Problem Set 1
IE411 Graphs and Network Flows
Dr. Ralphs
Due September 12, 2017

1. Paper and wood products companies need to define cutting schedules that will maximize the total wood yield of their forests over some planning period. Suppose that a company with control of p forest units wants to identify the best cutting schedule over a planning horizon of k years. Forest unit i has a total acreage of a_i units and studies the company has undertaken predict that this unit will have w_{ij} tons of wood available for harvesting in period in year j. Based on its predictions of economic conditions, the company believes that it should harvest l_j tons of wood in year j. Due to the availability of of equipment and personnel, the company can harvest at most u_j tons of wood in year j.

   (a) Create an abstract model for the problem of determining maximum wood yield as a network flow problem.

   (b) Randomly generate some data for this problem and construct a graph in GiMPy representing the problem. Produce a picture of this graph and include it in your write-up.

   (c) Solve a few instances of this problem of reasonable size using GiMPy. Include the code for generating the data, constructing the graphs, and solving the instances in your write-up.

2. Several families are planning a shared car trip on scenic drive in the White Mountains of New Hampshire, To minimize the possibility of any quarrels, they want to assign individuals to cars so that no two members of a family are in the same car.

   (a) Create an abstract model for the problem of determining the allocation of individuals to cars as a network flow problem.

   (b) Randomly generate some data for this problem and construct a graph in GiMPy representing the problem. Produce a picture of this graph and include it in your write-up.

   (c) Solve a few instances of this problem of reasonable size using GiMPy. Include the code for generating the data, constructing the graphs, and solving the instances in your write-up.

3. Show that a directed graph is acyclic if and only if we can renumber its nodes so that its node-node adjacency matrix is lower triangular.

4. Consider a directed graph $G = (N, A)$. For any $S \subseteq N$, let

$$\text{neighbor}(S) = \{j \in N \mid \exists i \in S \text{ such that } (i,j) \in A\} \tag{1}$$

   Show that G is strongly connected if and only if for every non-empty $S \subset N$, neighbor$(S) \neq \emptyset$.

5. Let $\mathcal{H}$ denote the node-node adjacency matrix of a network G and let $\mathcal{H}^k = \mathcal{H} \cdot \mathcal{H}^{k-1}$ for $k = 2, \ldots, n$. Show that $\mathcal{H}^k_{ij}$ is the number of distinct walks from i to j with exactly k arcs.