The COIN-OR High-Performance Parallel Search Framework (CHiPPS)

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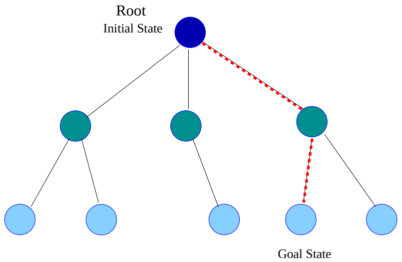
CPAIOR, June 15, 2010

Thanks: Work supported in part by the National Science Foundation and IBM

Outline

Tree Search Algorithms

- Tree search algorithms systematically search the nodes of an acyclic graph for certain *goal nodes*.



- Tree search algorithms have been applied in many areas such as
  - Constraint satisfaction,
  - Game search,
  - Constraint Programming, and
  - Mathematical programming.

Elements of Tree Search Algorithms

- A generic tree search algorithm consists of the following elements:

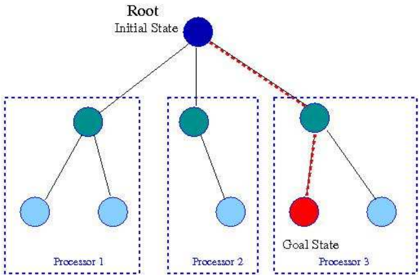
Generic Tree Search Algorithm

- Processing method: Is this a goal node?
- Fathoming rule: Can node can be fathomed?
- Branching method: What are the successors of this node?
- Search strategy: What should we work on next?

- The algorithm consists of choosing a candidate node, processing it, and either fathoming or branching.
- During the course of the search, various information (*knowledge*) is generated and can be used to guide the search.

Parallelizing Tree Search Algorithms

- In general, the search tree can be very large.
- The generic algorithm appears very easy to parallelize, however.



- The appearance is deceiving, as the search graph is not generally known a priori and naïve parallelization strategies are not generally effective.

Parallel Overhead

- The amount of *parallel overhead* determines the scalability.

Major Components of Parallel Overhead in Tree Search

- Communication Overhead (cost of sharing knowledge)
- Idle Time
  - Handshaking/Synchronization (cost of sharing knowledge)
  - Task Starvation (cost of *not* sharing knowledge)
  - Ramp Up Time
  - Ramp Down Time
- Performance of Redundant Work (cost of *not* sharing knowledge)

- Knowledge sharing is the main driver of efficiency.
- This breakdown highlights the tradeoff between centralized and decentralized knowledge storage and decision-making.

Previous Work

Previous tree search codes:

- Game tree search: ZUGZWANG and APHID
- Constraint programming: ECLiPSe, G12, etc.
- Optimization:
  - Commercial: CPLEX, Lindo, Mosek, SAS/OR, Xpress, etc.
  - Serial: ABACUS, bc-opt, COIN/CBC, GLPK, MINTO, SCIP, etc.
  - Parallel: COIN/BCP, FATCOP, PARINO, PICO, SYMPHONY, etc.

However, to our knowledge:

- Few studies of general tree search algorithms, and only one framework (PIGSeL).
- No study has emphasized scalability for *data-intensive* applications.
- Many packages are not open source or not easy to specialize for particular problem classes.

The COIN-OR High-Performance Parallel Search Framework

- CHiPPS has been under development since 2000 in partnership with IBM, NSF, and the COIN-OR Foundation.
- The broad goal was to develop a successor to SYMPHONY and BCP, two previous parallel MIP solvers.
- It consists of a hierarchy of C++ class libraries for implementing general parallel tree search algorithms.
- It is an open source project hosted by COIN-OR.
- Design goals
  - Scalability
  - Usability

COIN-OR

- The software discussed in this talk is available for free download from the Computational Infrastructure for Operations Research Web site

  `projects.coin-or.org/CHiPPS`
- The COIN-OR Foundation (`www.coin-or.org`)
  - An non-profit educational foundation promoting the development and use of interoperable, open-source software for operations research.
  - A consortium of researchers in both industry and academia dedicated to improving the state of computational research in OR.
- The COIN-OR Repository
  - A library of interoperable software tools for building optimization codes, as well as some stand-alone packages.
  - A venue for peer review of OR software tools.
  - A development platform for open source projects, including an SVN repository, project management tools, etc.

CHiPPS: Design Goals

- Intuitive object-oriented class structure.
  - `AlpsModel`
  - `AlpsTreeNode`
  - `AlpsNodeDesc`
  - `AlpsSolution`
  - `AlpsParameterSet`
- Minimal algorithmic assumptions in the base class.
  - Support for a wide range of problem classes and algorithms.
  - Support for constraint programming.
- Easy for user to develop a custom solver.
- Design for *parallel scalability*, but operate effective in a sequential environment.
- Explicit support for *memory compression* techniques (packing/differencing) important for implementing optimization algorithms.

CHiPPS: Overview of Features

- The design is based on a very general concept of *knowledge*.
- Knowledge is shared asynchronously through *pools* and *brokers*.
- Management overhead is reduced with the *master-hub-worker* paradigm.
- Overhead is decreased using dynamic task granularity.
- Two static load balancing techniques are used.
- Three dynamic load balancing techniques are employed.
- Uses asynchronous messaging to the highest extent possible.
- A scheduler on each process manages tasks like
  - node processing,
  - load balancing,
  - update search states, and
  - termination checking, etc.

CHiPPS Library Hierarchy

ALPS (Abstract Library for Parallel Search)

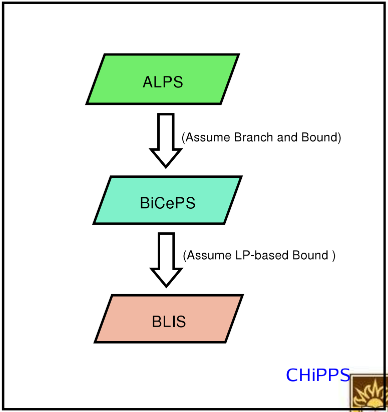
- search-handling layer
- prioritizes based on quality

BiCePS (Branch, Constrain, and Price Software)

- data-handling layer for relaxation-based optimization
- variables and constraints
- iterative bounding procedure

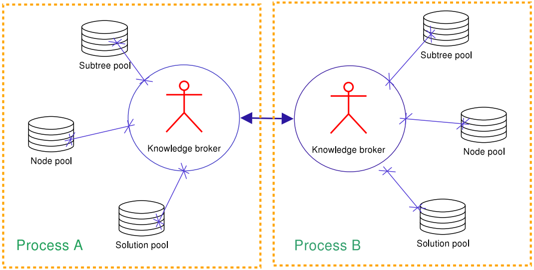
BLIS (BiCePS Linear Integer Solver)

- concretization of BiCePS
- linear constraints and objective

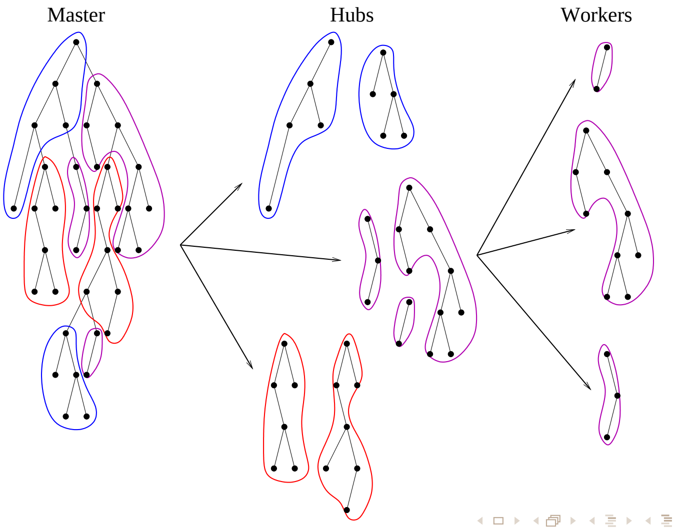


ALPS: Knowledge Sharing

- All knowledge to be shared is stored in classes derived from a single base class and has an associated *encoded form*.
- Encoded form is used for identification, storage, and communication.
- Knowledge is maintained by one or more *knowledge pools*.
- The knowledge pools communicate through *knowledge brokers*.

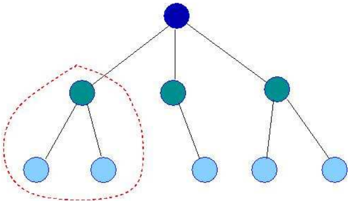


ALPS: Master-Hub-Worker Paradigm



ALPS: Task Granularity

- Task granularity is a crucial element of parallel efficiency.
- In CHiPPS, each worker is capable of exploring an entire subtree autonomously.
- By stopping the search prematurely, the task granularity can be adjusted dynamically.
- As granularity increases, communication overhead decreases, but other sources of overhead increase.



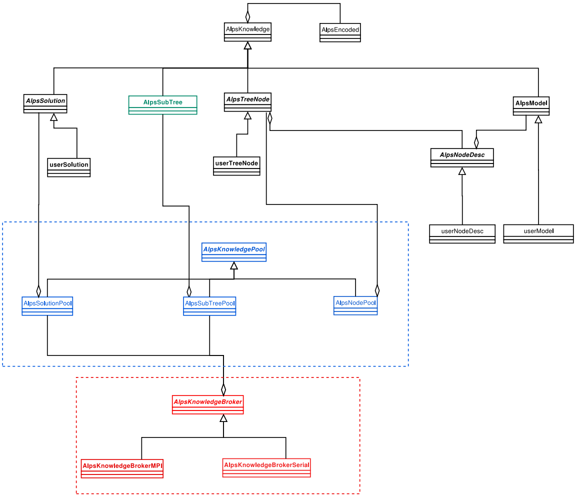
ALPS: Synchronization

- As much as possible, we have eliminated handshaking and synchronization.
- A knowledge broker can work completely asynchronously, as long as its local node pool is not empty.
- This asynchronism can result in an increase in the performance of redundant work.
- To overcome this, we need good load balancing.

ALPS: Load Balancing

- Static
  - Performed at startup
  - Two types
    - Two-level root initialization.
    - Spiral initialization.
- Dynamic
  - Performed periodically and as needed.
  - Balance by quantity and quality.
  - Keep subtrees together to enable differencing.
  - Three types
    - Inter-cluster dynamic load balancing,
    - Intra-cluster dynamic load balancing, and
    - Worker-initiated dynamic load balancing.
  - Workers do not know each others' workloads.
  - Donors and receivers are matched at both the hub and master level.
  - Three schemes work together to ensure workload is balanced.

ALPS: Class Hierarchy



BiCePS: Basic Notions

- BiCePS introduces the notion of *variables* and *constraints* (generically referred to as *objects*).
- Objects are abstract entities with *values* and *bounds*.
- They are used to build mathematical programming *models*.
- Search tree nodes consist of subproblems described by sets of variables and constraints.
- Key assumptions
  - Algorithm is relaxation-based branch-and-bound.
  - Bounding is an iterative procedure involving generation of variables and constraints.

BiCePS: Differencing Scheme

- Descriptions of search tree nodes can be extremely large.
- For this reason, subtrees are stored using a *differencing scheme*.
- Nodes are described using differences from the parent is this description is smaller.
- Again, there is a tradeoff between memory savings and additional computation.
- This approach requires keeping subtrees whole as much as possible.
- This impacts load balancing significantly.

BLIS: A Generic Solver for MILP

MILP

$$\min \quad c^T x \tag{1}$$
$$\text{s.t.} \quad Ax \leq b \tag{2}$$
$$x_i \in \mathbb{Z} \quad \forall\, i \in I \tag{3}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $I \subseteq \{1, 2, \ldots, n\}$.

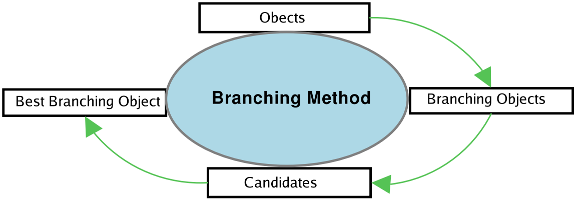
Basic Algorithmic Elements

- Search strategy.
- Branching scheme.
- Object generators.
- Heuristics.

BLIS: Branching Scheme

BLIS Branching scheme comprises three components:

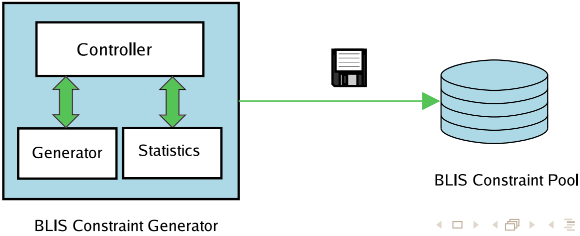
- Branching object: has feasible region and can be branched on.
- Branching candidate:
  - created from objects not in their feasible regions or
  - contains instructions for how to conduct branching.
- Branching method:
  - specifies how to create a set of branching candidates.
  - has the method to compare objects and choose the best one.



BLIS: Constraint Generators

BLIS constraint generator:

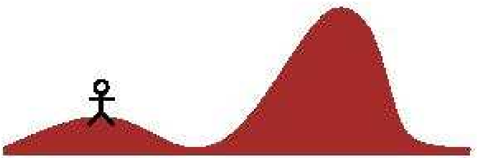
- provides an interface between BLIS and the algorithms in COIN/Cgl.
- provides a base class for deriving specific generators.
- has the ability to specify rules to control generator:
  - where to call: root, leaf?
  - how many to generate?
  - when to activate or disable?
- contains the statistics to guide generating.



BLIS: Heuristics

BLIS primal heuristic:

- defines the functionality to heuristically search for solutions.
- has the ability to specify rules to control heuristics.
  - where to call: before root, after bounding, at solution?
  - how often to call?
  - when to activate or disable?
- collects statistics to guide the heuristic.
- provides a base class for deriving specific heuristics.



Implementing a Knapsack Solver

- As a demonstration application, we implemented a solver for the knapsack problem using ALPS.
- The solver uses the closed form solution of the LP relaxation as a bound.
- Branching is on the fractional variable.
- Implementation consists of deriving a few classes to specify the algorithm.
  - `KnapModel`
  - `KnapTreeNode`
  - `KnapSolution`
  - `KnapParams`
- Once the classes have been implemented, the user writes a `main` function.
- The only difference between parallel and serial code is the knowledge broker class that is used.

Sample main() Function

```
int main(int argc, char* argv[])
{
    KnapModel model;
#if defined(SERIAL)
    AlpsKnowledgeBrokerSerial knap(argc, argv, model);
#elif defined(PARALLEL_MPI)
    AlpsKnowledgeBrokerMPI knap(argc, argv, model);
#endif
    knap.registerClass("MODEL", new KnapModel);
    knap.registerClass("SOLUTION", new KnapSolution);
    knap.registerClass("NODE", new KnapTreeNode);
    knap.search();
    knap.printResult();
    return 0;
}
```

The Vehicle Routing Problem

The VRP is a combinatorial problem whose *ground set* is the edges of a graph $G(V, E)$. Notation:

- V is the set of customers and the depot (0).
- d is a vector of the customer demands.
- k is the number of routes.
- C is the capacity of a truck.

A feasible solution is composed of:

- a partition $\{R_1, \ldots, R_k\}$ of V such that $\sum_{j \in R_i} d_j \leq C,\ 1 \leq i \leq k$;
- a permutation σ_i of $R_i \cup \{0\}$ specifying the order of the customers on route i.

Standard IP Formulation for the VRP

VRP Formulation

$$\begin{array}{rcll} \sum_{j=1}^{n} x_{0j} & = & 2k & \\ \sum_{j=1}^{n} x_{ij} & = & 2 & \forall i \in V \setminus \{0\} \\ \sum_{\substack{i \in S \\ j \notin S}} x_{ij} & \geq & 2b(S) & \forall S \subset V \setminus \{0\},\ |S| > 1. \end{array}$$

- $b(S)$ = lower bound on the number of trucks required to service S (normally $\lceil (\sum_{i \in S} d_i)/C \rceil$).
- The number of constraints in this formulation is exponential.
- We must therefore generate the constraints dynamically.
- A solver can be implemented in BLIS by deriving just a few classes.

Implementing the VRP Solver

- The algorithm is defined by deriving the following classes.
  - `VrpModel`
  - `VrpSolution`
  - `VrpCutGenerator`
  - `VrpHeuristic`
  - `VrpVariable`
  - `VrpsParams`
- Once the classes have been implemented, the user writes a `main` function as before.

Computational Results: Platforms

Clemson Cluster

Machine:	Beowulf cluster with 52 nodes
Node:	dual core PPC, speed 1654 MHz
Memory:	4G RAM each node
Operating System:	Linux
Message Passing:	MPICH

SDSC Blue Gene System

Machine:	IBM Blue Gene with 3,072 compute nodes
Node:	dual processor, speed 700 MHz
Memory:	512 MB RAM each node
Operating System:	Linux
Message Passing:	MPICH

KNAP Scalability on Difficult Instances

- Tested the 26 instances on the SDSC Blue Gene.
- The default algorithm was used except that
  - the static load balancing scheme is the two-level root initialization,
  - the number of nodes generated by the master varies from 10000 to 30000 depends on individual instance,
  - the number of nodes generated by a hub varies from 10000 to 20000 depends on individual instance,
  - the size a unit work is 300 nodes; and
  - multiple hubs were used.

P	Node	Ramp-up	Idle	Ramp-down	Wallclock	Eff
64	14733745123	0.69%	4.78%	2.65%	6296.49	1.00
128	14776745744	1.37%	6.57%	5.26%	3290.56	0.95
256	14039728320	2.50%	7.14%	9.97%	1672.85	0.94
512	13533948496	7.38%	4.30%	14.83%	877.54	0.90
1024	13596979694	8.33%	3.41%	16.14%	469.78	0.84
2048	14045428590	9.59%	3.54%	22.00%	256.22	0.77

- KNAP scales well even when using several thousand processors.
- Ramp-up and ramp-down overhead inevitably increase.

BLIS Scalability for Moderately Difficult Instances

- Selected 18 MILP instances from Lehigh/CORAL, MIPLIB 3.0, MIPLIB 2003, BCOL, and markshare.
- Tested on the Clemson cluster.

Instance	Nodes	Ramp -up	Idle	Ramp -down	Comm Overhead	Wallclock	Eff
1 P Per Node	11809956	– –	– –	– –	– –	33820.53 0.00286	1.00
4P Per Node	11069710	0.03% 0.03%	4.62% 4.66%	0.02% 0.00%	16.33% 16.34%	10698.69 0.00386	0.79
8P Per Node	11547210	0.11% 0.10%	4.53% 4.52%	0.41% 0.53%	16.95% 16.95%	5428.47 0.00376	0.78
16P Per Node	12082266	0.33% 0.27%	5.61% 5.66%	1.60% 1.62%	17.46% 17.45%	2803.84 0.00371	0.75
32P Per Node	12411902	1.15% 1.22%	8.69% 8.78%	2.95% 2.93%	21.21% 21.07%	1591.22 0.00410	0.66
64P Per Node	14616292	1.33% 1.38%	11.40% 11.46%	6.70% 6.72%	34.57% 34.44%	1155.31 0.00506	0.46

BLIS Scalability for Very Difficult Instances

- Tests on Clemson's palmetto cluster (60 on the Top 500 list, 11/2008, Linux, MPICH, 8-core 2.33GHz Xeon/Opteron mix, 12-16GB RAM).
- Tests use one processor per node.

Raw Computational Results

Name	256	128	64	1
mcf2	926	1373	2091	43059
neos-1126860	2184	1830	2540	39856
neos-1122047	1676	1125	1532	NS
neos-1413153	4230	3500	2990	20980
neos-1456979		78.06%	NS	NS
neos-1461051	396	1082	536	NS
neos-1599274		1500	8108	9075
neos-548047		137.29%	376.48%	482%
neos-570431	1034	1255	1308	21873
neos-611838	712	956	886	8005
neos-612143	565	1716	1315	4837
neos-693347		1.28%	1.70%	NS
neos-912015	538	438	275	10674
neos-933364		6.67%	6.79%	11.80%
neos-933815		6.54%	8.77%	32.85%
neos-934184		6.67%	6.76%	9.15%
neos18		30.78%	30.78%	79344

Speedups

Name	256	128	64
mcf2	46.5	31.36	20.59
neos-1126860	18.25	21.78	15.69
neos-1413153	4.96	5.99	7.02
neos-1599274		6.05	1.12
neos-570431	21.15	17.43	16.72
neos-611838	11.24	8.37	9.03
neos-612143	8.56	2.82	3.68
neos-912015	19.84	24.37	38.81

Efficiency

Name	256	128	64
mcf2	0.18	0.25	0.32
neos-1126860	0.07	0.17	0.25
neos-1413153	0.02	0.05	0.11
neos-1599274		0.05	0.02
neos-570431	0.08	0.14	0.26
neos-611838	0.04	0.07	0.14
neos-612143	0.03	0.02	0.06
neos-912015	0.08	0.19	0.61

ALPS

- Our methods implemented in ALPS seem effective in improving scalibility.
- The framework is useful for implementing serial or parallel tree search applications.
- The KNAP application achieves very good scalability.
- There is still much room for improvement
  - load balancing,
  - fault tolerance,
  - hybrid architectures,
  - grid enable.

BLIS

- The performance of BLIS in serial mode is favorable when compared to state of the art non-commercial solvers.
- The scalability for solving generic MILPs is highly dependent on properties of individual instances.
- Based on BLIS, applications like VRP/TSP can be implemented in a straightforward way.
- Much work is still needed
  - Callable library API
  - Support for column generation
  - Enhanced heuristics
  - Additional capabilities

Obtaining CHiPPS

The CHiPPS framework is available for download at

`https://projects.coin-or.org/CHiPPS`

Thank You!

Questions?