Problem Set #4
IE 417
Due November 5, 2002

1. a. A matrix $P \in \mathbf{R}^{n \times n}$ is called a *projection matrix* if $P = P^T$ and $PP = P$. Prove that if P is a projection matrix, then

   - The matrix I -P is also a projection matrix,
   - P is positive semi-definite,
   - $||Px|| \leq ||x|| \ \forall x \in \mathbf{R}^n$.

   b. Show that if $A \in \mathbf{R}^{m \times n}$, then $P = A^T(AA^T)^{-1}A$ is a projection matrix and that $Px \in \text{range}(A^T) = \{A^Ty: y \in \mathbf{R}^m\} \ \forall x \in \mathbf{R}^n$ (i.e., P is the matrix that projects $\mathbf{R}^n$ onto range(A^T).

   c. Show that I-P is also a projection matrix such that $(I-P)x \in \text{null}(A) = \{x \in \mathbf{R}^n : Ax = 0\} \ \forall x \in \mathbf{R}^n$ (i.e., (I-P) is the matrix that projects $\mathbf{R}^n$ onto null(A)).

2. 8.8

3. 8.22 (compare Hooke and Jeeves, Rosenbrock, steepest descent, and DFP)

4. 8.27

5. 8.40