Algorithms in Systems Engineering
IE172

Lecture 23

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References for Today's Lecture

- Required reading
  - Section 8.6
- References
  - CLRS Chapter 32
  - R. Sedgewick, *Algorithms in C++* (Third Edition), 1998.

String Matching

- The *string-matching problem* is to determine whether a given string occurs as part of a larger string.
- The smaller string is usually called the *search pattern*.
- This problem is also referred to as *pattern matching*.
- String matching has numerous important applications.
  - Finding words or phrases in large documents (text editors, search engines).
  - Finding the occurrence of a given genetic sequence in a large genome.

Terminology

- The set of allowable characters will be denoted Σ (usually the ASCII characters.
- The set of strings composed of these characters is denoted $\Sigma*$.
- For convenience, we will view a given string as an array of members of Σ.
- As in previous settings, we may also convert strings to integers in order to work with them more efficiently.
- Let T be a given string of length n and P a search pattern of length m.
- We say P occurs with shift s in T if $T[s..s+m-1] = P[0..m-1]$.
- If P occurs with shift s in T, then we call s a *valid shift*.
- We can restate the string-matching problem as that of locating all valid shifts in T with respect to P.

The Naive Approach

- The most obvious approach to string matching is to simply compare $T[s..s+m-1]$ to $P[0..m-1]$ for each $0 \leq s \leq n-m$.
- What is the running time of this algorithm?

The Rabin-Karp Algorithm

- To apply the Rabin-Karp Algorithm, we first convert P to its integer equivalent p.
- Recall that this can be done in $\Theta(m)$ steps using Horner's rule.
- Let t_s be the integer value of the substring $T[s..s+m-1]$.
- Note that s is a valid shift if and only if $t_s = p$.
- If we can calculate t_s for all $0 \leq s \leq n-m$, then we can find all valid shifts in $\Theta(n-m)$ comparisons in theory.
- Problem: As before, the integers we are talking about can be HUGE!

Implementing the Rabin-Karp Algorithm

- We can calculate t_0 using Horner's rule, as we did with p.
- Let $K = |\Sigma|$ be the number of characters in our alphabet.
- Give t_s, we can calculate t_{s+1} by the formula

$$t_{s+1} = K(t_s - K^{m-1}T[s]) + T[s+m]$$

- Hence, we can calculate t_{s+1} from t_s in constant time, in theory.
- This means that in theory, we can calculate t_s for all $0 \leq s \leq n - m$ in $\Theta(n - m)$ steps.
- Again, however, the numbers we are dealing with can be very large.

Modifying the Rabin-Karp Algorithm

- To deal with the very large numbers that can occur, we apply a simple hash function.
- This approach is similar to that for generating compact digital signatures.
- Our simple hash function for a string S will be $S \mod q$, where q is usually the size of the maximum integer that fits into a computer word.
- Using modular arithmetic, we can calculate $t_s \mod q$ for all $0 \leq s \leq n - m$ in $\Theta(n - m)$ steps.
- We then compare $t_s \mod q$ to $p \mod q$ for all $0 \leq s \leq n - m$.
- If $t_s \not\equiv p \mod q$, then $t_s \neq p$.
- Otherwise, we need to calculate t_s and compare it to p.
- What is the running time of this modification in the worst case?

Average Case Analysis for Rabin-Karp

- In this case, we can do a simplistic average case analysis.
- We need to know the number of shifts s for which $t_s \equiv p \mod q$.
- As usual, we will assume a random string is equally likely to hash to any value between 0 and q-1.
- In this case, an invalid shift will produce a *spurious hit* with probability $1/q$.
- The overall running time is then

$$O(n + m(v + n/q)),$$

  where v is the number of valid shifts).
- If v is "small" (constant) and $q \geq m$, then this simplifies to $O(n)$.