IE 417: Advanced Mathematical Programming

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Office Hours: M 10:00-11:00, TR 2:30-3:30
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Course web page: http://www.lehigh.edu/~tkr2/teaching/ie417/

Course meeting time: TR 4:10-5:25 453 Mohler Lab

Description of Course

This course will address a number of advanced topics in mathematical programming with particular emphasis on optimization problems with non-linear objective function and/or non-linear constraints. Topics will include convex analysis, unconstrained and constrained optimization, non-linear duality theory, Lagrangian relaxation, and algorithmic methods for solving non-linear programs. The algorithmic methods covered will include descent methods, Newton's method, conjugate gradient methods, and penalty and barrier methods. If time permits, we will also consider interior point methods for linear programming.

Course Objectives

The goal of this course is for students to

1. **Continue to improve their ability to rigorously prove mathematical statements.**
2. **Cultivate an ability to perform accurate self-assessment of their work.**
3. **Review and extend knowledge and understanding of the underlying mathematical foundations of the field of optimization.**
4. **Develop an understanding of optimality conditions for both constrained and unconstrained nonlinear optimization problems.**
5. **Learn how and when to apply optimality conditions for practical problem solving.**
6. **Learn and apply algorithmic and computational techniques for solving mathematical programs, particularly non-linear.**
7. **Understand the computational issues involved in solving non-linear programs.**

Required Text

Mokhtar S. Bazaraa, Hanif D. Sherali, and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms,* Wiley (1993).

Suggested Supplementary Materials

Daniel Solow, *How to Read and Do Proofs: An Introduction to Mathematical Thought Processes,* Wiley (2001).

Daniel J. Velleman, *How to Prove It: A Structured Approach*, Cambridge University Press (1994).

Course Requirements

1. **<u>Lectures</u>:: Students will be expected to attend and participate in the lectures. Part of the grade will be determined by overall class participation. Lecture materials, most likely in the form of Power Point slides, will be available for reference before the lecture on the course web page.**

2. **<u>Reading</u>: There will be readings associated with each lecture. Most readings will be from the required text, but there may be supplementary material. Students will be responsible for completing these readings.**

3. **<u>Problem Sets</u>: There will be a problem set due approximately every two weeks according to the schedule below. Students are encouraged to work together, but each student should write up his/her solutions independently.**

4. **<u>Exams</u>: There will be a mid-term and a final. Both exams will be administered in a take-home format. You will be allowed to consult only a given list of references, during the exam period.**

Schedule of Homeworks and Exams (subject to change)

<u>Homework/Exam</u>	**<u>Date</u>**
Homework #1	**Sept 10**
Homework #2	**Sept 24**
Homework #3	**Oct 8**
Mid-term	**Oct 22-24**
Homework #4	**Nov 5**
Homework #5`	**Nov 19**
Homework #6	**Dec 3**
Final	**??**

Schedule of Topics (subject to change)

1. **Review of Mathematical Proof Techniques (1 lecture)**
2. **Convex Analysis (4-6 lectures)**
   - **Convex Sets (2-3 lectures)**
   - **Convex Functions (2-3 lectures)**
3. **Optimality Conditions (4-6 lectures)**
   - **Unconstrained Optimization (2-3 lectures)**
   - **Constrained Optimization (2-3 lectures)**
4. **Nonlinear Duality Theory (3-4 lectures)**

5. **Computational Methods for Nonlinear Programming (8-10 lectures)**

- **Fundamentals of Algorithms (1 lecture)**
- **Descent Methods (2 lectures)**
- **Newton's Method (1-2 lectures)**
- **Conjugate Gradient Methods (2 lectures)**
- **Subgradient Optimization (1 lecture)**
- **Penalty and Barrier Methods (2 lectures)**

6. Interior Point Methods for Linear Programming (2 lectures)

Rough Textbook Coverage

<u>Chapters to be covered in lecture</u>:

Chapter 2-4, 6-10

<u>Suggested supplementary reading</u>:

Chapter 1
Appendix A
Appendix B

Grading Scheme

30% Homework
30% Mid-term
30% Final
10% Class Participation

Teaching Philosophy and Policies

Group Work

You are encouraged to work together on problem sets. However, you must ultimately demonstrate your understanding of the material by writing up your own solutions without the help of other students or their written work.

Referencing the Work of Others

You should attempt the problem sets on your own before consulting outside references. However, I do not want to discourage the use of research materials as a way to supplement your understanding of the course material. If you do use external references in developing your solutions, and especially if you quote directly from these references, please cite them! Failure to cite references will result in severe penalty.

Lateness

I will allow a total of 7 days of lateness on assignments throughout the semester. These 7 days can be split up in any way you choose. In other words, you can have one assignment late by 7 days or 7 assignments each late by one day. After that, there is a penalty of 10% off per late-day on each assignment. No assignment will be accepted more than 7 days late. Exceptions to this rule will be made on a case-by-case basis. Please let me know if you will be turning in an assignment late.

Grading

I do not believe in giving a formula for determining grades (but I will give one anyway). I believe your grade should reflect the actual learning that took place in the course. Hence, the way to maximize your grade in the course is to learn and understand the material. Most formulaic grading systems allow you (even encourage you) to maximize your grade without necessarily maximizing your learning. I want to discourage you from disconnecting these two goals.

Higher learning involves not just acquiring knowledge, but developing the ability to "know what you don't know." I call this ability *self-assessment of knowledge*. Among other things, it involves knowing when you do and do not have a rigorous proof or an accurate answer. One of the goals of this course is to cultivate your ability to perform an accurate self-assessment of your work. Hence, you are encouraged to think about and state accurately not only the parts that you *do* understand from each homework, but also the parts that you *do not*. Please do not muddle your way through proofs and other exercises in the hope that I will not read them carefully. You will get additional credit for an accurate self-assessment of your answer or approach. Hence, if you have gotten most of the way through a proof and just cannot complete the last step or even if you are missing a step in the middle but know how to do the rest, just try to write down what you have done so far and what it is that you don't know how to do. This will help me to better gauge where your understanding is incomplete so that we can review these areas in class. It will also demonstrate your understanding of your own work.

Effective learning also involves knowing where to go to get help when you realize that your knowledge or understanding of a topic is incomplete. This could mean asking a classmate some questions, consulting external references, or coming to office hours. It can also mean asking a question in class when you don't understand part of the lecture. Chances are, other people don't understand it either. This is all an important part of *class participation*.

Another area in which you will be evaluated is by the *level of detail* and *rigor* in your proofs and homework answers. In general, you should err on the side of giving too much detail in your written work. One mistake students make is assuming that since I assigned the problem, I have already thought of every possible approach to it (including the one you chose). Many times, however, I will not have thought of the approach you are using and will therefore need some help in understanding your thought process. The more explicit you are, the easier it will be for me to grade and the more you will demonstrate your understanding. If you spend hours coming up with the answer to a problem, don't short-change yourself by spending only a few minutes writing it down. Take some time to think about how best to present your thoughts. Otherwise, you may be throwing your hard work away.

Another mistake students make, especially with respect to the level of detail that should be provided in proofs, is to use journal articles and/or textbooks as examples. In general, the proofs that appear in journal articles are not detailed enough for our purposes. Journal articles are inherently space-constrained and hence the level of detail is sacrificed and the reader asked to fill in the blanks in the interest of brevity. You should not write your problem sets as if you are space-constrained.

To summarize, you will be graded according to my overall assessment of your learning in the course and your understanding of the course material. This includes your ability to perform self-assessment, your ability to ask questions to increase your understanding, and your ability to express your ideas in written form rigorously and with an appropriate level of detail.

Learning Styles

There are many different styles of learning. Some people gain better understanding from listening to something being explained orally. Some get better understanding from written material. Some like a combination of both. I do my best to accommodate various styles of learning. However, feel free to let me know what your learning style is so that I can take that into account when determining the future direction of the course.

Office Hours and Appointments

My door is always open, but I would appreciate it if you could try to utilize office hours as much as possible. If you would like to make an appointment outside office hours, just call or send an e-mail.