The COIN-OR Optimization Suite:

Python Tools for Optimization

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COIN fORgery: Developing Open Source Tools for OR

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Outline

1. Introduction to Python

2. Python Tools in COIN-OR
   - CyLP
   - yaposib
   - PuLP and Dippy
   - Pyomo
   - GiMPy
   - GrUMPy
   - CuPPy

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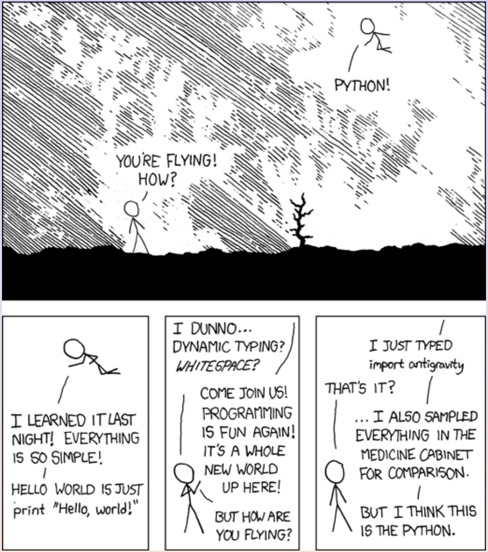
Why Python?

- Pros
  - As with many high-level languages, development in Python is quick and painless (relative to C++!).
  - Python is popular in many disciplines and there is a dizzying array of packages available.
  - Python's syntax is very clean and naturally adaptable to expressing mathematical programming models.
  - Python has the primary data structures necessary to build and manipulate models built in.
  - There has been a strong movement toward the adoption of Python as the high-level language of choice for (discrete) optimizers.
  - Sage is quickly emerging as a very capable open-source alternative to Matlab.
- Cons
  - Python's one major downside is that it can be very slow.
  - Solution is to use Python as a front-end to call lower-level tools.

Drinking the Python Kool-Aid



Introduction to Python

Adapted from a Tuturial by Guido van Rossum
Director of PythonLabs at Zope Corporation

Presented at
LinuxWorld - New York City - January 2002

Why Python?

- Interpreted language
- Intuitive syntax
- Dynamic typing
- **Loads** of built-in libraries and available extensions
- Shallow learning curve
- Easy to call C/C++ for efficiency
- Object-oriented
- Simple, but extremely powerful

Tutorial Outline

- interactive "shell‟
- basic types: numbers, strings
- container types: lists, dictionaries, tuples
- variables
- control structures
- functions & procedures
- classes & instances
- modules
- exceptions
- files & standard library

Interactive "Shell"

- Great for learning the language
- Great for experimenting with the library
- Great for testing your own modules
- Two variations: IDLE (GUI), python (command line)
- Type statements or expressions at prompt:

```
>>> print "Hello, world"
Hello, world
>>> x = 12**2
>>> x/2
72
>>> # this is a comment
```

Python Program

- To write a program, put commands in a file

```
#hello.py
print "Hello, world"
x = 12**2
x/2
print x
```

- Execute on the command line

```
~> python hello.py
Hello, world
72
```

Variables

- No need to declare
- Need to assign (initialize)
  - use of uninitialized variable raises exception
- Not typed

```
if friendly: greeting = "hello world"
else: greeting = 12**2
print greeting
```

- ***Everything*** is an “object":
  - Even functions, classes, modules

Control Structures

if *condition*:
    statements
[elif *condition*:
    statements] ...
else:
    statements

while *condition*:
    statements

for *var* in *sequence*:
    statements

break
continue

Grouping Indentation

In Python:

```
for i in range(20):
    if i%3 == 0:
        print i
        if i%5 == 0:
            print "Bingo!"
    print "---"
```

In C:

```
for (i = 0; i < 20; i++)
{
    if (i%3 == 0) {
        printf("%d\n", i);
        if (i%5 == 0) {

   printf("Bingo!\n"); }
      }
      printf("---\n");
}
```

```
0
Bingo!
---
---
---
3
---
---
---
6
---
---
---
9
---
---
---
12
---
---
---
15
Bingo!
---
---
---
18
---
---
```

Numbers

- The usual suspects
  - 12, 3.14, 0xFF, 0377, (-1+2)*3/4**5, abs(x), 0<x<=5
- C-style shifting & masking
  - 1<<16, x&0xff, x|1, ~x, x^y
- Integer division truncates :-(
  - 1/2 -> 0 # 1./2. -> 0.5, float(1)/2 -> 0.5
  - Will be fixed in the future
- Long (arbitrary precision), complex
  - 2L**100 -> 1267650600228229401496703205376L
    - In Python 2.2 and beyond, 2**100 does the same thing
  - 1j**2 -> (-1+0j)

Strings

- "hello"+"world" "helloworld" # concatenation
- "hello"*3 "hellohellohello" # repetition
- "hello"[0] "h" # indexing
- "hello"[-1] "o" # (from end)
- "hello"[1:4] "ell" # slicing
- len("hello") 5 # size
- "hello" < "jello" 1 # comparison
- "e" in "hello" 1 # search
- "escapes: \n etc, \033 etc, \if etc"
- 'single quotes' """triple quotes""" r"raw strings"

Lists

- Flexible arrays, *not* Lisp-like linked lists
  - a = [99, "bottles of beer", ["on", "the", "wall"]]
- Same operators as for strings
  - a+b, a*3, a[0], a[-1], a[1:], len(a)
- Item and slice assignment
  - a[0] = 98
  - a[1:2] = ["bottles", "of", "beer"]
    -> [98, "bottles", "of", "beer", ["on", "the", "wall"]]
  - del a[-1]   # -> [98, "bottles", "of", "beer"]

More List Operations

```
>>> a = range(5)          # [0,1,2,3,4]
>>> a.append(5)           # [0,1,2,3,4,5]
>>> a.pop()               # [0,1,2,3,4]
5
>>> a.insert(0, 42)       # [42,0,1,2,3,4]
>>> a.pop(0)              # [0,1,2,3,4]
5.5
>>> a.reverse()           # [4,3,2,1,0]
>>> a.sort()              # [0,1,2,3,4]
```

Dictionaries

- Hash tables, "associative arrays"
  - d = {"duck": "eend", "water": "water"}
- Lookup:
  - d["duck"] -> "eend"
  - d["back"] # raises KeyError exception
- Delete, insert, overwrite:
  - del d["water"] # {"duck": "eend", "back": "rug"}
  - d["back"] = "rug" # {"duck": "eend", "back": "rug"}
  - d["duck"] = "duik" # {"duck": "duik", "back": "rug"}

More Dictionary Ops

- Keys, values, items:
  - d.keys() -> ["duck", "back"]
  - d.values() -> ["duik", "rug"]
  - d.items() -> [("duck","duik"), ("back","rug")]
- Presence check:
  - d.has_key("duck") -> 1; d.has_key("spam") -> 0
- Values of any type; keys almost any
  - {"name":"Guido", "age":43, ("hello","world"):1, 42:"yes", "flag": ["red","white","blue"]}

Dictionary Details

- Keys must be **immutable**:
  - numbers, strings, tuples of immutables
    - these cannot be changed after creation
  - reason is *hashing* (fast lookup technique)
  - **not** lists or other dictionaries
    - these types of objects can be changed "in place"
  - no restrictions on values
- Keys will be listed in **arbitrary order**
  - again, because of hashing

Tuples

- key = (lastname, firstname)
- point = x, y, z   # parentheses optional
- x, y, z = point   # unpack
- lastname = key[0]
- singleton = (1,)   # trailing comma!!!
- empty = ()   # parentheses!
- tuples vs. lists; tuples immutable

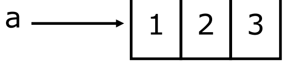
Reference Semantics

- Assignment manipulates references
  - x = y **does not make a copy** of y
  - x = y makes x **reference** the object y references
- Very useful; but beware!
- Example:

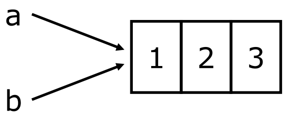
```
>>> a = [1, 2, 3]
>>> b = a
>>> a.append(4)
>>> print b
[1, 2, 3, 4]
```

Changing a Shared List

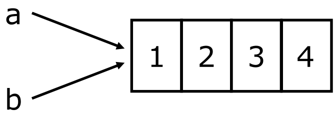
```
a = [1, 2, 3]
```




```
b = a
```




```
a.append(4)
```

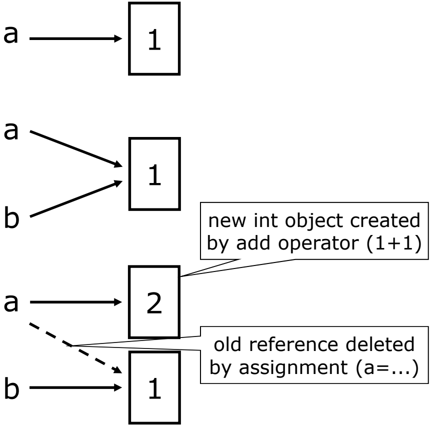


Changing an Integer

a = 1

b = a

a = a+1



Functions, Procedures

def *name*(*arg1*, *arg2*, ...):
  """*documentation*"""   # optional doc string
  statements

return   # from procedure
return *expression*   # from function

Example Function

```
def gcd(a, b):
    "greatest common divisor"
    while a != 0:
        a, b = b%a, a    # parallel assignment
    return b

>>> gcd.__doc__
'greatest common divisor'
>>> gcd(12, 20)
4
```

Classes

class *name*:
    "*documentation*"
    statements

-or-

class *name*(*base1*, *base2*, ...):
    ...

Most, *statements* are method definitions:
    def *name*(self, *arg1*, *arg2*, ...):
        ...

May also be *class variable* assignments

Example Class

```
class Stack:
    "A well-known data structure…"
    def __init__(self):          # constructor
        self.items = []
    def push(self, x):
        self.items.append(x)     # the sky is the limit
    def pop(self):
        x = self.items[-1]                 # what happens if it's empty?
        del self.items[-1]
        return x
    def empty(self):
        return len(self.items) == 0        # Boolean result
```

Using Classes

- To create an instance, simply call the class object:

```
x = Stack() # no 'new' operator!
```

- To use methods of the instance, call using dot notation:

```
x.empty()   # -> 1
x.push(1)                # [1]
x.empty()   # -> 0
x.push("hello")                # [1, "hello"]
x.pop()        # -> "hello"    # [1]
```

- To inspect instance variables, use dot notation:

```
x.items        # -> [1]
```

Modules

- Collection of stuff in *foo*.py file
  - functions, classes, variables
- Importing modules:
  - import re; print re.match("[a-z]+", s)
  - from re import match; print match("[a-z]+", s)
- Import with rename:
  - import re as regex
  - from re import match as m

Getting Python

- There are many different flavors of Python, all of which support the same basic API, but have different backends and performance.
- The “original flavor” is CPython, but there is also Jython, Iron Python, Pyjs, PyPy, RubyPython, and others.
- If you are going to use a package with a C extensions, you probably need to get CPython.
- For numerical computational, some additional packages are almost certainly required, NumPy and SciPy being the most obvious.
  - On Linux, Python and the most important packages will be pre-installed, with additional ones installed easily via a package manager.
  - On OS X, Python comes pre-installed, but it is easier to install Python and any additional packages via Homebrew.
  - On Windows, it’s easiest to install a distribution that includes the scientific software, such as PythonXY or Portable Python.
- Another option is to use Sage, a Matlab-like collection of Python packages (including COIN).

In Class Exercise: Install Python!

Getting an IDE

- An additional requirement for doing development is an IDE.
- My personal choice is Eclipse with the PyDev plug-in.
- This has the advantage of being portable and cross-platform, as well as supporting most major languages.

Python Extensions

- It is possible to implement extensions to the basic language in C/C++.
- Calls into these extensions libraries are then executed efficiently as native C/C++ code.
- Although it is possible in theory to provide binary packages for these extensions, this is a headache on OS X and Linux.
- It is likely you will have to build your own versions, but this is relatively easy.
- On Windows, building extensions is harder, but working binaries are usually easier to obtain.

Basic Build Steps

- First, build and install the relevant project using the autotools.
  - You can avoid some potential complications by configuring with `-enable-static -disable-shared`.
  - Otherwise, you need to set either `LD_LIBRARY_PATH` (Linux) or `DYLD_LIBRARY_PATH` (OS X) to point to `${prefix}/lib`.
- Next, set some environment variables.
  - For `yaposib`, you need to have `pkg-config` installed and set `PKG_CONFIG_PATH=${prefix}/lib/pkgconfig`.
  - For `CyLP` and `DipPy`, you need to set `COIN_INSTALL_DIR=${prefix}`.
- Finally, just execute `python setup.py install`.

Outline

CyLP: Low-level Modeling and API for Cbc/Clp/Cgl

- CyLP provides a low-level modeling language for accessing details of the algorithms and low-level parts of the API.
- The included modeling language is "close to the metal", works directly with numerical data with access to low-level data structures.
- Clp
  - Pivot-level control of algorithm in Clp.
  - Access to fine-grained results of solve.
- Cbc
  - Python classes for customization
- Cgl
  - Python class for building cut generators wrapped around Cgl.
- Developers: Mehdi Towhidi and Dominique Orban

CyLP: Accessing the Tableaux

```
lp = CyClpSimplex()
x = lp.addVariable('x', numVars)
lp += x_u >= x >= 0

lp += A * x <= b if cons_sense == '<=' else A * x >= b

lp.objective = -c * x if obj_sense == 'Max' else c * x
lp.primal(startFinishOptions = 1)
numCons = len(b)
print 'Current solution is', lp.primalVariableSolution['x']
print 'Current tableaux is', lp.tableaux
for row in range(lp.nConstraints):
    print 'Variables basic in row', row, 'is', lp.basicVariables[r
    print 'and has value' lp.rhs[row]
```

yaposib: Python Bindings for OSI

Provides Python bindings to any solver with an OSI interface

```
solver = yaposib.available_solvers()[0]

for filename in sys.argv[1:]:

    problem = yaposib.Problem(solver)

    print("Will now solve %s" % filename)
    err = problem.readMps(filename)
    if not err:
        problem.solve()
        if problem.status == 'optimal':
            print("Optimal value: %f" % problem.obj.value)
            for var in problem.cols:
                print("\t%s = %f" % (var.name, var.solution))
        else:
            print("No optimal solution could be found.")
```

PuLP: Algebraic Modeling in Python

- PuLP is a modeling language in COIN-OR that provides data types for Python that support algebraic modeling.
- PuLP only supports development of linear models.
- Main classes
  - `LpProblem`
  - `LpVariable`
- Variables can be declared individually or as “dictionaries” (variables indexed on another set).
- We do not need an explicit notion of a parameter or set here because Python provides data structures we can use.
- In PuLP, models are technically “concrete,” since the model is always created with knowledge of the data.
- However, it is still possible to maintain a separation between model and data.

PuLP Basics: Facility Location Example

```
from products   import REQUIREMENT, PRODUCTS
from facilities import FIXED_CHARGE, LOCATIONS, CAPACITY

prob = LpProblem("Facility_Location")

ASSIGNMENTS = [(i, j) for i in LOCATIONS for j in PRODUCTS]
assign_vars = LpVariable.dicts("x", ASSIGNMENTS, 0, 1, LpBinary)
use_vars    = LpVariable.dicts("y", LOCATIONS, 0, 1, LpBinary)

prob += lpSum(use_vars[i] * FIXED_COST[i] for i in LOCATIONS)

for j in PRODUCTS:
    prob += lpSum(assign_vars[(i, j)] for i in LOCATIONS) == 1

for i in LOCATIONS:
    prob += lpSum(assign_vars[(i, j)] * REQUIREMENT[j]
                  for j in PRODUCTS) <= CAPACITY * use_vars[i]

prob.solve()

for i in LOCATIONS:
    if use_vars[i].varValue > 0:
        print "Location ", i, " is assigned: ",
        print [j for j in PRODUCTS if assign_vars[(i, j)].varValue > 0]
```

DipPy: Modeling Decomposition (Mike O'Sullivan)

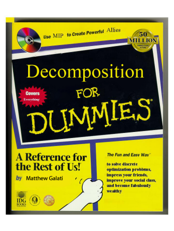
DIP Framework

DIP is a software framework and stand-alone solver for implementation and use of a variety of decomposition-based algorithms.

- Decomposition-based algorithms have traditionally been extremely difficult to implement and compare.
- DIP abstracts the common, generic elements of these methods.
  - Key: API is in terms of the compact formulation.
  - The framework takes care of reformulation and implementation.
  - DIP is now a *fully generic* decomposition-based parallel MILP solver.

Methods

- Column generation (Dantzig-Wolfe)
- Cutting plane method
- Lagrangian relaxation (not complete)



⇐ *Joke*!

DipPy Basics: Facility Location Example

```
from products   import REQUIREMENT, PRODUCTS
from facilities import FIXED_CHARGE, LOCATIONS, CAPACITY

prob = dippy.DipProblem("Facility_Location")

ASSIGNMENTS = [(i, j) for i in LOCATIONS for j in PRODUCTS]
assign_vars = LpVariable.dicts("x", ASSIGNMENTS, 0, 1, LpBinary)
use_vars    = LpVariable.dicts("y", LOCATIONS, 0, 1, LpBinary)

prob += lpSum(use_vars[i] * FIXED_COST[i] for i in LOCATIONS)

for j in PRODUCTS:
    prob += lpSum(assign_vars[(i, j)] for i in LOCATIONS) == 1

\color{red}for i in LOCATIONS:
\color{red}    prob.relaxation[i] += lpSum(assign_vars[(i, j)] * REQUIREMENT[j]
\color{red}                                for j in PRODUCTS) <= CAPACITY * use_vars[i]

dippy.Solve(prob, {doPriceCut:1})

for i in LOCATIONS:
    if use_vars[i].varValue > 0:
        print "Location ", i, " is assigned: ",
        print [j for j in PRODUCTS if assign_vars[(i, j)].varValue > 0]
```

In Class Exercise: Install DipPy!

Pyomo

- An algebraic modeling language in Python similar to PuLP.
- Can import data from many sources, including AMPL data files.
- More powerful, includes support for nonlinear modeling.
- Allows development of both concrete models (like PuLP) and abstract models (like AMPL).
- Also include PySP for stochastic Programming.
- Primary classes
  - `ConcreteModel`, `AbstractModel`
  - `Set`, `Parameter`
  - `Var`, `Constraint`
- Developers: Bill Hart, John Siirola, Jean-Paul Watson, David Woodruff, and others...

Pyomo Basics: Dedication Model

```
model = ConcreteModel()

Bonds, Features, BondData, Liabilities = read_data('ded.dat')

Periods = range(len(Liabilities))

model.buy = Var(Bonds, within=NonNegativeReals)
model.cash = Var(Periods, within=NonNegativeReals)
model.obj = Objective(expr=model.cash[0] +
            sum(BondData[b, 'Price']*model.buy[b] for b in Bonds),
            sense=minimize)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t]
             + sum(BondData[b, 'Coupon'] * model.buy[b]
               for b in Bonds if BondData[b, 'Maturity'] >= t)
             + sum(BondData[b, 'Principal'] * model.buy[b]
               for b in Bonds if BondData[b, 'Maturity'] == t)
             == Liabilities[t])
model.cash_balance = Constraint(Periods[1:], rule=cash_balance_rule)
```

In Class Exercise: Install Pyomo!

```
pip install pyomo
```

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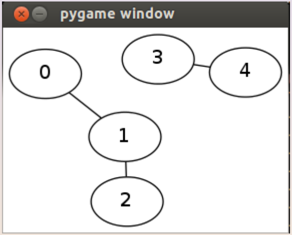
- GiMPy
- GrUMPy
- CuPPy

GiMPy (with Aykut Bulut)

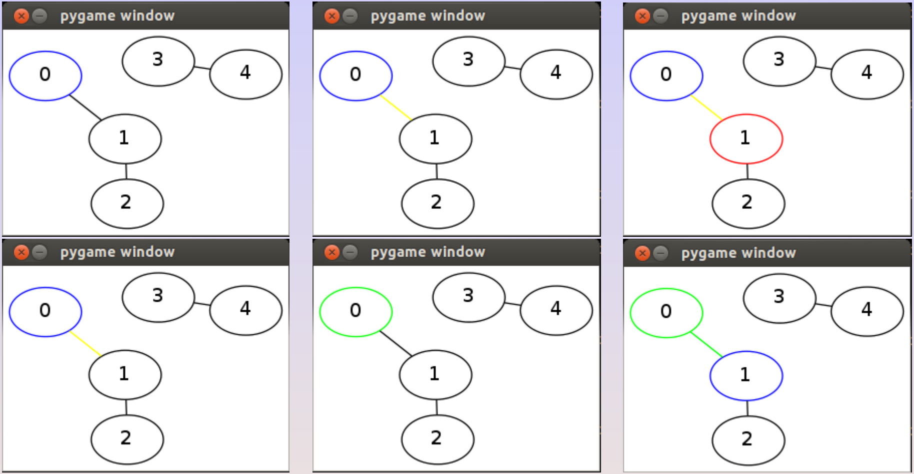
- A graph class for Python $2.*$.
- Builds, displays, and saves graphs (many options)
- Focus is on *visualization* of well-known graph algorithms.
  - Priority in implementation is on *clarity* of the algorithms.
  - Efficiency is *not* the goal (though we try to be as efficient as we can).

```
easy_install install coinor.grumpy
```

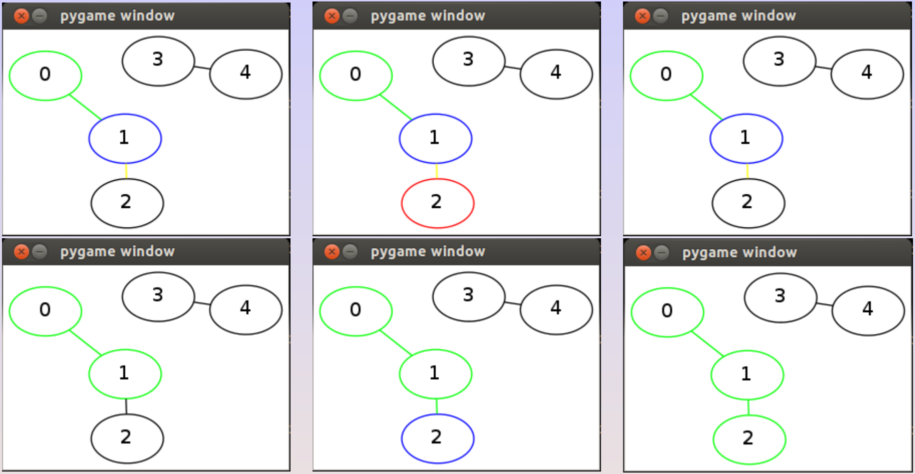
```
g = Graph(display='xdot')
g.add_edge(0,1)
g.add_edge(1,2)
g.add_edge(3,4)
g.display()
g.search(0)
```



GIMPy Example



GiMPy Example



GiMPy: Graph Methods in Python

The following problem/algorithm pairs with similar visualization options exist.

- Graph Search:
  - BFS
  - DFS
  - Prim's
  - Component Labeling,
  - Dijkstra's
  - Topological Sort
- Shortest path: Dijkstra's, Label Correcting
- Maximum flow: Augmenting Path, Preflow Push
- Minimum spanning tree: Prim's Algorithm, Kruskal Algorithm
- Minimum Cost Flow: Network Simplex, Cycle Canceling
- Data structures: Union-Find (quick union, quick find), Binary Search Tree, Heap

GiMPy Tree

- `Tree` class derived from `Graph` class.
- `BinaryTree` class derived from `Tree` class.
- Has binary tree specific API and attributes.

GrUMPy Overview

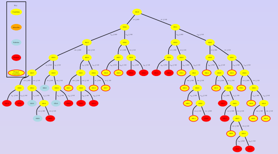
- Visualizations for solution methods for linear models.
  - Branch and bound
  - Cutting plane method
- `BBTree` derived from GiMPy `Tree`.
  - Reads branch-and-bound data either dynamically or statically.
  - Builds dynamic visualizations of solution process.
  - Includes a pure Python branch and bound implementation.
- `Polyhedron2D` derived from `pypolyhedron`.
  - Can construct 2D polyhedra defined by generators or inequalities.
  - Displays convex hull of integer points.
  - Can produce animations of the cutting plane method.
- GrUMPy is an expansion and continuation of the BAK project (Brady Hunsaker and Osman Ozaltin).

```
easy_install coinor.grumpy
```

GrUMPy: `BBTree` Branch and Bound Implementation

```
T = BBTree()
#T.set_layout('dot2tex')
#T.set_display_mode('file')
T.set_display_mode('xdot')
CONSTRAINTS, VARIABLES, OBJ, MAT, RHS = \
    T.GenerateRandomMIP(rand_seed = 19)
T.BranchAndBound(CONSTRAINTS, VARIABLES, OBJ, MAT, RHS,
                 branch_strategy = PSEUDOCOST_BRANCHING,
                 search_strategy = BEST_FIRST,
                 display_interval = 1)
```

GrUMPy: `BBTree` Branch and Bound Implementation



GrUMPy: Dynamic Branch and Bound Visualizations

- GrUMPy provides four visualizations of the branch and bound process.
- Can be used dynamically or statically with any instrumented solver.
  - BB tree
  - Histogram
  - Scatter plot
  - Incumbent path

GrUMPy Branch and Bound Tree

Figure: BB tree generated by GrUMPy

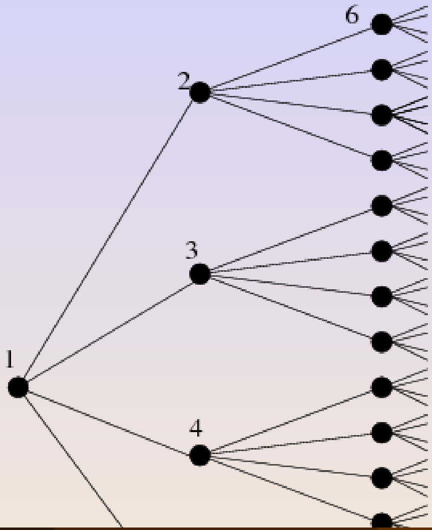
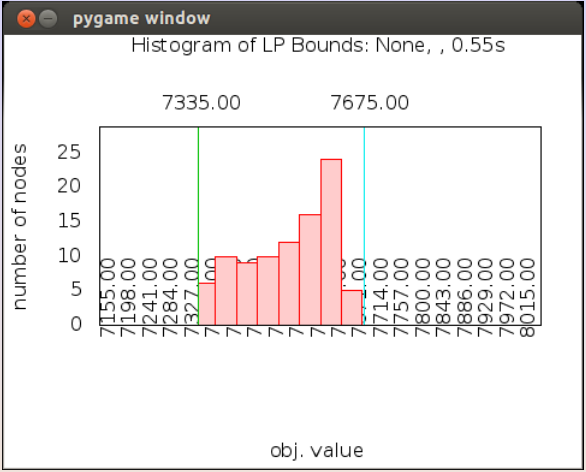
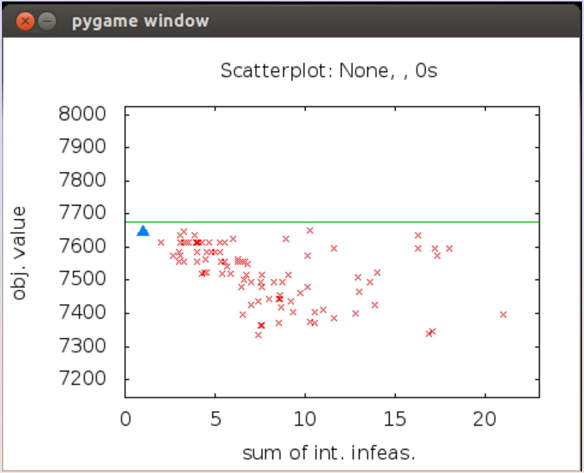


Figure: BB histogram generated by GrUMPy



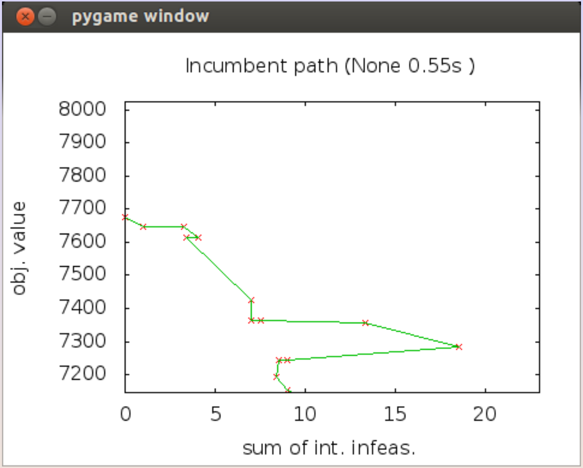
GrUMPy Scatter Plot

Figure: Scatter plot generated by GrUMPy



GrUMPy Incumbent Path

Figure: Incumbent path generated by GrUMPy



GrUMPy: `Polyhedron2D`

```
f = Figure()
p = Polyhedron2D(A = [[4, 1], [1, 4], [1, -1], [-1, 0], [0, -1]],
                 b = [28, 27, 1, 0, 0])
#p = Polyhedron2D(points = [[0, 0], [2, 2], [3.75, 2.75], [3, 1]])
f.add_polyhedron(p, color = 'blue', linestyle = 'solid', label = 'p',
                 show_int_points = True)
f.set_xlim(p.plot_min[0], p.plot_max[0])
f.set_ylim(p.plot_min[1], p.plot_max[1])
pI = p.make_integer_hull()
f.add_polyhedron(pI, color = 'red', linestyle = 'dashed', label = 'pI')
f.add_point((5.666,5.333), 0.02, 'red')
f.add_text(5.7, 5.4, r'$(17/3, 16/3)$')
f.add_line([3, 2], 27, p.plot_max, p.plot_min,
           color = 'green', linestyle = 'dashed')
f.show()
```

`Polyhedron2D`: Visualzing Polyhedra

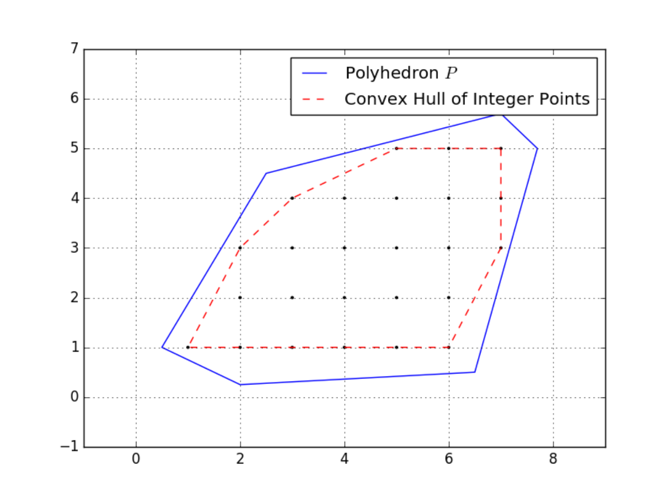


Figure: Convex hull of $\mathcal{S}$

CuPPy: Cutting Planes in Python

- Simple implementations and visualizations of cutting plane procedures.
- Uses CyLP to access the tableaux of the underlying Clp model.
- Currently has visualizations for GMI and split cuts.

```
f0 = getFraction(sol[basicVarInd])
f = [getFraction(lp.tableau[row, i]) for i in range(lp.nVariables]
pi = np.array([f[j]/f0 if f[j] <= f0
               else (1-f[j])/(1-f0) for j in range(lp.nVariables)])
pi_slacks = np.array([x/f0 if x > 0 else -x/(1-f0)
                      for x in lp.tableau[row, lp.nVariables:]])
pi -= pi_slacks * lp.coefMatrix
pi0 = (1 - np.dot(pi_slacks, lp.constraintsUpper) if sense == '<='
       else 1 + np.dot(pi_slacks, lp.constraintsUpper))
```

```
easy_install coinor.grumpy
```

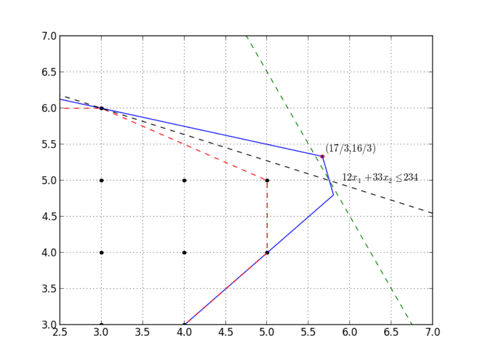
GrUMPy + CuPPy: Visualizing GMI and Gomory Cuts

The GMI cut from the first row is

$$\frac{1}{10}s_1 + \frac{8}{10}s_2 \geq 1, \qquad (1)$$

In terms of x_1 and x_2, we have

$$12x_1 + 33x_2 \leq 234, \qquad \text{(GMI-C1)}$$



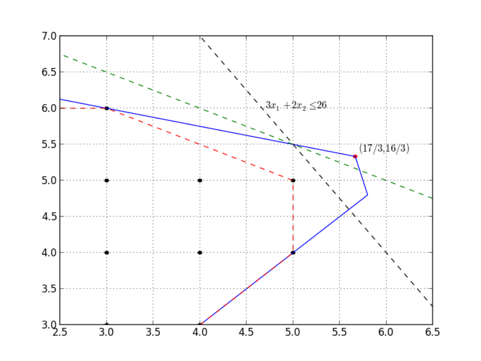
GrUMPy + CuPPy: Visualizing GMI and Gomory Cuts

The GMI cut from the third row is

$$\frac{4}{10}s_1 + \frac{2}{10}s_2 \geq 1, \tag{2}$$

In terms of x_1 and x_2, we have

$$3x_1 + 2x_2 \leq 26, \tag{GMI-C3}$$



GrUMPy + CuPPy: Visualizing Intersection Cuts

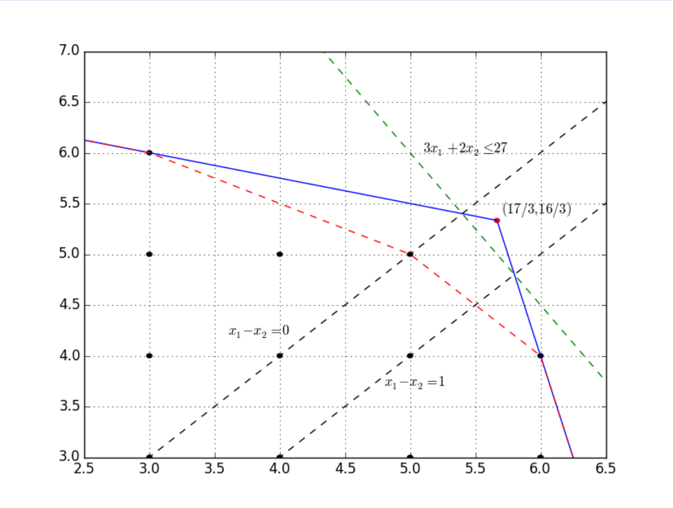


Figure: GMI Cut from row 2 as an intersection cut

End of Part 3!

Questions?