

Problem Set 2
IE411 Graphs and Network Flows
Dr. Ralphs
Due February 11, 2014

1. Show formally that in a breadth-first search tree, the unique path in the tree from the source to any other node is the shortest path (in terms of number of edges).
2. Consider a set of n scalar numbers a_1, a_2, \dots, a_n arranged in non-decreasing order of their values. We wish to partition these numbers into clusters (groups) so that (1) each cluster contains at least p numbers; (2) each cluster contains consecutive numbers from the original list; and (3) the sum of the squared deviations of the cluster members from their cluster means is as small as possible. Show how to formulate this problem as a shortest path problem.
3. Given a DAG, does there exist a topological sort that cannot result from applying a DFS-based algorithm, no matter in which order the vertices are chosen?
4. Do an empirical study comparing depth-first search using the simple recursive method and depth-first search using general graph search with a stack. Compare using performance profiles.
5. Do an empirical study of the following properties of graphs. In all cases, you should produce figures to show your results visually.
 - (a) Study the number of components a random undirected graph has, as a function of size and density. You can use whatever methodology you like for producing the random graphs and doing the experiments. You should produce some sort of summary table as an output.
 - (b) Study the number of strong components a random directed graph has, as a function of size and density without allowing parallel (but oppositely directed) arcs.
 - (c) Study the diameter of randomly generated undirected graphs. This can be done using breadth-first search.