The COIN-OR Optimization Suite:

Algegraic Modeling Ted Ralphs

COMPUTATIONAL INFRASTRUCTURE FOR OPERATIONS RESEARCH
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COIN fORgery: Developing Open Source Tools for OR

Institute for Mathematics and Its Applications, Minneapolis, MN

Outline

1. Introduction
2. Solver Studio
3. Traditional Modeling Environments
4. Python-Based Modeling
5. Comparative Case Studies

Outline

Algebraic Modeling Languages

- Generally speaking, we follow a four-step process in modeling.
  - Develop an abstract model.
  - Populate the model with data.
  - Solve the model.
  - Analyze the results.
- These four steps generally involve different pieces of software working in concert.
- For mathematical programs, the modeling is often done with an *algebraic modeling system*.
- Data can be obtained from a wide range of sources, including spreadsheets.
- Solution of the model is usually relegated to specialized software, depending on the type of model.

Modeling Software

Most existing modeling software can be used with COIN solvers.

- Commercial Systems
    - GAMS
    - MPL
    - AMPL
    - AIMMS
- Python-based Open Source Modeling Languages and Interfaces
    - Pyomo
    - PuLP/Dippy
    - CyLP (provides API-level interface)
    - yaposib

Modeling Software (cont'd)

- Other Front Ends (mostly open source)
  - FLOPC++ (algebraic modeling in C++)
  - CMPL
  - MathProg.jl (modeling language built in Julia)
  - GMPL (open-source AMPL clone)
  - ZMPL (stand-alone parser)
  - SolverStudio (spreadsheet plug-in: www.OpenSolver.org)
  - Open Office spreadsheet
  - R (RSymphony Plug-in)
  - Matlab (OPTI)
  - Mathematica

COIN-OR Solvers with Modeling Language Support

- COIN-OR is an open source project dedicated to the development of open source software for solving operations research problems.
- COIN-OR distributes a free and open source suite of software that can handle all the classes of problems we'll discuss.
  - Clp (LP)
  - Cbc (MILP)
  - Ipopt (NLP)
  - SYMPHONY (MILP, BMILP)
  - DIP (MILP)
  - Bonmin (Convex MINLP)
  - Couenne (Non-convex MINLP)
  - Optimization Services (Interface)
- COIN also develops standards and interfaces that allow software components to interoperate.
- Check out the Web site for the project at `http://www.coin-or.org`

How They Interface

- Although not required, it’s useful to know something about how modeling languages interface with solvers.
- In many cases, modeling languages interface with solvers by writing out an intermediate file that the solver then reads in.
- It is also possible to generate these intermediate files directly from a custom-developed code.
- Common file formats
    - MPS format: The original standard developed by IBM in the days of Fortran, not easily human-readable and only supports (integer) linear modeling.
    - LP format: Developed by CPLEX as a human-readable alternative to MPS.
    - .nl format: AMPL’s intermediate format that also supports non-linear modeling.
    - OSIL: an open, XML-based format used by the Optimization Services framework of COIN-OR.
    - Python C Extension: Several projects interface through a Python extension that can be easily

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2. **Solver Studio**
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SolverStudio (Andrew Mason)

- Spreadsheet optimization has had a (deservedly) bad reputation for many years.
- SolverStudio will change your mind about that!
- SolverStudio provides a full-blown modeling environment inside a spreadsheet.
  - Edit and run the model.
  - Populate the model from the spreadsheet.
- In many of the examples in the remainder of the tutorial, I will show the models in SolverStudio.

In Class Exercise: Install Solver Studio!

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Traditional Modeling Environments

- AMPL is one of the most commonly used modeling languages, but many other languages, including GAMS, are similar in concept.
- AMPL has many of the features of a programming language, including loops and conditionals.
- Most available solvers will work with AMPL.
- GMPL and ZIMPL are open source languages that implement subsets of AMPL.
- The Python-based languages to be introduced later have similar functionality, but a more powerful programming environment.
- AMPL will work with all of the solvers we've discussed so far.
- You can also submit AMPL models to the NEOS server.
- Student versions can be downloaded from `www.ampl.com`.

Example: Simple Bond Portfolio Model (`bonds_simple.mod`)

- A bond portfolio manager has $\$100K$ to allocate to two different bonds.

Bond	Yield	Maturity	Rating
A	4	3	A (2)
B	3	4	Aaa (1)

- The goal is to maximize total return subject to the following limits.
  - The average rating must be at most 1.5 (lower is better).
  - The average maturity must be at most 3.6 years.
- Any cash not invested will be kept in a non-interest bearing account and is assumed to have an implicit rating of 0 (no risk).

AMPL Concepts

- In many ways, AMPL is like any other programming language.
- Example: Bond Portfolio Model

```
ampl: option solver clp;
ampl: var X1;
ampl: var X2;
ampl: maximize yield: 4*X1 + 3*X2;
ampl: subject to cash: X1 + X2 <= 100;
ampl: subject to rating: 2*X1 + X2 <= 150;
ampl: subject to maturity: 3*X1 + 4*X2 <= 360;
ampl: subject to X1_limit: X1 >= 0;
ampl: subject to X2_limit: X2 >= 0;
ampl: solve;
...
ampl: display X1;
X1 = 50
ampl: display X2;
X2 = 50
```

Storing Commands in a File (`bonds_simple.run`)

- You can type the commands into a file and then load them.
- This makes it easy to modify your model later.
- Example:

```
ampl: option solver clp;
ampl: model bonds_simple.mod;
ampl: solve;
...
ampl: display X1;
X1 = 50
ampl: display X2;
X2 = 50
```

Generalizing the Model

- Suppose we don't know ahead of time what bonds we want to include or what the input data describing each bond will be.
- For this purpose, we can develop an abstract algebraic model without specifying values for the input data.
- Components of an abstract algebraic model are
  - Data
    - Sets: Lists of stocks and other investment options
    - Parameters: Numerical inputs such as budget restrictions, historical returns, etc.
  - Model
    - Variables: Values in the model that need to be decided upon.
    - Objective Function: A function of the variable values to be maximized or minimized.
    - Constraints: Functions of the variable values that must lie within given bounds.

Example: General Bond Portfolio Model (`bonds.mod`)

```
set bonds;                    # bonds available

param yield {bonds};          # yields
param rating {bonds};         # ratings
param maturity {bonds};       # maturities
param max_rating;             # Maximum average rating allowed
param max_maturity;           # Maximum maturity allowed
param max_cash;               # Maximum available to invest

var buy {bonds} >= 0;         # amount to invest in bond i

maximize total_yield : sum {i in bonds} yield[i] * buy[i];

subject to cash_limit : sum {i in bonds} buy[i] <= max_cash;
subject to rating_limit :
   sum {i in bonds} rating[i]*buy[i] <= max_cash*max_rating;
subject to maturity_limit :
   sum {i in bonds} maturity[i]*buy[i] <= max_cash*max_maturity;
```

Getting the Data (`bonds.dat`)

- The data to populate the model can come from a number of sources.
- AMPL has its own format for specifying the data in the model.

```
set bonds := A B;

param : yield rating maturity :=
  A        4      2        3
  B        3      1        4;

param max_cash := 100;
param max_rating 1.5;
param max_maturity 3.6;
```

Solving the Model (`bonds.run`)

```
ampl: model bonds.mod;
ampl: data bonds.dat;
ampl: solve;
...
ampl: display buy;
buy [*] :=
A  50
B  50
;
```

Modifying the Data (`bonds_alt.dat`)

- Suppose we want to increase available production hours by 2000.
- To resolve from scratch, simply modify the data file and reload.

```
ampl: reset data;
ampl: data bonds_alt.dat;
ampl: solve;
...
ampl: display buy;
buy [*] :=
A   30
B   70
;
```

Modifying Individual Data Elements

- Instead of resetting all the data, you can modify one element.

```
ampl: reset data max_cash;
ampl: data;
ampl data: param max_cash := 150;
ampl data: solve;
...
ampl: display buy;
buy [*] :=
A  45
B  105
;
```

Extending the Model (`bonds_extended.dat`)

- Now suppose we want to add another type of bond.

```
set bonds := A B C;

param : yield rating maturity :=
  A        4     2      3
  B        3     1      4
  C        6     3      2;

param max_cash := 100;
param max_rating 1.3;
param max_maturity 3.8;
```

Solving the Extended Model

```
ampl: reset data;
ampl: data bonds_extended.dat;
ampl: solve;
..
ampl: display buy;
buy [*] :=
A   0
B  85
C  15
;
```

Getting Data from a Spreadsheet (`FinancialModels.xlsx:Bonds1-AMPL`)

- Another obvious source of data is a spreadsheet, such as Excel.
- AMPL has commands for accessing data from a spreadsheet directly from the language.
- An alternative is to use SolverStudio.
- SolverStudio allows the model to be composed within Excel and imports the data from an associated sheet.
- Results can be printed to a window or output to the sheet for further analysis.

Further Generalization (`FinancialModels.xlsx:Bonds2-AMPL`)

- Note that in our AMPL model, we essentially had three “features” of a bond that we wanted to take into account.
  - Maturity
  - Rating
  - Yield
- We constrained the level of two of these and then optimized the third one.
- The constraints for the features all have the same basic form.
- What if we wanted to add another feature?
- We can make the list of features a set and use the concept of a two-dimensional parameter to create a table of bond data.

The Generalized Model (`bonds_features.mod`)

```
set bonds;
set features;

param bond_data {bonds, features};
param limits{features};
param yield{bonds};

param max_cash;

var buy {bonds} >= 0;

maximize obj : sum {i in bonds} yield[i] * buy[i];

subject to cash_limit : sum {i in bonds} buy[i] <= max_cash;

subject to limit_constraints {f in features}:
sum {i in bonds} bond_data[i, f]*buy[i] <=  max_cash*limits[f];
```

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Example: Facility Location Problem

- We have n locations and m customers to be served from those locations.
- There is a fixed cost c_j and a capacity W_j associated with facility j.
- There is a cost d_{ij} and demand w_{ij} for serving customer i from facility j.

- We have two sets of binary variables.
  - y_j is 1 if facility j is opened, 0 otherwise.
  - x_{ij} is 1 if customer i is served by facility j, 0 otherwise.

Capacitated Facility Location Problem

$$
\begin{array}{lll}
\min & \sum_{j=1}^{n} c_j y_j + \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij} = 1 & \forall i \\
 & \sum_{i=1}^{m} w_{ij} x_{ij} \leq W_j & \forall j \\
 & x_{ij} \leq y_j & \forall i, j \\
 & x_{ij}, y_j \in \{0, 1\} & \forall i, j
\end{array}
$$

PuLP Basics: Facility Location Example

```
from products   import REQUIREMENT, PRODUCTS
from facilities import FIXED_CHARGE, LOCATIONS, CAPACITY

prob = LpProblem("Facility_Location")

ASSIGNMENTS = [(i, j) for i in LOCATIONS for j in PRODUCTS]
assign_vars = LpVariable.dicts("x", ASSIGNMENTS, 0, 1, LpBinary)
use_vars    = LpVariable.dicts("y", LOCATIONS, 0, 1, LpBinary)

prob += lpSum(use_vars[i] * FIXED_COST[i] for i in LOCATIONS)

for j in PRODUCTS:
    prob += lpSum(assign_vars[(i, j)] for i in LOCATIONS) == 1

for i in LOCATIONS:
    prob += lpSum(assign_vars[(i, j)] * REQUIREMENT[j]
                  for j in PRODUCTS) <= CAPACITY * use_vars[i]

prob.solve()

for i in LOCATIONS:
    if use_vars[i].varValue > 0:
        print "Location ", i, " is assigned: ",
        print [j for j in PRODUCTS if assign_vars[(i, j)].varValue > 0]
```

PuLP Basics: Facility Location Example

```
# The requirements for the products
REQUIREMENT = {
    1 : 7,
    2 : 5,
    3 : 3,
    4 : 2,
    5 : 2,
}
# Set of all products
PRODUCTS = REQUIREMENT.keys()
PRODUCTS.sort()
# Costs of the facilities
FIXED_COST = {
    1 : 10,
    2 : 20,
    3 : 16,
    4 : 1,
    5 : 2,
}
# Set of facilities
LOCATIONS = FIXED_COST.keys()
LOCATIONS.sort()
# The capacity of the facilities
CAPACITY = 8
```

Dippy Basics

- DiPPy is an extension of PuLP that provides the ability to model decomposition-based structure.
- With DipPy, one can implement customized subroutines for column generation, cut generation, heuristics, branching, etc. in Python.
- The framework handles the incorporation of these into an overall branch and Xxx algorithm.
- This makes it easy to get up and running with relatively sohisticated methodology.
- It also makes it easy to compare methodologies with as many variables fixed as possible.
- Switching from branch and cut to branch and price is as easy as changing a parameter.
- With SolverStudio, it can even be done from a spreadsheet!.
- There are defaults for all methods—the user need not implement anything to utilize the underlying solver.

DipPy Basics: Facility Location Example

```
from products   import REQUIREMENT, PRODUCTS
from facilities import FIXED_CHARGE, LOCATIONS, CAPACITY

prob = dippy.DipProblem("Facility_Location")

ASSIGNMENTS = [(i, j) for i in LOCATIONS for j in PRODUCTS]
assign_vars = LpVariable.dicts("x", ASSIGNMENTS, 0, 1, LpBinary)
use_vars    = LpVariable.dicts("y", LOCATIONS, 0, 1, LpBinary)

prob += lpSum(use_vars[i] * FIXED_COST[i] for i in LOCATIONS)

for j in PRODUCTS:
    prob += lpSum(assign_vars[(i, j)] for i in LOCATIONS) == 1

for i in LOCATIONS:
    prob.relaxation[i] += lpSum(assign_vars[(i, j)] * REQUIREMENT[j]
                          for j in PRODUCTS) <= CAPACITY * use_vars[i]

dippy.Solve(prob, {doPriceCut:1})

for i in LOCATIONS:
    if use_vars[i].varValue > 0:
        print "Location ", i, " is assigned: ",
        print [j for j in PRODUCTS if assign_vars[(i, j)].varValue > 0]
```

«««< Updated upstream

Dippy Callbacks

```
def solve_subproblem(prob, index, redCosts, convexDual):
   ...
   return knapsack01(obj, weights, CAPACITY)
def knapsack01(obj, weights, capacity):
    ...
    return solution
def first_fit(prob):
    ...
    return bvs
prob.init_vars = first_fit
def choose_branch(prob, sol):
   ...
   return ([], down_branch_ub, up_branch_lb, [])
def generate_cuts(prob, sol):
    ...
    return new_cuts
def heuristics(prob, xhat, cost):
    ...
    return sols
dippy.Solve(prob, {’doPriceCut’: ’1’})
```

Dippy Callbacks

```
def solve_subproblem(prob, index, redCosts, convexDual):
   ...
   return knapsack01(obj, weights, CAPACITY)
def knapsack01(obj, weights, capacity):
    ...
    return solution
def first_fit(prob):
    ...
    return bvs
prob.init_vars = first_fit
def choose_branch(prob, sol):
   ...
   return ([], down_branch_ub, up_branch_lb, [])
def generate_cuts(prob, sol):
    ...
    return new_cuts
def heuristics(prob, xhat, cost):
    ...
    return sols
dippy.Solve(prob, {’doPriceCut’: ’1’})
```

Pyomo Basics

- In contrast to PuLP, Pyomo allows the creaton of “abstract” models, like other AMLs.
- Note, however, that it can also be used to create concrete models, like PuLP.
- Like, it can read data from a wide range of source.
- It also allows constraints to involve more general functions.
- As we will see, this power comes with some increased complexity.

Pyomo Basics: Dedication Model

```
model = ConcreteModel()

Bonds, Features, BondData, Liabilities = read_data('ded.dat')

Periods = range(len(Liabilities))

model.buy = Var(Bonds, within=NonNegativeReals)
model.cash = Var(Periods, within=NonNegativeReals)
model.obj = Objective(expr=model.cash[0] +
            sum(BondData[b, 'Price']*model.buy[b] for b in Bonds),
            sense=minimize)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t]
             + sum(BondData[b, 'Coupon'] * model.buy[b]
               for b in Bonds if BondData[b, 'Maturity'] >= t)
             + sum(BondData[b, 'Principal'] * model.buy[b]
               for b in Bonds if BondData[b, 'Maturity'] == t)
             == Liabilities[t])
model.cash_balance = Constraint(Periods[1:], rule=cash_balance_rule)
```

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Bond Portfolio Example: Simple PuLP Model (`bonds_simple-PuLP.py`)

```
from pulp import LpProblem, LpVariable, lpSum, LpMaximize, value

prob = LpProblem("Dedication Model", LpMaximize)

X1 = LpVariable("X1", 0, None)
X2 = LpVariable("X2", 0, None)

prob += 4*X1 + 3*X2
prob += X1 + X2 <= 100
prob += 2*X1 + X2 <= 150
prob += 3*X1 + 4*X2 <= 360

prob.solve()

print 'Optimal total cost is: ', value(prob.objective)

print "X1 :", X1.varValue
print "X2 :", X2.varValue
```

Notes About the Model

- Like the simple AMPL model, we are not using indexing or any sort of abstraction here.
- The syntax is very similar to AMPL.
- To achieve separation of data and model, we use Python's `import` mechanism.

Bond Portfolio Example: Abstract PuLP Model (`bonds-PuLP.py`)

```
from pulp import LpProblem, LpVariable, lpSum, LpMaximize, value

from bonds import bonds, max_rating, max_maturity, max_cash

prob = LpProblem("Bond Selection Model", LpMaximize)

buy = LpVariable.dicts('bonds', bonds.keys(), 0, None)

prob += lpSum(bonds[b]['yield'] * buy[b] for b in bonds)

prob += lpSum(buy[b] for b in bonds) <= max_cash, "cash"

prob += (lpSum(bonds[b]['rating'] * buy[b] for b in bonds)
         <= max_cash*max_rating, "ratings")

prob += (lpSum(bonds[b]['maturity'] * buy[b] for b in bonds)
         <= max_cash*max_maturity, "maturities")
```

Notes About the Model

- We can use Python's native `import` mechanism to get the data.
- Note, however, that the data is read and stored *before* the model.
- This means that we don't need to declare sets and parameters.
- Carriage returns are syntactic (parentheses imply line continuation).
- Constraints
  - Naming of constraints is optional and only necessary for certain kinds of post-solution analysis.
  - Constraints are added to the model using a very intuitive syntax.
  - Objectives are nothing more than expressions that are to be optimized rather than explicitly constrained.
- Indexing
  - Indexing in Python is done using the native dictionary data structure.
  - Note the extensive use of comprehensions, which have a syntax very similar to quantifiers in a mathematical model.

Bond Portfolio Example: Solution in PuLP

```
prob.solve()

epsilon = .001

print 'Optimal purchases:'
for i in bonds:
    if buy[i].varValue > epsilon:
        print 'Bond', i, ":", buy[i].varValue
```

Bond Portfolio Example: Data Import File (`bonds_data.py`)

```
bonds = {'A' : {'yield'     : 4,
                'rating'    : 2,
                'maturity'  : 3,},

         'B' : {'yield'     : 3,
                'rating'    : 1,
                'maturity'  : 4,},
        }

max_cash = 100
max_rating = 1.5
max_maturity = 3.6
```

Notes About the Data Import

- We are storing the data about the bonds in a “dictionary of dictionaries.”
- With this data structure, we don’t need to separately construct the list of bonds.
- We can access the list of bonds as `bonds.keys()`.
- Note, however, that we still end up hard-coding the list of features and we must repeat this list of features for every bond.
- We can avoid this using some advanced Python programming techniques, but SolverStudio makes this easy.

Bond Portfolio Example: PuLP Model in SolverStudio (`FinancialModels.xlsx:Bonds-PuLP`)

```
buy = LpVariable.dicts('bonds', bonds, 0, None)
for f in features:
    if limits[f] == "Opt":
        if sense[f] == '>':
            prob += lpSum(bond_data[b, f] * buy[b] for b in bonds)
        else:
            prob += lpSum(-bond_data[b, f] * buy[b] for b in bonds)
    else:
        if sense[f] == '>':
            prob += (lpSum(bond_data[b,f]*buy[b] for b in bonds) >=
                              max_cash*limits[f], f)
        else:
            prob += (lpSum(bond_data[b,f]*buy[b] for b in bonds) <=
                               max_cash*limits[f], f)
prob += lpSum(buy[b] for b in bonds) <= max_cash, "cash"
```

Notes About the SolverStudio PuLP Model

- We've explicitly allowed the option of optimizing over one of the features, while constraining the others.
- Later, we'll see how to create tradeoff curves showing the tradeoffs among the constraints imposed on various features.

Portfolio Dedication

Definition

Dedication or *cash flow matching* refers to the funding of known future liabilities through the purchase of a portfolio of risk-free non-callable bonds.

Notes:

- Dedication is used to eliminate interest rate risk.
- Dedicated portfolios do not have to be managed.
- The goal is to construct such portfolio at a minimal price from a set of available bonds.
- This is a multi-period model.

Example: Portfolio Dedication

- A pension fund faces liabilities totalling ℓ_j for years $j = 1, ..., T$.
- The fund wishes to dedicate these liabilities via a portfolio comprised of n different types of bonds.
- Bond type i costs c_i, matures in year m_i, and yields a yearly coupon payment of d_i up to maturity.
- The principal paid out at maturity for bond i is p_i.

LP Formulation for Portfolio Dedication

- We assume that for each year j there is at least one type of bond i with maturity $m_i = j$, and there are none with $m_i > T$.
- Let x_i be the number of bonds of type i purchased, and let z_j be the cash on hand at the beginning of year j for $j = 0, \ldots, T$. Then the dedication problem is the following LP.

$$
\begin{aligned}
&\min_{(x,z)} z_0 + \sum_i c_i x_i \\
\text{s.t. } & z_{j-1} - z_j + \sum_{\{i:m_i \geq j\}} d_i x_i + \sum_{\{i:m_i = j\}} p_i x_i = \ell_j, \quad (j = 1, \ldots, T-1) \\
& z_T + \sum_{\{i:m_i = T\}} (p_i + d_i) x_i = \ell_T. \\
& z_j \geq 0, j = 1, \ldots, T \\
& x_i \geq 0, i = 1, \ldots, n
\end{aligned}
$$

Basic AMPL Model for Dedication (`dedication.mod`)

Here is the model for the portfolio dedication example.

```
set Bonds;
param T > 0 integer;
param Liabilities {1..T};
param Price {Bonds};
param Maturity {Bonds};
param Coupon {Bonds};
param Principal {Bonds};

var buy {Bonds} >= 0;
var cash {0..T} >= 0;

minimize total_cost : cash[0] + sum {i in Bonds} Price[i] * buy[i]

subject to cash_balance {t in 1..T}: cash[t-1] - cash[t] +
        sum{i in Bonds : Maturity[i] >= t} Coupon[i] * buy[i] +
        sum{i in Bonds : Maturity[i] = t} Principal[i] * buy[i] =
        Liabilities[t];
```

Notes on AMPL Model

- In multi-period models, we have to somehow represent the set of periods.
- Such a set is different from a generic set because it involves *ranged data*.
- We must somehow do arithmetic with elements of this set in order to express the model.
- In AMPL, a ranged set can be constructed using the syntax `1..T`.
- Both endpoints are included in the range.
- Another important feature of the above model is the use of conditionals in the limits of the sum.
- Conditionals can be used to choose a subset of the items in a given set satisfying some condition.

PuLP Model for Dedication (`dedication-PuLP.py`)

```
Bonds, Features, BondData, Liabilities = read_data('ded.dat')

prob = LpProblem("Dedication Model", LpMinimize)

buy  = LpVariable.dicts("buy", Bonds, 0, None)
cash = LpVariable.dicts("cash", range(len(Liabilities)), 0, None)

prob += cash[0] + lpSum(BondData[b, 'Price']*buy[b] for b in Bonds)

for t in range(1, len(Liabilities)):
    prob += (cash[t-1] - cash[t]
             + lpSum(BondData[b, 'Coupon'] * buy[b]
               for b in Bonds if BondData[b, 'Maturity'] >= t)
             + lpSum(BondData[b, 'Principal'] * buy[b]
               for b in Bonds if BondData[b, 'Maturity'] == t)
             == Liabilities[t], "cash_balance_%s"%t)
```

Notes on PuLP Model

- We are parsing the AMPL data file with a custom-written function `read_data` to obtain the data.
- The data is stored in a two-dimensional table (dictionary with tuples as keys).
- The *range* operator is used to create ranged sets in Python.
- The upper endpoint is not included in the range and ranges start at 0 by default (`range(3) = [0, 1, 2]`).
- The `len` operator gets the number of elements in a given data structure.
- Python also supports conditions in comprehensions, so the model reads naturally in Python's native syntax.
- See also `FinancialModels.xlsx:Dedication-PuLP`.

Concrete Pyomo Model for Dedication (`dedication-PyomoConcrete.py`)

```
Bonds, Features, BondData, Liabilities = read_data(’ded.dat’)

Periods = range(len(Liabilities))

model.buy = Var(Bonds, within=NonNegativeReals)
model.cash = Var(Periods, within=NonNegativeReals)
model.obj = Objective(expr=model.cash[0] +
            sum(BondData[b, ’Price’]⋆model.buy[b] for b in Bonds),
            sense=minimize)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t]
             + sum(BondData[b, ’Coupon’] ⋆ model.buy[b]
               for b in Bonds if BondData[b, ’Maturity’] >= t)
             + sum(BondData[b, ’Principal’] ⋆ model.buy[b]
               for b in Bonds if BondData[b, ’Maturity’] == t)
             == Liabilities[t])
model.cash_balance = Constraint(Periods[1:], rule=cash_balance_rule)
```

Notes on the Concrete Pyomo Model

- This model is almost identical to the PuLP model.
- The only substantial difference is the way in which constraints are defined, using “rules.”
- Indexing is implemented by specifying additional arguments to the rule functions.
- When the rule function specifies an indexed set of constraints, the indices are passed through the arguments to the function.
- The model is constructed by looping over the index set, constructing each associated constraint.
- Note the use of the Python slice operator to extract a subset of a ranged set.

Instantiating and Solving a Pyomo Model

- The easiest way to solve a Pyomo Model is from the command line.

```
pyomo -solver=cbc -summary dedication-PyomoConcrete.py
```

- It is instructive, however, to see what is going on under the hood.
  - Pyomo explicitly creates an “instance” in a solver-independent form.
  - The instance is then translated into a format that can be understood by the chosen solver.
  - After solution, the result is imported back into the instance class.
- We can explicitly invoke these steps in a script.
- This gives a bit more flexibility in post-solution analysis.

Instantiating and Solving a Pyomo Model

```
epsilon = .001

opt = SolverFactory("cbc")
instance = model.create()
results = opt.solve(instance)
instance.load(results)

print "Optimal strategy"
for b in instance.buy:
    if instance.buy[b].value > epsilon:
        print ’Buy %f of Bond %s’ %(instance.buy[b].value,
                                     b)
```

Abstract Pyomo Model for Dedication (`dedication-PyomoAbstract.py`)

```
model = AbstractModel()

model.Periods = Set()
model.Bonds = Set()
model.Price = Param(model.Bonds)
model.Maturity = Param(model.Bonds)
model.Coupon = Param(model.Bonds)
model.Principal = Param(model.Bonds)
model.Liabilities = Param(range(9))

model.buy = Var(model.Bonds, within=NonNegativeReals)
model.cash = Var(range(9), within=NonNegativeReals)
```

Abstract Pyomo Model for Dedication (cont'd)

```
def objective_rule(model):
    return model.cash[0] + sum(model.Price[b]*model.buy[b]
                               for b in model.Bonds)
model.objective = Objective(sense=minimize, rulre=objective_rule)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t]
             + sum(model.Coupon[b] * model.buy[b]
               for b in model.Bonds if model.Maturity[b] >= t)
             + sum(model.Principal[b] * model.buy[b]
               for b in model.Bonds if model.Maturity[b] == t)
             == model.Liabilities[t])

model.cash_balance = Constraint(range(1, 9), rue=cash_balance_rule
```

Notes on the Abstract Pyomo Model

- In an abstract model, we declare sets and parameters abstractly.
- After declaration, they can be used without instantiation, as in AMPL.
- When creating the instance, we explicitly pass the name of an AMPL-style data file, which is used to instantiate the concrete model.

```
instance = model.create('dedication.dat')
```

- See also `FinancialModels.xlsx:Dedication-Pyomo`.

Example: Short Term Financing

A company needs to make provisions for the following cash flows over the coming five months: $-150K$, $-100K$, $200K$, $-200K$, $300K$.

- The following options for obtaining/using funds are available,
  - The company can borrow up to $\$100K$ at 1% interest per month,
  - The company can issue a 2-month zero-coupon bond yielding 2% interest over the two months,
  - Excess funds can be invested at 0.3% monthly interest.
- How should the company finance these cash flows if no payment obligations are to remain at the end of the period?

Example (cont.)

- All investments are risk-free, so there is no stochasticity.
- What are the decision variables?
  - x_i, the amount drawn from the line of credit in month i,
  - y_i, the number of bonds issued in month i,
  - z_i, the amount invested in month i,
- What is the goal?
  - To maximize the cash on hand at the end of the horizon.

Example (cont.)

The problem can then be modeled as the following linear program:

$$
\begin{aligned}
\max_{(x,y,z,v)\in\mathbb{R}^{12}} \; & f(x,y,z,v) = v \\
\text{s.t. } & x_1 + y_1 - z_1 = 150 \\
& x_2 - 1.01x_1 + y_2 - z_2 + 1.003z_1 = 100 \\
& x_3 - 1.01x_2 + y_3 - 1.02y_1 - z_3 + 1.003z_2 = -200 \\
& x_4 - 1.01x_3 - 1.02y_2 - z_4 + 1.003z_3 = 200 \\
& -1.01x_4 - 1.02y_3 - v + 1.003z_4 = -300 \\
& 100 - x_i \geq 0 \quad (i = 1, \ldots, 4) \\
& x_i \geq 0 \quad (i = 1, \ldots, 4) \\
& y_i \geq 0 \quad (i = 1, \ldots, 3) \\
& z_i \geq 0 \quad (i = 1, \ldots, 4) \\
& v \geq 0.
\end{aligned}
$$

AMPL Model for Short Term Financing (`short_term_financing.mod`)

```
param T > 0 integer;
param cash_flow {0..T};
param credit_rate;
param bond_yield;
param invest_rate;

maximize wealth : invest[T];

subject to balance {t in 0..T} :
credit[t] - (1 + credit_rate)* credit[t-1] +
bonds[t] - (1 + bond_yield) * bonds[t-bond_maturity] -
invest[t] + (1 + invest_rate) * invest[t-1] = cash_flow[t];

subject to initial_credit : credit[-1] = 0;
subject to final_credit   : credit[T]  = 0;
subject to initial_invest : invest[-1] = 0;
subject to initial_bonds {t in 1..bond_maturity}: bonds[-t] = 0;
subject to final_bonds {t in T+1-bond_maturity..T}: bonds[t] = 0;
```

AMPL Data for Short Term Financing (`short_term_financing.dat`)

These are the data for the example.

```
param T := 5;

param : cash_flow :=
0    150
1    100
2   -200
3    200
4    -50
5   -300;

param credit_rate    := .01;
param bond_yield     := .02;
param bond_maturity := 3;
param invest_rate    := .003;
```

Notes on AMPL Model for Short Term Financing

- Note that we've created some "dummy" variables for use of bonds and credit and investment before time zero.
- These are only for convenience to avoid edge cases when expressing the constraints.
- Again, we see the use of the parameter `T` to capture the number of periods.
- See also `FinancialModels.xlsx:Short-term-financing-AMPL`.

PuLP Model for Short Term Financing (`short_term_financing-PuLP.py`)

```
from short_term_financing_data import cash, c_rate, b_yield
from short_term_financing_data import b_maturity, i_rate

T = len(cash)
credit = LpVariable.dicts("credit", range(-1, T), 0, None)
bonds = LpVariable.dicts("bonds", range(-b_maturity, T), 0, None)
invest = LpVariable.dicts("invest", range(-1, T), 0, None)

prob += invest[T-1]
for t in range(0, T):
    prob += (credit[t] - (1 + c_rate)* credit[t-1] +
            bonds[t] - (1 + b_yield) * bonds[t-int(b_maturity)] -
            invest[t] + (1 + i_rate) * invest[t-1] == cash[t])
prob += credit[-1] == 0
prob += credit[T-1] == 0
prob += invest[-1] == 0
for t in range(-int(b_maturity), 0):  prob += bonds[t] == 0
for t in range(T-int(b_maturity), T): prob += bonds[t] == 0
```