Priority Queues

IE 496 Lecture 12

Reading for This Lecture

- Horowitz and Sahni, Chapter 2
- Kozen, Lecture 8

Priority Queues

- A queue where each item has a specified *priority*.
- Additional operations for priority queues
  - find_min()
  - delete_min()
- Applications
  - sorting
  - greedy algorithms
- We will discuss these in future lectures

Heaps

- A *heap* is a scheme for implementing a priority queue.
- A *heap* is a binary tree in which the value at each node is at least as large as the values in each of its children.
- Hence, a largest element is always at the root.
- Heaps support the following operations
  - `insert()`
  - `delete_max()`
  - `make_heap()`
  - `adjust_heap()`

Inserting into a heap

- Insert the value into node $n+1$ and "bubble up".
- Compare the value to its parent and swap if necessary.
- Continue swapping until heap property is restored.
- One way to make a heap from n elements is to simply insert them one at a time.
- Analysis
  - `insert()`
  - `make_heap()`

Adjusting a heap

- If only the root of a heap is out of order, we can restore order by "bubbling down" (`adjust()`).
  - Swap the root with the larger child.
  - Continue swapping process until heap property is restored.
- Heapify (create a heap by iterative adjusting)

  For each node $i = \lfloor n/2 \rfloor \rightarrow 1$

  adjust i w.r.t. the subtrees rooted at its children

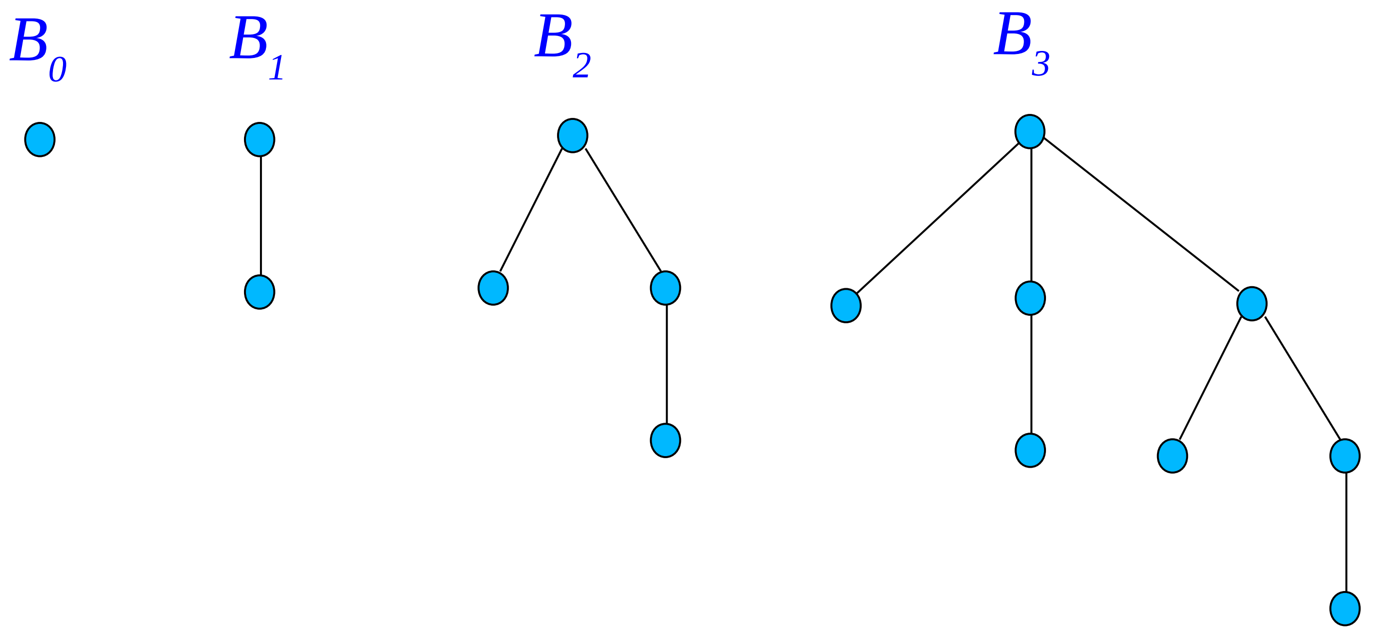
- Analysis

Deleting from a heap

- To delete the root node,
  - exchange node 1 with node *n*.
  - adjust the heap.
- Heapsort
  - First heapify.
  - Iteratively delete the root node.
- Analysis

Binomial Trees

- The *binomial tree* of rank i (B_i) is defined recursively.
- B_i consists of a *root* with i children $B0$, . . ., B_{i-1}.



Binomial Heaps

- A *binomial heap* is a collection of heap ordered binomial trees and a pointer to the overall max/min.
- No more than one tree of each rank is allowed.
- The children of each vertex are maintained in a circular linked list.
- The basic operation is *linking*.
- Two trees of rank i can be combined into one tree of rank $i+1$ in constant time.

Eager Meld

- We can combine two heaps by performing a `meld()` reminiscent of binary addition.
- Successively link trees of equal rank and "carry" one if necessary.
- Must track the position of the new min/max element.
- This operation takes $O(\log n)$ time.

Inserting into a Binomial Heap

- To `insert()` an element:
  - Make a new heap from the single element to be inserted.
  - Meld the new heap with the old one.
- To `make_heap()` from scratch, perform a sequence of inserts.
- To `delete()` the min/max element:
  - The children of this element form a new binomial heap.
  - Meld the old heap and the new one.

Amortized Analysis

- `meld()` and `delete()` both take O(log n).
- We will use *amortized analysis* to show that `insert()` is constant time overall.
- Each time, we create a new tree, we charge an extra unit of time to that operation.
- We use those “credits” to “pay” for operations that link trees later on.
- In this way, we can justify that any sequence of inserts takes constant time *on average*.