) + (1 - \omega) f_2(x). \tag{7}$$

for $0 \leq \omega \leq 1$. It is well-known that

- Solutions to (7) are efficient for any $0 \leq \omega \leq 1$.
- Not all efficient outcomes are solutions to (7) for some $0 \leq \omega \leq 1$.

A Biobjective Algorithm

Let $P(\omega)$ denote the problem defined by (BODP) with the objective replaced by

$$f = \omega f_1 + (1-\omega) f_2 = \omega \sum_{k=1}^{K} f_k + (1-\omega) \sum_{t \in S} \sum_{i \in N} p_i^t .$$

Let $\pi = (p, m, x)$ and

$$y^\omega = f(\pi^\omega) = (f_1(\pi^\omega), f_2(\pi^\omega)).$$

Let

$$\omega_{pq} = \frac{y_2^* - y_2^q}{y_1^* - y_1^p + y_2^* - y_2^q}, \tag{8}$$

and M be the cardinality of the set of efficient solutions.

A Biobjective Algorithm

Weighted Sums Algorithm

Initialization Solve $P(1)$ and $P(0)$ to identify optimal outcomes y^1 and y^N, respectively, and the *ideal point* $y^* = (y^1, y^N)$. Set $\mathcal{I} = \{(y^1, y^N)\}$ and $\mathcal{L} = \{(\pi^1, y^1,), (\pi^N, y^N)\}$.

Iteration While $\mathcal{I} \neq \emptyset$ do:

1. Remove any (y^p, y^q) from I.
2. Compute ω_{pq} as in (8) and solve $P(\omega_{pq})$. If the outcome is y^p or y^q, then y^p and y^q are adjacent in the list $(y^1, y^2, \ldots, y^N)$.
3. Otherwise, a new outcome y^r is generated. Add (π^r, y^r) to L. Add (y^p, y^r) and (y^r, y^q) to $\mathcal{I}$.

- This algorithm can be used to a subset of efficient solutions.
- For many applications, this is sufficient.
- To generate the entire set of efficient solutions, we can use the WCN algorithm of Ralphs et al. (2004).

Current Work

We have shown

- how to combine two previously separate models into one using multicriteria optimization
- a solution method that generates a large portion of the efficient set of solutions

Currently, we are working on

- development of a linear model (with Pietro Belotti)
  - standard linearization techniques
  - simplifying assumptions
  - approximations
- determining the implications of our simplifying assumptions
- approaches to including the variable B_{ij}^{t} in the optimization models
- computational experimentation
- other methods for incorporating multiple objectives in this network design model

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