Problem Set 7
IE406 Introduction to Mathematical Programming
Dr. Ralphs
Due October 31, 2007

1. Bertsimas 5.12

2. Bertsimas 5.13

3. Consider the following linear programming problem and its optimal final tableau shown below.

$$\begin{aligned} \min\ & -2x_1 - x_2 + x_3 \\ \text{s.t.}\quad & x_1 + 2x_2 + x_3 \le 12 \\ & -x_1 + x_2 - 2x_3 \le 3 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Final tableau:

x_1	x_2	x_3	x_4	x_5	
0	3	3	2	0	24
1	2	1	1	0	12
0	3	-1	1	1	15

(a) Determine the optimal dual solution by examining the tableau.

(b) Determine the range of values of the right hand side of the first constraint for which the basis shown above remains optimal.

(c) Suppose that after obtaining the optimal solution depicted in the final tableau above, it was revealed that the following set of constraints were left out and must also be satisfied:

$$\begin{aligned} 2x_1 + 3x_2 &\le 20 \\ x_1 - x_2 + x_3 &\le 11 \\ 2x_1 - 3x_3 &\le 23 \end{aligned}$$

Use constraint generation to obtain an optimal solution after augmenting the original LP with these three new constraints. (Hint: This only requires a few calculations.)

4. The output of a paper mill consists of standard rolls 110 inches (110") wide, which are cut into smaller rolls to meet orders. This week, there are orders for smaller rolls of the following widths:

Width	Orders
20”	48
45”	35
50”	24
55”	10
75”	8

The owner of the mill wants to know what cutting patterns to apply so as to fill the orders using the smallest number of 110” rolls.

A cutting pattern consists of a certain number of smaller rolls of each width that can be cut from one larger roll, such as two of 45” and one of 20”, or one of 50” and one of 55” (and 5” of waste). Notice that the sum of the widths of the smaller rolls in the pattern must be less than 110”. For example, we could consider the following six patterns:

Width	1	2	3	4	5	6
20”	3	1	0	2	1	3
45”	0	2	0	0	0	1
50”	1	0	1	0	0	0
55”	0	0	1	1	0	0
75”	0	0	0	0	1	0

The pattern in the first column represents 3 20” rolls and one 50” roll.

(a) Develop an AMPL model, allowing for any set of order widths and any set of patterns, that minimizes the total number of 110” rolls used, assuming that the number of smaller rolls produced need only be greater than or equal to the number ordered (ignore the fact that the solutions may be fractional).

(b) Using the given data, how many rolls should be cut according to which pattern to minimize the total number of 110” rolls used in this example?

(c) Find another pattern that, when added to those above, improves the optimal solution.

(d) As noted, all of the solutions above use fractional numbers of rolls. Can you find solutions that satisfy the constraints, but also uses an integer number of rolls in each pattern? How much does your integer solution cause the value of the objective function value to go up?