Problem Set 3
IE411 Graphs and Network Flows
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Due February 25, 2014

1. The *money-changing problem* requires that we determine whether it is possible to change a given amount of currency p into coins of known denominations $a_1, a_2, \ldots, a_k$. In other words, the money-changing problem asks whether there are non-negative integers $x_1, x_2, \ldots, x_k$ such that $p = \sum_{i=1}^{k}$.

   (a) Describe a method of determining all numbers in a given range $[l, u]$ that we can successfully change.

   (b) Describe a method of determining whether we can change a given number p and if so, identify the number of coins of each denomination required.

2. Construct an example of a shortest path problem with some negative arc lengths, but no negative cycle, that Dijkstra's algorithm will solve correctly. Construct a second example that it will not solve correctly. Be sure to explain your examples in detail.

3. Let $c_{ij} \geq 0$ be interpreted as the capacity of arc (i, j) rather than its cost. Then the capacity of a given path is the minimum capacity of any one arc in the path. The *maximum capacity path problem* is to determine a path of maxiumum capacity from a given source node to each other node. Describe a modification of Dijkstra's algorithm that solves the maximum capacity path problem. Prove correctness and perform a complexity analysis.

4. Sedgewick Problems 20.33, 20.71, and 20.72.