Problem Set 1
IE 496 – Computational Methods in Optimization
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1. Consider an interconnection scheme based on a tree, but in which neighboring nodes on each level are connected. That is, each interior node has two additional links, one to each of its left and right neighbors. Nodes on the outer edge of the tree have one additional link. We will call such a scheme an X-tree.

   (a) What is the degree of an X-tree? Explain.

   (b) What is the communication diameter? Explain.

   (c) What is the bisection width? Explain.

   (d) Give a lower bound for sorting on an X-tree. Explain.

2. Define a *linear array of size n with a bus* to be a linear array augmented with a single global bus. In other words, every processor is connected to the bus and in one unit of time, one processor can write to the bus and all other processors can read from it.

   (a) Give an efficient algorithm to sum n values, initially distributed one per processor on such an architecture. Perform an analysis of the algorithm for the case of finding the minimum of n values on a regular linear array.

   (b) Can the efficiency of the algorithm be improved by increasing the size of the data with respect to the number of processors?

   (c) Give an efficient algorithm to compute n partial sums under the same conditions and perform a similar analysis. Compare to your results in the other two parts.

3. A matrix is said to have a saddle point if some position has the smallest entry in its row and the largest entry in its column. Design an efficient algorithm for determining if a matrix has a saddle point and determining its location if there is one. Discuss the efficiency with which such an algorithm could be implemented on a linear array and a mesh, starting with one array value on each processor.

4. Consider the simple sorting algorithm known as *insertion sort*. Propose a version of insertion sort that is optimized to take into account the presence of a memory cache and suggest what improvement in efficiency could be expected by implementing it.

5. Replicate the experiments discussed in the lecture slides for cache optimization of the initialization of a matrix and the multiplication of two matrices. Carefully describe the architecture on which your experiments were performed and compare your results to those discussed in the article.