Problem Set 4
IE411 Graphs and Network Flows
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Due April 1, 2014

1. (a) An arc is *upward critical* if increasing the capacity of this arc increases the maximum flow value. Describe an algorithm for identifying all upward critical arcs in a network and analyze its running time. Your algorithm should have a better running time than that for solving m maximum flow problems.

   (b) Does every network have an upward critical?

   (c) An arc is *downward critical* if decreasing the capacity of this arc decreases the maximum flow value. Is the set of downward critical arcs the same as the set of upward critical arcs? If not, give an algorithm for determining the set of downward critical arcs and analyze its running time.

2. Using the potential function $\max\{d(i) \mid i \text{ is an active node}\}$ show that the highest-label preflow-push algorithm performs $O(n^3)$ nonsaturating pushes.

3. Show that if a flow x and a set of node potentials π satisfy complementary slackness conditions with respect to a given network, then they also satisfy reduced cost optimality conditions.

4. In an instance of the minimum cost network flow problem, suppose that one arc (p, q) has no lower and upper bounds. Describe how to transform this instance into one in which all arcs have zero lower bound and finite upper bound.

5. Perform an empiricial comparison of the following algorithms for maximum flow using GiMPy.

   (a) Augmenting path with depth-first search

   (b) Shortest augmenting path with breadth-first search in each iteration

   (c) Shortest augmenting path with labeling (preflow push)

   (d) Preflow-push with FIFO node selection

   (e) Preflow-push with highest-label node selection

   You should generate at least two sets of test networks with different properties and construct performance profiles showing the comparison for execution time for all methods. Separately compare the number of paths generated for the augmenting path methods and the number of nonsaturting pushes for the preflow push methods.