A Framework for Decomposition in Integer Programming

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Outline

The Decomposition Principle in Integer Programming

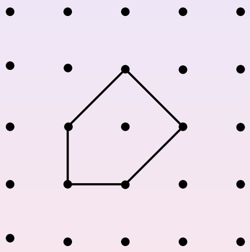
Basic Idea: By leveraging our ability to solve the optimization/separation problem for a relaxation, we can improve the bound yielded by the LP relaxation.

$$z_{IP} = \min_{x \in \mathbb{Z}^n} \{c^\top x \mid A'x \geq b', A''x \geq b''\}$$

$$z_{LP} = \min_{x \in \mathbb{R}^n} \{c^\top x \mid A'x \geq b', A''x \geq b''\}$$

$$z_D = \min_{x \in \mathcal{P}'} \{c^\top x \mid A''x \geq b''\}$$

$$z_{IP} \geq z_D \geq z_{LP}$$



$\mathcal{P} = \text{conv}\{x \in \mathbb{Z}^n \mid A'x \geq b', A''x \geq b''\}$

Assumptions:

- $OPT(c, \mathcal{P})$ and $SEP(x, \mathcal{P})$ are *"hard"*.
- $OPT(c, \mathcal{P}')$ and $SEP(x, \mathcal{P}')$ are *"easy"*.
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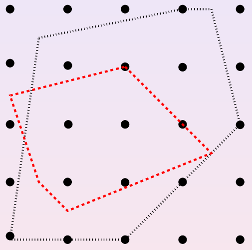

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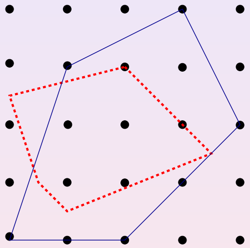

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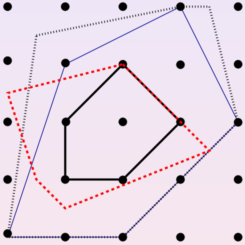

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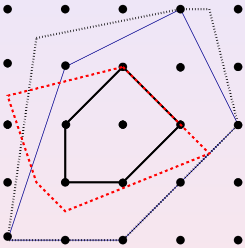

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Example - Traveling Salesman Problem

Classical Formulation

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\begin{array}{lll}
x(\delta(\{u\})) & = & 2 \quad\quad \forall u \in V \\
x(E(S)) & \leq & |S| - 1 \quad \forall S \subset V,\ 3 \leq |S| \leq |V| - 1 \\
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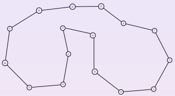

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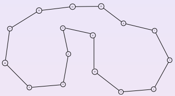

Two Relaxations

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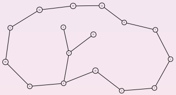

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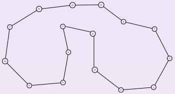

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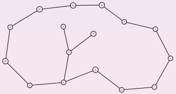

2-Matching

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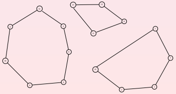
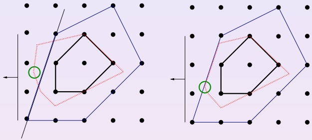


Traditional Decomposition Methods

The **Cutting Plane Method (CP)** iteratively builds an *outer* approximation of $\mathcal{P}'$ by solving a cutting plane generation subproblem.



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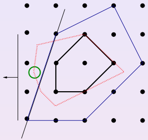
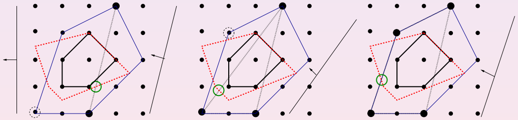


The **Dantzig-Wolfe Method (DW)** iteratively builds an *inner* approximation of $\mathcal{P}'$ by solving a column generation subproblem.



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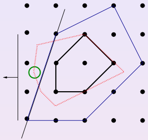

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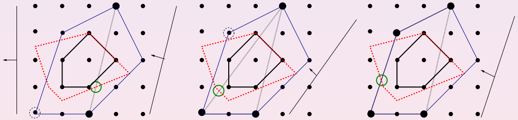
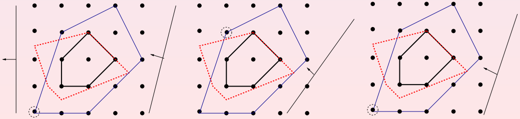


The **Lagrangian Method (LD)** iteratively solves a Lagrangian relaxation subproblem.



Common Threads

- The LP bound is obtained by optimizing over the intersection of two explicitly defined polyhedra.

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- Traditional decomposition-based bounding methods contain two primary steps
  - **Master Problem:** Update the primal/dual solution information.
  - **Subproblem:** Update the approximation of $\mathcal{P}'$: $SEP(x, \mathcal{P}')$ or $OPT(c, \mathcal{P}')$.
- Integrated decomposition methods further improve the bound by considering two implicitly defined polyhedra whose descriptions are iteratively refined.
  - Price and Cut (PC)
  - Relax and Cut (RC)
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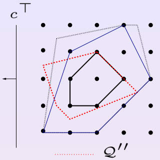

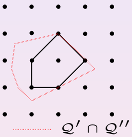

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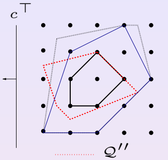

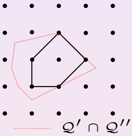

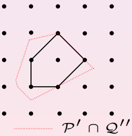

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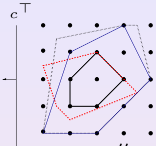

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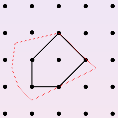
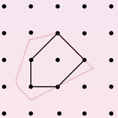


$\mathcal{Q}' \cap \mathcal{Q}''$



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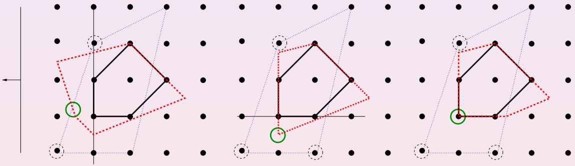
Price and Cut

Price and Cut: Use **DW** as the bounding method. If we let $\mathcal{F}' = \mathcal{P}' \cap \mathbb{Z}^n$, then

$$z_{DW} = \min_{\lambda \in \mathbb{R}_+^{\mathcal{F}'}} \{c^\top(\sum_{s \in \mathcal{F}'} s\lambda_s) : A''(\sum_{s \in \mathcal{F}'} s\lambda_s) \geq b'', \sum_{s \in \mathcal{F}'} \lambda_s = 1\}$$

- As in the cutting plane method, separate $\hat{x} = \sum_{s \in \mathcal{F}'} s\hat{\lambda}_s$ from $\mathcal{P}$ and add cuts to $[A'', b'']$.
- **Advantage**: Cut generation takes place in the space of the compact formulation (the original space), maintaining the structure of the column generation subproblem.

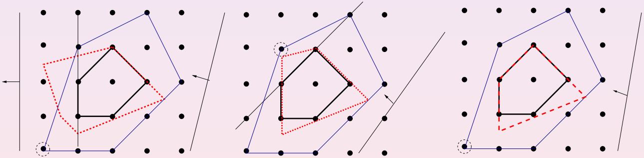


Relax and Cut

Relax and Cut: Use **LD** as the bounding method.

$$z_{LD} = \max_{u \in \mathbb{R}^n_+} \min_{s \in \mathcal{F}'} \{(c^\top - u^\top A'')s + u^\top b''\}$$

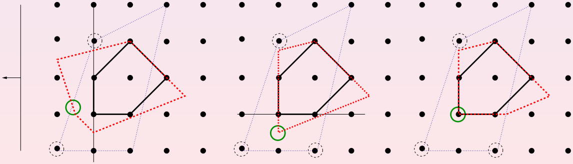
- In each iteration, separate $\hat{s} \in \operatorname{argmin}_{s \in \mathcal{F}'} \{(c^\top - u^\top A'')s + u^\top b''\}$, a solution to the Lagrangian relaxation.
- **Advantage**: It is often much easier to separate a member of $\mathcal{F}'$ from $\mathcal{P}$ than an arbitrary real vector, such as $\hat{x}$.



Decompose and Cut

Decompose and Cut: As in price and cut, use **DW** as the bounding method, but use the decomposition obtained in each iteration to generate improving inequalities as in **RC**.

- Rather than (or in addition to) separating $\hat{x}$, separate each member of $D = \{s \in \mathcal{F}' \mid \hat{\lambda}_s > 0\}$.
- As with **RC**, it is often much easier to separate a member of $\mathcal{F}'$ from $\mathcal{P}$ than an arbitrary real vector, such as $\hat{x}$.
- **RC** only gives us one member of $\mathcal{F}'$ to separate, while **PC** gives us a set, one of which must be violated by any inequality violated by $\hat{x}$.
- We can also use CP and decompose the fractional solution obtained in each iteration into a convex combination of members of $\mathcal{F}'$ and apply the same technique.
- In case this decomposition fails, we still get a Farkas cut for free.

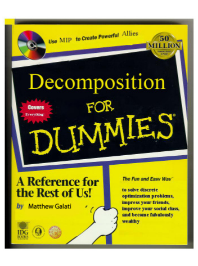


DECOMP Framework: Motivation

DECOMP Framework

DECOMP is a software framework that provides a virtual sandbox for testing and comparing various decomposition-based bounding methods.

- It's very difficult to compare the variants discussed here in a controlled way.
- The method for separation/optimization over $\mathcal{P}'$ is the primary application-dependent component of any of these algorithms.
- DECOMP abstracts the common, generic elements of these methods.
  - Key: The user defines application-specific components in the space of the compact formulation.
  - The framework takes care of reformulation and implementation for all variants described here.



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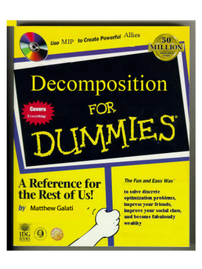

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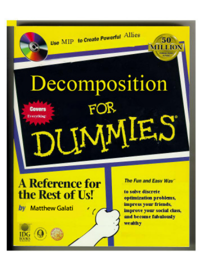

DECOMP Framework: Implementation

COmputational INfrastructure for Operations Research

- **DECOMP** was built around data structures and interfaces provided by COIN-OR.
- The **DECOMP** framework, written in C++, is accessed through two user interfaces:
  - Applications Interface: `DecompApp`
  - Algorithms Interface: `DecompAlgo`
- **DECOMP** provides the bounding method for branch and bound.
- **ALPS** (Abstract Library for Parallel Search) provides the framework for parallel tree search.
  - `AlpsDecompModel : public AlpsModel`
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DECOMP Features

- One interface to all default algorithms: CP/DC, DW, LD, PC, RC.
- Automatic reformulation allows users to deal with variables and constraints in the original space.
- Built on top of the OSI interface, so easy to swap solvers (simplex to interior point).
- Can utilize CGL cuts in all algorithms (since cut generation is always done in the original space).
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- The base class `DecompApp` provides an interface for the user to define the application-specific components of their algorithm.
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- `DecompApp::APPcreateModel()`. Define $[A'', b'']$ and $[A', b']$ (optional).
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- `DecompApp::isUserFeasible()`. Does x^* define a feasible solution?
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- `DecompApp::APPsolveRelaxed()`. Provide a subroutine for $OPT(c, \mathcal{P}')$.
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  - Alternative methods for solving the master LP in DW, such as **interior point methods** or **ACCPM**.
  - Add stabilization to the dual updates in LD, as in **bundle methods**.
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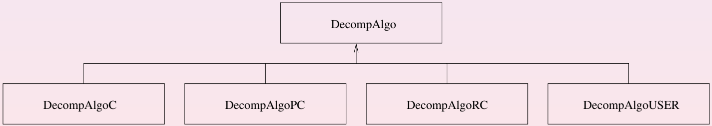

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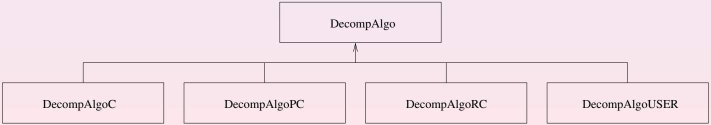

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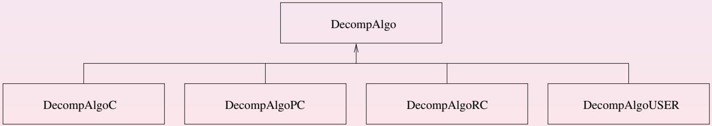

DECOMP - TSP Example

TSP_Main

```
int main(int argc, char ** argv){
   //create the utility class for parsing parameters
   UtilParameters utilParam(argc, argv);

   //create the user application (a DecompApp)
   TSP_DecompApp tsp(utilParam);
   tsp.createModel();

   //create the algorithm(s) (a DecompAlgo)
   DecompAlgoC  * cut       = new DecompAlgoC(&tsp, &utilParam);
   DecompAlgoPC * pcOneTree  = new DecompAlgoPC(&tsp, &utilParam,
                                  TSP_DecompApp::MODEL_ONETREE);
   DecompAlgoPC * pcTwoMatch = new DecompAlgoPC(&tsp, &utilParam,
                                  TSP_DecompApp::MODEL_TWOMATCH);
   DecompAlgoRC * rcOneTree  = new DecompAlgoRC(&tsp, &utilParam,
                                  TSP_DecompApp::MODEL_ONETREE);
   DecompAlgoRC * rcTwoMatch = new DecompAlgoRC(&tsp, &utilParam,
                                  TSP_DecompApp::MODEL_TWOMATCH);

   //create the driver AlpsDecomp model
   AlpsDecompModel alpsModel(utilParam);

   //install the algorithms
   //alpsModel.addDecompAlgo(cut);
   alpsModel.addDecompAlgo(pcOneTree);

   //solve
   alpsModel.solve();
}
```

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  - $\mathcal{P}' \supset \mathcal{P}$ may have a *large* description.
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