Homework 8
IE316 – Advanced Operations Research Techniques
Dr. Ralphs
Due December 5, 2001

General Instructions: This assignment is somewhat open-ended because there are undoubtedly many possible models that could be used to solve these problems. The decisions to be made are well-defined, however. **Because of the open-ended nature of this assignment, it is important that your explanations be very clear and detailed. Explain your model in detail, including what your decision variables are, what each constraint means, and what the objective function is. A significant part of your grade will be based on how well you explain your model and your solution. Please leave yourself plenty of time to write up the solutions and do it well. I strongly encourage you to type your solutions for additional clarity.**

Grading: This assignment will have the weight of two regular homework assignments, so for those of you who need to make up ground, this is one place to do it. Developing a correct model and solving it will be worth approximately half the points for each problem. Performing the analysis will be the other half. The point values for each part are indicated.

Group Work: I encourage you to work in groups of no more than four on this assignment. If you work in a group, please turn in one write-up for the entire group.

1. A food is manufactured by refining raw oils and blending them together. The raw oils come in two categories.

vegetable oils	VEG1
	VEG2
non-vegetable oils	OIL1
	OIL2
	OIL3

Each oil may be purchased for immediate delivery (January) or bought on the futures market for delivery in a subsequent month. Prices now and in the futures market (in \$/ton) are:

	VEG1	VEG2	OIL1	OIL2	OIL3
January	110	120	130	110	115
February	130	130	110	90	115
March	110	140	130	100	95
April	120	110	120	120	125
May	100	120	150	110	105
June	90	100	140	80	135

The final product sells for \$150/ton.

Vegetable oils and non-vegetable oils require different production lines for refining. In any month, it is not possible to refine more than 200 tons of vegetable oils and more than 250 tons of non-vegetable oils. There is no loss of weight in the refining process and the cost of refining may be ignored.

It is possible to store up to 1000 tons of each raw oil for use laster. The cost of storage for both vegetable and non-vegetable oils is \$5 per ton per month. The final product cannot be stored, nor can refined oils be stored.

There is a technological restriction of “hardness” on the final product. In the units in which hardness is measured, it must lie between 3 and 6. It is assumed that hardness blends linearly and that the hardnesses of the raw oils are:

VEG1	8.8
VEG2	6.1
OIL1	2.0
OIL2	4.2
OIL3	5.0

At present, there are 500 tons of each type of raw oil in storage. It is required that these stocks also exist at the end of June. Your model should determine how much of the various oils to buy, use, and store during each month.

(a) (10 points) Develop a linear programming model using AMPL that determines a profit-maximizing strategy.

(b) (10 points) Solve your model and create a table that summarizes the actions to be taken in each month under the optimal strategy.

(c) (10 points) Determine the marginal worth of additional units of refining capacity for each kind of oil in each month, i.e., consider the effect of temporarily increasing capacity during a given month while keeping capacity the same in other months. What can you say about the effect of increasing capacity for all months of the plan by an equal amount?

(d) (10 points) Discuss the sensitivity of the solution to the prices of the oils.

(e) (15 points) Change the model to incorporate the following real-world constraints (this may require the use of integer variables).

   i. The food may never be made up of more than three oils in any one month.

   ii. If an oil is used in a given month, at least 20 tons must be used.

   iii. If either of VEG1 or VEG2 are used in a month, then so must OIL3.

   Solve your model using AMPL and create a table of the results. Comment on the difficulty of this new model as compared to the old one.

2. An economy consists of three industries: coal, steel, and transport. Each unit produced by one of the industries (a unit will be taken to be one dollar's worth of value of production) requires input from possibly its own industry as well as other industries. There is a time lag in the economy, so that output in year $t+1$ requires an input in year t. The required inputs, as well as the manpower requirements (also measured in dollars) are:

	Outputs (year $t+1$)		
Inputs (year t)	Coal	Steel	Transport
Coal	0.1	0.5	0.4
Steel	0.1	0.1	0.2
Transport	0.2	0.1	0.2
Manpower	0.6	0.3	0.2

Output from an industry may also be used to build productive capacity for itself or other industries in future years. The inputs required to give unit increases (capacity for one dollar's worth of extra production) in productive capacity are:

	Increase (year $t+2$)		
Inputs (year t)	Coal	Steel	Transport
Coal	0.0	0.7	0.9
Steel	0.1	0.1	0.2
Transport	0.2	0.1	0.2
Manpower	0.4	0.2	0.1

Stocks of goods may be held from year to year. At present (year 0), the stocks and productive capacities (per year) are (in $ million):

	Stocks	Productive capacity
Coal	150	300
Steel	80	350
Transport	100	280

There is also a limited yearly manpower capacity of $470 million.

For your analysis, you should consider a five year growth period. However, to develop a reasonable model, it is necessary to think beyond the end of the horizon. Ignoring external demand in year six and beyond would result in no inputs in the fifth year. You may therefore assume that external demand remains constant in all years, the stock level remains constant after year 5, and that there is no increase in productive capacity after year five. In other words, after year 5, the economy remains in "steady state." Your model should take this into account.

We are interested in the growth patterns that would result from pursuing various objectives (stated below). The output of your model should be the following, for years 1-5:

- the total capacity of each industry,
- the output of each industry,
- the amount of manpower required, and
- the stock of each industry at the end of the year.

Note that during the current year (year 0), we assume there is no demand and no production. However, the stock existing at the beginning of year zero (given above) can be used as inputs to production in year 1 and to increase in capacity in year 2, if desired. This initial stock can thus be thought of as the initial production level in year zero.

(a) (15 points) Develop a linear programming model using AMPL to determine the optimal strategy for growth of the economy with the following three different objectives in mind.

   i. Maximize total productive capacity at the end of year five while meeting an external demand of \$60 million of coal, \$60 million of steel, and \$30 million of transport each year (apart from year 0).

   ii. Maximizing total production (rather than productive capacity) in years four and five. In this case, you can ignore external demand.

   iii. Maximizing total manpower requirements (ignoring the manpower capacity limitation) over the five years, while meeting the same yearly demand as in (i).

(b) (10 points) Produce tables summarizing the output of the model under each of these three scenarios.

(c) (10 months) Determine the marginal worth of additional manpower in each month under the first objective.

(d) (10 points) Suppose you wanted to simultaneously consider two of the objective functions (this is called multi-objective optimization).

   i. Describe a method for finding, among all solutions that are optimal with respect to a primary objective function, a solution that maximizes a secondary objective function.

   ii. Describe how to optimize over a linear combination of two or more objective functions.