IE 495 Lecture 2

August 31, 2000

Reading for this lecture

- Primary
  - Miller and Boxer, Chapter 5
  - Aho, Hopcroft, and Ullman, Chapter 1
  - Fountain, Chapter 4
- Secondary
  - Roosta, Chapter 2
  - Cosnard and Trystram, Chapters 4

Interconnection Networks

Aside: Introduction to Graphs

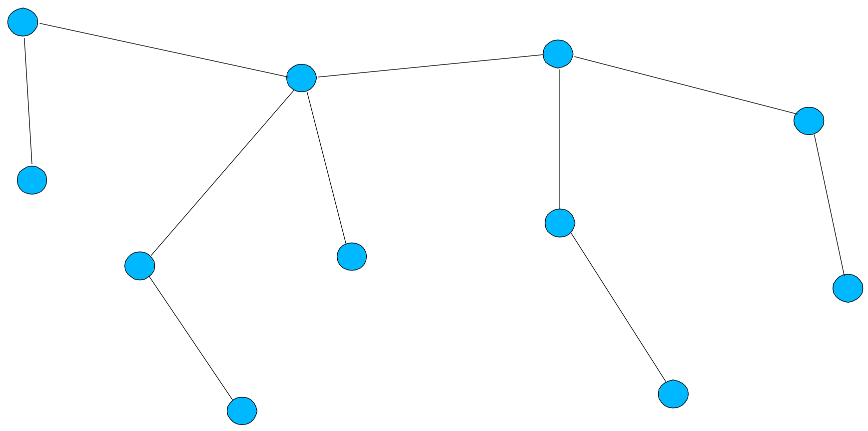
- A graph $G = (V, E)$ is defined by two sets, a finite, nonempty set V of *vertices* (or nodes) and a set $E \subseteq V \times V$ of *edges*.
- Example: A road network.
- The edges can be either ordered pairs or unordered pairs.
- If the edges are ordered pairs, then they are usually called *arcs* and the graph is called a *directed graph*.
- Otherwise, the graph is called *undirected.*
- See AHU, Section 2.3

(Undirected) Graph Terms

- Vertices u and v are *endpoints* of the edge (u, v).
- We say an edge e = (u, v) is *incident to* its endpoints.
- Two vertices u and v are *adjacent* if $(u, v) \in E$.
- The *degree* of a vertex is the number of edges incident to it (equivalently, the number of vertices adjacent to it).
- A *path* is a sequence of edges $(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n)$
- The *length* of such a path is $n-1$.
- Often, we represent a path simply as a sequence of vertices.

Applications of Graph Theory

- Graph theory is a very rich subject area
- Sample Applications
  - Shortest Path Problem
  - Minimum Spanning Tree
  - Traveling Salesman Problem



What is an interconnection network?

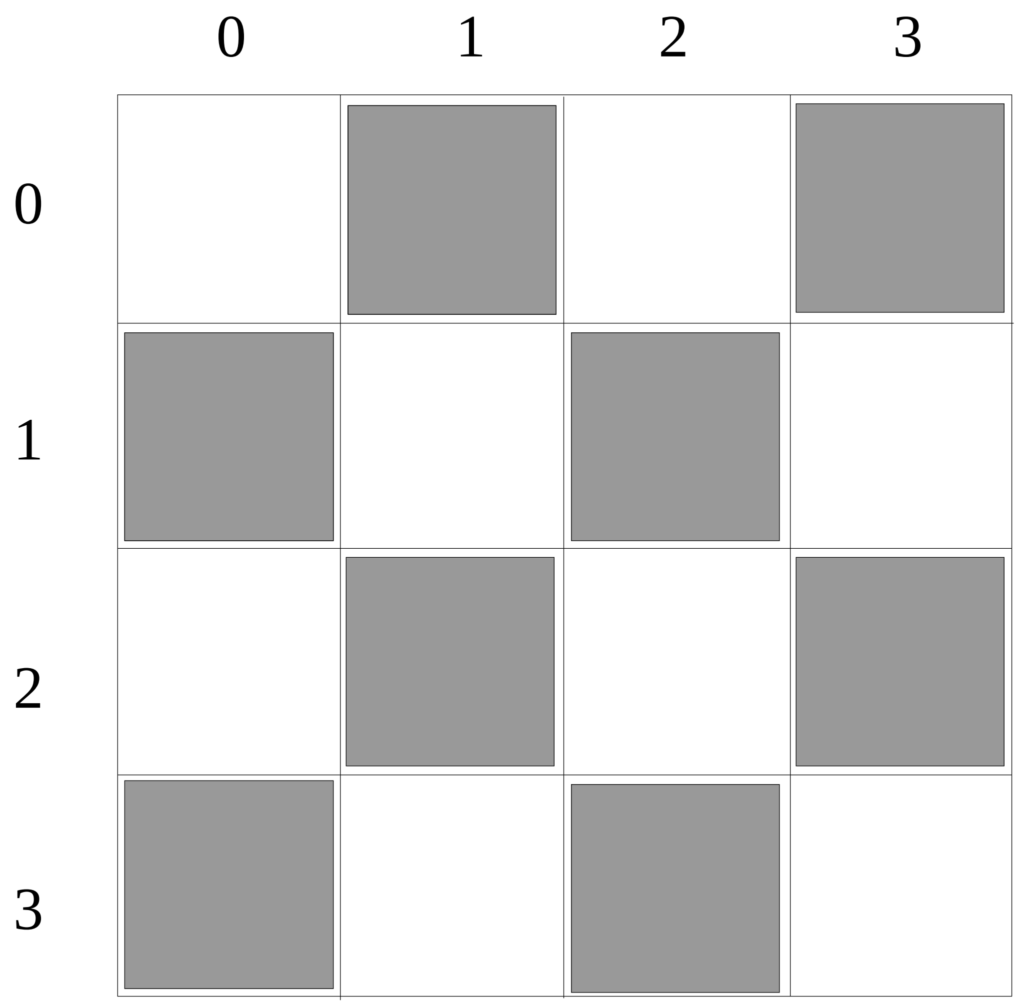
- A graph (directed or undirected)
  - The nodes are the processors
  - The edges represent direct connections
- Properties and Terms
  - Degree of the Network
  - Communication Diameter
  - Bisection Width
  - Processor Neighborhood
  - Connectivity Matrix
  - Adjacency Matrix

Measures of Goodness

- Communication diameter: The maximum shortest path between two processors.
- Bisection width: The minimum cut such that the two resulting sets of processors have the same order of magnitude.
- Connectivity Matrix
- Adjacenecy Matrix

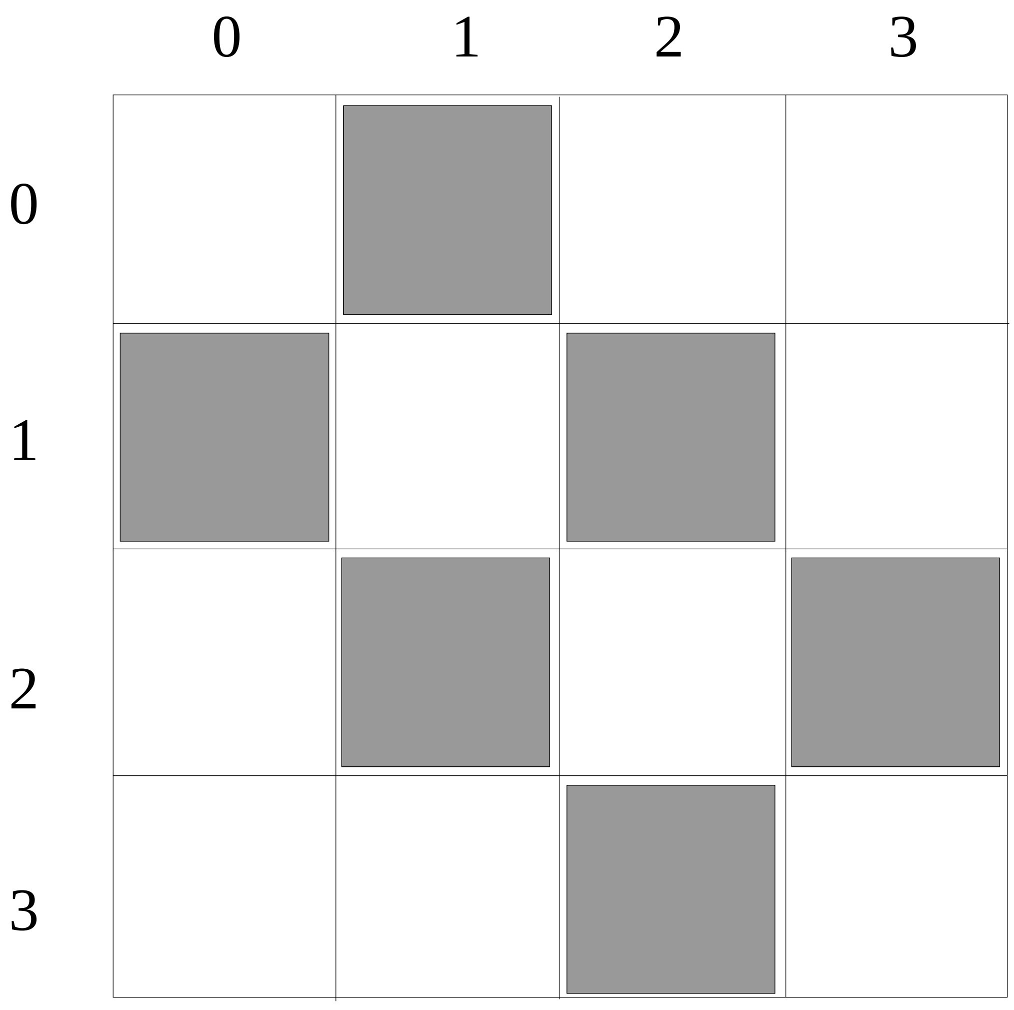
Connectivity Matrices

Example 1



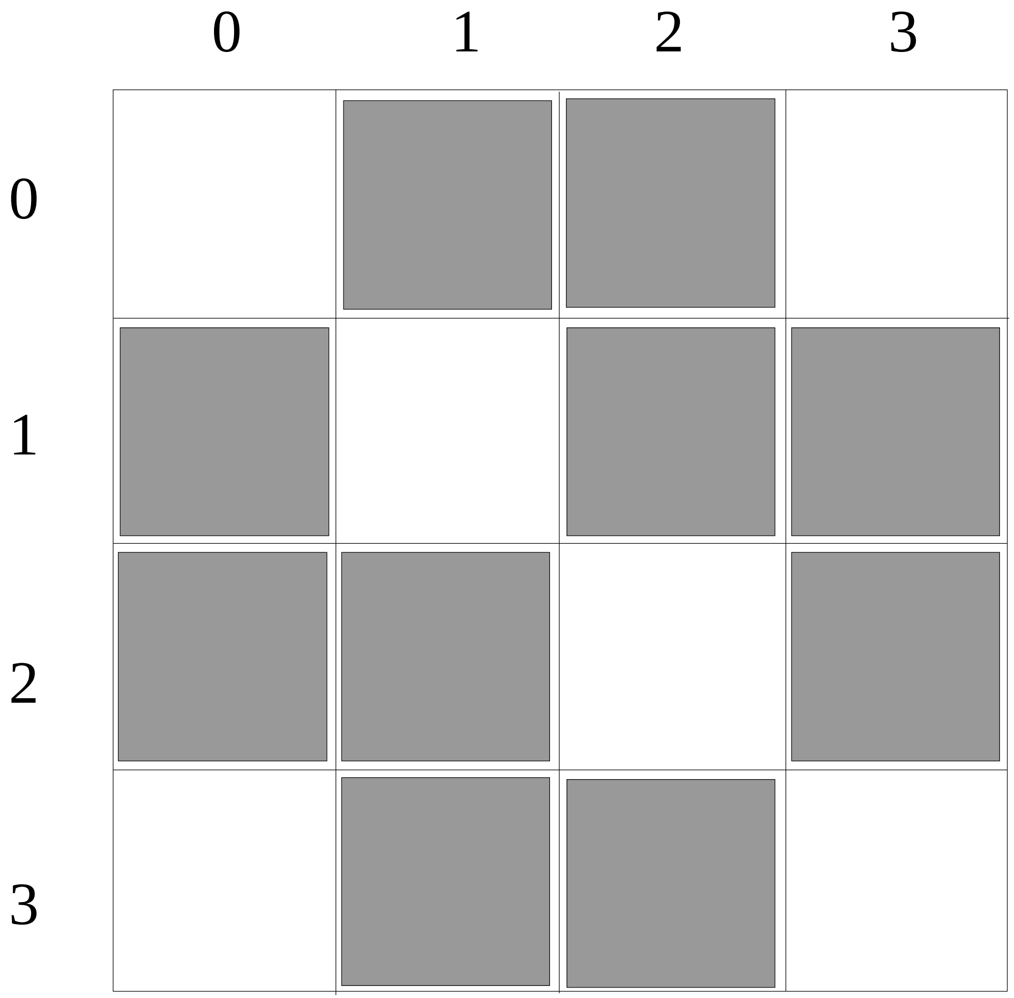
Connectivity Matrices

Example 2



2-step Connectivity Matrices

Example 2



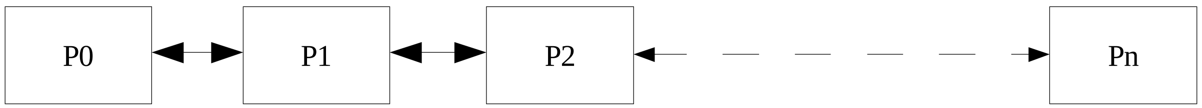
N-step Connectivity Matrices

- Indicates the processor pairs that can reach each other in N steps
- Computed using Boolean matrix multiplication
- The corresponding adjacency matrix indicates how many disjoint paths connect each pair.

	0	1	2	3
0	1	1	1	
1	1	1	1	1
2	1	1	1	1
3		1	1	1

	0	1	2	3
0	1	1	2	1
1	1	1	1	2
2	2	1	1	1
3	1	2	1	1

Linear Array

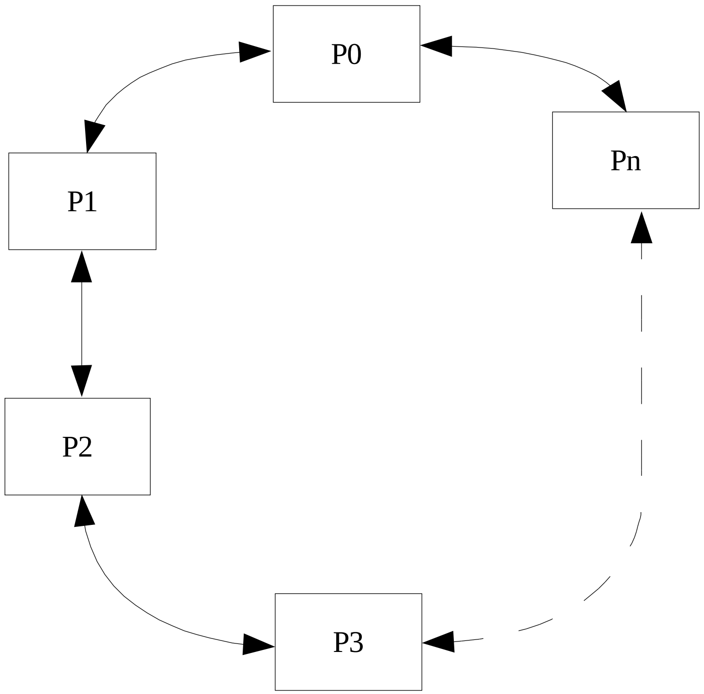


Diameter

Bisection Width

Degree

Ring



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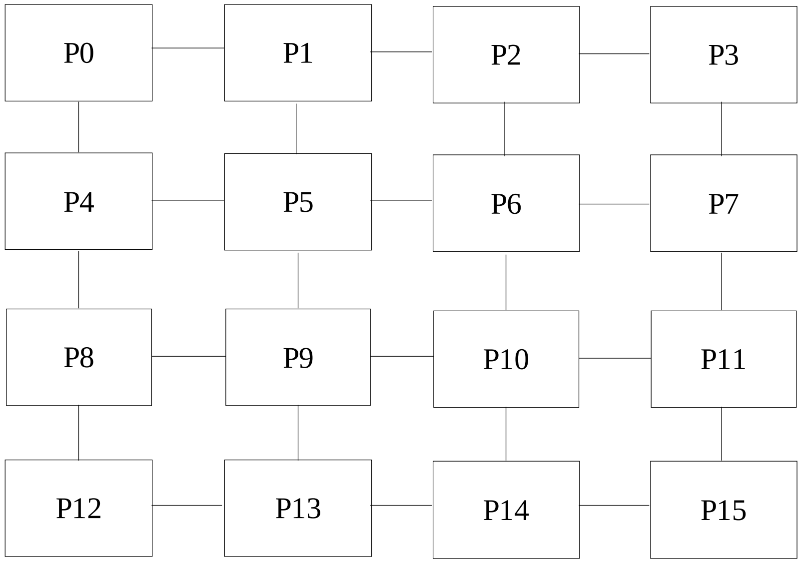
Diameter

Bisection Width

Degree

Mesh



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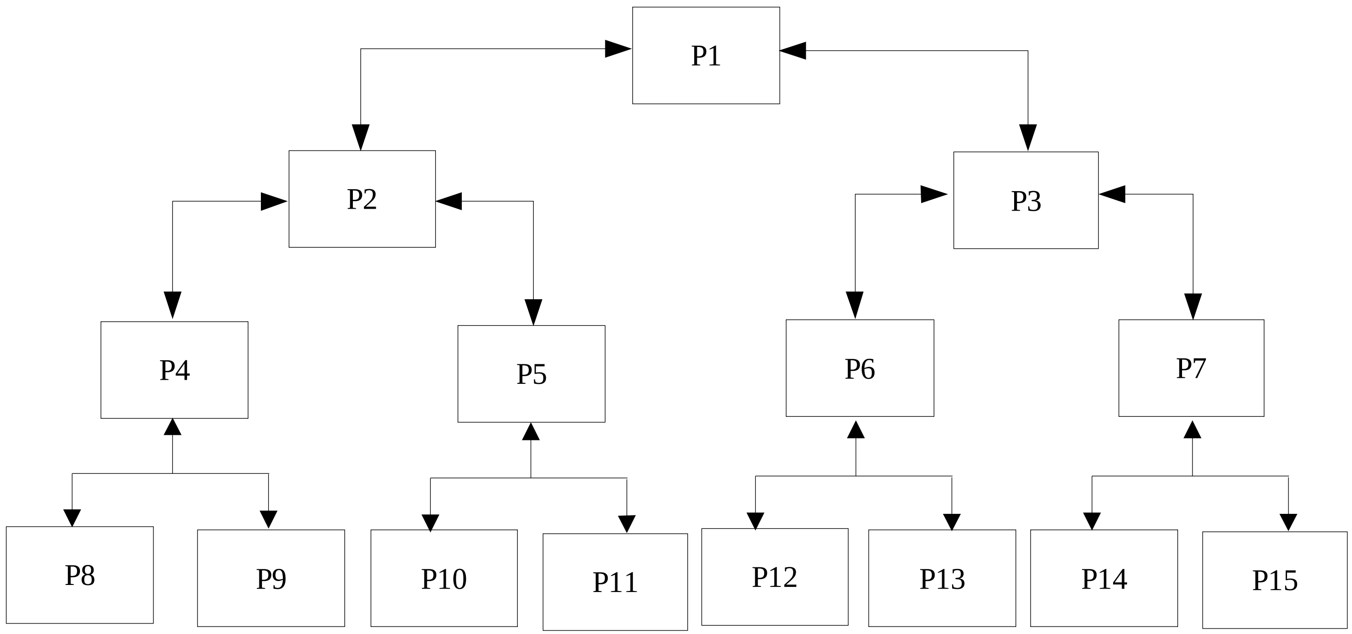
Diameter

Bisection Width

Degree

Tree



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Diameter

Bisection Width

Degree

Other Schemes

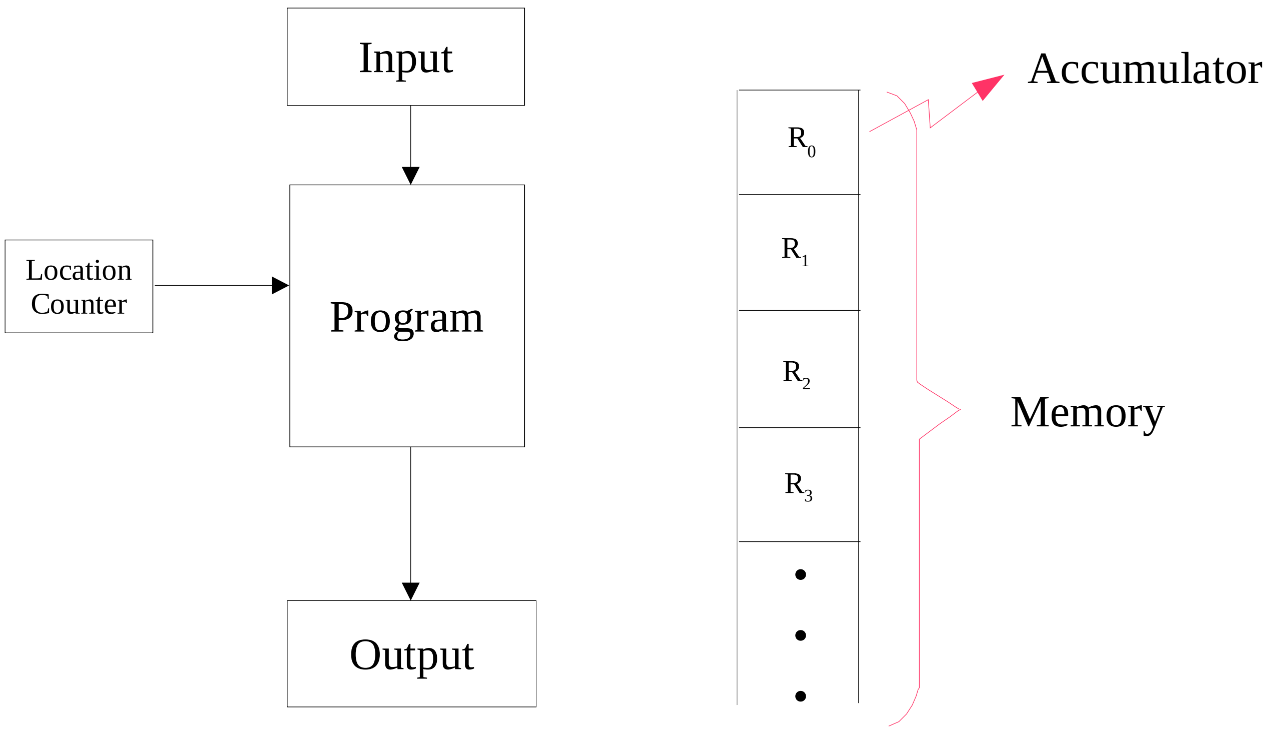
- Pyramid: A 4-ary tree where each level is connected as a mesh
- Hypercube: Two processors are connected if and only if their ID #'s differ in exactly one bit.
  - Low communications diameter
  - High bisection width
  - Doesn't have constant degree
- Perfect Shuffle: Processor *i* is connected *one*-*way* to processor *2i mod(N-1)*.
- Others: Star, De Bruijn, Delta, Omega, Butterfly

Models of Computation

Analysis of Algorithms

- We are interested in the time and space needed to perform an algorithm.
- There are several ways of approaching this analysis.
  - Worst case
  - Average case
  - Best case
- Worst case is the most common type of analysis (why?).
- Generally speaking, time is the most constraining resource.

Random Access Machine Model



A RAM Program

- At each time step, one elementary operation is completed.
- Sample list of elementary operations

- LOAD	- READ
- STORE	- WRITE
- ADD	- JUMP
- SUB	- JGTZ
- MULT	- JZERO
- DIV	- HALT