Extra Credit Assignment
IE316 – Advanced Operations Research Techniques
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Due December 5, 2001

General Instructions: This extra credit assignment is intended to give those who did not do well on the second quiz a chance to make up some ground. It is to treated as a take-home exam, rather than as a homework assignment. You cannot work with anyone and you are only allowed to consult me, the textbook, the lecture slides, and your class notes.
Scoring: The points you earn on this extra credit assignment will be added directly to your second quiz score. The maximum number of points you will be allowed to receive is .3(50 – your quiz score). The percentage of that maximum that you actually receive depends on how well you answer the questions below. In other words, if you get a grade of 50% on this assignment and your maximum allowed point total is 10 points, then 5 points will be added to your quiz score. These are difficult problems, but you will make headway on them if you work at it and I will give partial credit for going through the right thought process and demonstrating your knowledge of the material. Good luck!

1. Consider the LP of Exercise 5.13. As in part (c), consider changing the right hand side vector b parametrically from $b = [1 \ \ 2]^T$ to $b = [1 - 2\theta \ \ 2 - 3\theta]$. Determine the optimal primal and dual solutions and the optimal value for all values of θ. Graph the optimal value as a function of θ.

2. Consider an LP in standard form with four decision variables (x_1, x_2, x_3, and x_4) and two constraints, in which x_1 and x_2 are basic variables in some optimal basic feasible solution. Suppose that the ranges given for the current basis to remain optimal when changing only one objective function coefficient are

$$L_1 \leq c_1 \leq U_1 \tag{1}$$
$$L_2 \leq c_2 \leq U_2 \tag{2}$$

Consider **simultaneously** changing c_1 and c_2 to $c_1 + \delta_1$ and $c_2 + \delta_2$, respectively, where $\delta_1 > 0$ and $\delta_2 < 0$. Let

$$r_1 = \frac{\delta_1}{U_1 - c_1} \tag{3}$$
$$r_2 = \frac{-\delta_2}{c_2 - L_2} \tag{4}$$

In other words, r_1 represents the increase in c_1 as a fraction of the total range available and r_2 is defined similarly. Show that if $r_1 + r_2 \leq 1$, the basis remains optimal. Can you generalize this result?