Advanced Operations Research Techniques
IE316

Lecture1

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Reading for This Lecture

- Operations Research Methods and Models
- Bertsimas 1.1-1.2, 1.4-1.5, 1.3 (optional)

What is a model?

mod·el: *A schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics.*

–American Heritage Dictionary of the English Language

- Two types of models
  - Concrete
  - Abstract
- Mathematical models
  - Are abstract models.
  - Describe the mathematical relationships among elements in a system.

Systems Modeling

- In ISE, we are mainly concerned with modeling *systems*.
- What is a system?
- sys·tem: A functionally related group of elements, especially:
  - The human body regarded as a functional physiological unit.
  - An organism as a whole, especially with regard to its vital processes or functions.
  - A group of physiologically or anatomically complementary organs or parts: the nervous system; the skeletal system.
  - A group of interacting mechanical or electrical components.
  - A network of structures and channels, as for communication, travel, or distribution.
  - A network of related computer software, hardware, and data transmission devices.

Why do we model systems?

- The exercise of building a model can provide insight.
- It's possible to do things with models that we can't do with "the real thing."
- Analyzing models can help us decide on a course of action.

Examples of Models

- Physical Models
- Simulation Models
- Probability Models
- Economic Models
- Biological Models
- Mathematical Programming Models

Mathematical Programming Models

- What does *mathematical programming* mean?
- Programming here means "planning."
- Literally, these are "mathematical models for planning."
- Also called *optimization models*.
- Essential elements
  - Decision variables
  - Constraints
  - Objective Function
  - Parameters and Data

Forming a Mathematical Programming Model

The general form of a *math programming model* is:

$$\begin{aligned} & min \text{ or } max \ f(x_1, \ldots, x_n) \\ s.t. \quad & g_i(x_1, \ldots, x_n) \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b_i \end{aligned}$$

We might also require the values of the variables to belong to a discrete set X.

Solutions

- A *solution* is an assignment of values to variables.
- A solution can be thought of as a vector.
- A *feasible solution* is an assignment of values to variables such that all the constraints are satisfied.
- The *objective function value* of a solution is obtained by evaluating the objective function at the given solution.
- An *optimal solution* (assuming minimization) is one whose objective function value is less than or equal to that of all other feasible solutions.

Types of Mathematical Programs

- The type of a mathematical program is determined primarily by
  - The form of the objective and the constraints.
  - The discrete set X.
  - Whether the input data is considered "known".
- We will consider mainly linear models.
  - The objective function is *linear*.
  - The constraints are *linear*.