The COIN-OR Optimization Suite: Algebraic Modeling Ted Ralphs

COIN fORgery: Developing Open Source Tools for OR

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Outline

1. Introduction
2. Solver Studio
3. Traditional Modeling Environments
4. Python-Based Modeling
5. Comparative Case Studies
Generally speaking, we follow a four-step process in modeling.

1. Develop an abstract model.
2. Populate the model with data.
3. Solve the model.
4. Analyze the results.

These four steps generally involve different pieces of software working in concert.

For mathematical programs, the modeling is often done with an algebraic modeling system.

Data can be obtained from a wide range of sources, including spreadsheets.

Solution of the model is usually relegated to specialized software, depending on the type of model.
Most existing modeling software can be used with COIN solvers.

- **Commercial Systems**
  - GAMS
  - MPL
  - AMPL
  - AIMMS

- **Python-based Open Source Modeling Languages and Interfaces**
  - Pyomo
  - PuLP/Dippy
  - CyLP (provides API-level interface)
  - yaposib
Other Front Ends (mostly open source)

- FLOPC++ (algebraic modeling in C++)
- CMPL
- MathProg.jl (modeling language built in Julia)
- GMPL (open-source AMPL clone)
- ZMPL (stand-alone parser)
- SolverStudio (spreadsheet plug-in: www.OpenSolver.org)
- Open Office spreadsheet
- R (RSymphony Plug-in)
- Matlab (OPTI)
- Mathematica
COIN-OR Solvers with Modeling Language Support

- **COIN-OR** is an open source project dedicated to the development of open source software for solving operations research problems.
- **COIN-OR** distributes a free and open source suite of software that can handle all the classes of problems we’ll discuss.
  - Clp (LP)
  - Cbc (MILP)
  - Ipopt (NLP)
  - SYMPHONY (MILP, BMILP)
  - DIP (MILP)
  - Bonmin (Convex MINLP)
  - Couenne (Non-convex MINLP)
  - Optimization Services (Interface)

COIN also develops **standards and interfaces** that allow software components to interoperate.

- Check out the Web site for the project at [http://www.coin-or.org](http://www.coin-or.org)
How They Interface

Although not required, it’s useful to know something about how modeling languages interface with solvers.

In many cases, modeling languages interface with solvers by writing out an intermediate file that the solver then reads in.

It is also possible to generate these intermediate files directly from a custom-developed code.

Common file formats

- **MPS format**: The original standard developed by IBM in the days of Fortran, not easily human-readable and only supports (integer) linear modeling.
- **LP format**: Developed by CPLEX as a human-readable alternative to MPS.
- **.nl format**: AMPL’s intermediate format that also supports non-linear modeling.
- **OSIL**: an open, XML-based format used by the Optimization Services framework of COIN-OR.
- **Python C Extension**: Several projects interface through a Python extension that can be easily
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SolverStudio (Andrew Mason)

- Spreadsheet optimization has had a (deservedly) bad reputation for many years.
- SolverStudio will change your mind about that!
- SolverStudio provides a full-blown modeling environment inside a spreadsheet.
  - Edit and run the model.
  - Populate the model from the spreadsheet.
- In many of the examples in the remainder of the tutorial, I will show the models in SolverStudio.
In Class Exercise: Install Solver Studio!
Traditional Modeling Environments

- AMPL is one of the most commonly used modeling languages, but many other languages, including GAMS, are similar in concept.
- AMPL has many of the features of a programming language, including loops and conditionals.
- Most available solvers will work with AMPL.
- GMPL and ZIMPL are open source languages that implement subsets of AMPL.
- The Python-based languages to be introduced later have similar functionality, but a more powerful programming environment.
- AMPL will work with all of the solvers we’ve discussed so far.
- You can also submit AMPL models to the NEOS server.
- Student versions can be downloaded from www.ampl.com.
A bond portfolio manager has $100K to allocate to two different bonds.

<table>
<thead>
<tr>
<th>Bond</th>
<th>Yield</th>
<th>Maturity</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>3</td>
<td>A (2)</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>4</td>
<td>Aaa (1)</td>
</tr>
</tbody>
</table>

The goal is to maximize total return subject to the following limits.

- The average rating must be at most 1.5 (lower is better).
- The average maturity must be at most 3.6 years.

Any cash not invested will be kept in a non-interest bearing account and is assumed to have an implicit rating of 0 (no risk).
In many ways, AMPL is like any other programming language.

Example: Bond Portfolio Model

```ampl
ampl: option solver clp;
ampl: var X1;
ampl: var X2;
ampl: maximize yield: 4*X1 + 3*X2;
ampl: subject to cash: X1 + X2 <= 100;
ampl: subject to rating: 2*X1 + X2 <= 150;
ampl: subject to maturity: 3*X1 + 4*X2 <= 360;
ampl: subject to X1_limit: X1 >= 0;
ampl: subject to X2_limit: X2 >= 0;
ampl: solve;
...
ampl: display X1;
X1 = 50
ampl: display X2;
X2 = 50
```
You can type the commands into a file and then load them.
This makes it easy to modify your model later.

**Example:**

ampl: option solver clp;
ampl: model bonds_simple.mod;
ampl: solve;
...
ampl: display X1;
X1 = 50
ampl: display X2;
X2 = 50
Suppose we don’t know ahead of time what bonds we want to include or what the input data describing each bond will be.

For this purpose, we can develop an abstract algebraic model without specifying values for the input data.

Components of an abstract algebraic model are

- **Data**
  - **Sets**: Lists of stocks and other investment options
  - **Parameters**: Numerical inputs such as budget restrictions, historical returns, etc.

- **Model**
  - **Variables**: Values in the model that need to be decided upon.
  - **Objective Function**: A function of the variable values to be maximized or minimized.
  - **Constraints**: Functions of the variable values that must lie within given bounds.
Example: General Bond Portfolio Model (bonds.mod)

set bonds;  # bonds available

param yield {bonds};  # yields
param rating {bonds};  # ratings
param maturity {bonds};  # maturities
param max_rating;  # Maximum average rating allowed
param max_maturity;  # Maximum maturity allowed
param max_cash;  # Maximum available to invest

var buy {bonds} >= 0;  # amount to invest in bond i

maximize total_yield : sum {i in bonds} yield[i] * buy[i];

subject to cash_limit : sum {i in bonds} buy[i] <= max_cash;
subject to rating_limit :
    sum {i in bonds} rating[i]*buy[i] <= max_cash*max_rating;
subject to maturity_limit :
    sum {i in bonds} maturity[i]*buy[i] <= max_cash*max_maturity;
The data to populate the model can come from a number of sources. AMPL has its own format for specifying the data in the model.

```ampl
define bonds := A B;

define param : yield rating maturity :=
  A  4  2  3
  B  3  1  4;

define param max_cash := 100;
define param max_rating 1.5;
define param max_maturity 3.6;
```
ampl: model bonds.mod;
ampl: data bonds.dat;
ampl: solve;
...
ampl: display buy;
buy [*] :=
A  50
B  50
;

Suppose we want to increase available production hours by 2000.

To resolve from scratch, simply modify the data file and reload.

```ampl
ampl: reset data;
ampl: data bonds_alt.dat;
ampl: solve;
...
ampl: display buy;
buy [ * ] :=
A  30
B  70
;
```
Modifying Individual Data Elements

Instead of resetting all the data, you can modify one element.

```
ampl: reset data max_cash;
ampl: data;
ampl data: param max_cash := 150;
ampl data: solve;
...
ampl: display buy;
buy [*] :=
A  45
B 105
;
```
Now suppose we want to add another type of bond.

```plaintext
set bonds := A B C;

param : yield rating maturity :=
  A   4   2   3
  B   3   1   4
  C   6   3   2;

param max_cash := 100;
param max_rating 1.3;
param max_maturity 3.8;
```
Solving the Extended Model

ampl: reset data;
ampl: data bonds_extended.dat;
ampl: solve;
..
ampl: display buy;
buy [*] :=
A  0
B  85
C  15
;
Another obvious source of data is a spreadsheet, such as Excel.

AMPL has commands for accessing data from a spreadsheet directly from the language.

An alternative is to use SolverStudio.

SolverStudio allows the model to be composed within Excel and imports the data from an associated sheet.

Results can be printed to a window or output to the sheet for further analysis.
Note that in our AMPL model, we essentially had three “features” of a bond that we wanted to take into account.

- Maturity
- Rating
- Yield

We constrained the level of two of these and then optimized the third one.

The constraints for the features all have the same basic form.

What if we wanted to add another feature?

We can make the list of features a set and use the concept of a two-dimensional parameter to create a table of bond data.
The Generalized Model (bonds_features.mod)

set bonds;
set features;

param bond_data {bonds, features};
param limits{features};
param yield{bonds};

param max_cash;

var buy {bonds} >= 0;

maximize obj : sum {i in bonds} yield[i] * buy[i];

subject to cash_limit : sum {i in bonds} buy[i] <= max_cash;

subject to limit_constraints {f in features}:
sum {i in bonds} bond_data[i, f]*buy[i] <= max_cash*limits[f];
Example: Facility Location Problem

- We have $n$ locations and $m$ customers to be served from those locations.
- There is a fixed cost $c_j$ and a capacity $W_j$ associated with facility $j$.
- There is a cost $d_{ij}$ and demand $w_{ij}$ for serving customer $i$ from facility $j$.
- We have two sets of binary variables.
  - $y_j$ is 1 if facility $j$ is opened, 0 otherwise.
  - $x_{ij}$ is 1 if customer $i$ is served by facility $j$, 0 otherwise.

### Capacitated Facility Location Problem

\[
\begin{align*}
\text{min} & \quad \sum_{j=1}^{n} c_j y_j + \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \\
& \quad \sum_{i=1}^{m} w_{ij} x_{ij} \leq W_j \quad \forall j \\
& \quad x_{ij} \leq y_j \quad \forall i, j \\
& \quad x_{ij}, y_j \in \{0, 1\} \quad \forall i, j
\end{align*}
\]
from products import REQUIREMENT, PRODUCTS
from facilities import FIXED_CHARGE, LOCATIONS, CAPACITY

prob = LpProblem("Facility_Location")

ASSIGNMENTS = [(i, j) for i in LOCATIONS for j in PRODUCTS]
assign_vars = LpVariable.dicts("x", ASSIGNMENTS, 0, 1, LpBinary)
use_vars = LpVariable.dicts("y", LOCATIONS, 0, 1, LpBinary)

prob += lpSum(use_vars[i] * FIXED_COST[i] for i in LOCATIONS)

for j in PRODUCTS:
    prob += lpSum(assign_vars[(i, j)] for i in LOCATIONS) == 1

for i in LOCATIONS:
    prob += lpSum(assign_vars[(i, j)] * REQUIREMENT[j] for j in PRODUCTS) <= CAPACITY * use_vars[i]

prob.solve()

for i in LOCATIONS:
    if use_vars[i].varValue > 0:
        print "Location ", i, " is assigned: ", 
        print [j for j in PRODUCTS if assign_vars[(i, j)].varValue > 0]
PuLP Basics: Facility Location Example

# The requirements for the products
REQUIREMENT = {
    1 : 7,
    2 : 5,
    3 : 3,
    4 : 2,
    5 : 2,
}

# Set of all products
PRODUCTS = REQUIREMENT.keys()
PRODUCTS.sort()

# Costs of the facilities
# Costs of the facilities
FIXED_COST = {
    1 : 10,
    2 : 20,
    3 : 16,
    4 : 1,
    5 : 2,
}

# Set of facilities
LOCATIONS = FIXED_COST.keys()
LOCATIONS.sort()

# The capacity of the facilities
CAPACITY = 8
Dippy Basics

- DiPPy is an extension of PuLP that provides the ability to model decomposition-based structure.
- With DipPy, one can implement customized subroutines for column generation, cut generation, heuristics, branching, etc. in Python.
- The framework handles the incorporation of these into an overall branch and Xxx algorithm.
- This makes it easy to get up and running with relatively sophisticated methodology.
- It also makes it easy to compare methodologies with as many variables fixed as possible.
- Switching from branch and cut to branch and price is as easy as changing a parameter.
- With SolverStudio, it can even be done from a spreadsheet!.
- There are defaults for all methods—the user need not implement anything to utilize the underlying solver.
DipPy Basics: Facility Location Example

```python
from products import REQUIREMENT, PRODUCTS
from facilities import FIXED_CHARGE, LOCATIONS, CAPACITY

prob = dippy.DipProblem("Facility_Location")

ASSIGNMENTS = [(i, j) for i in LOCATIONS for j in PRODUCTS]
assign_vars = LpVariable.dicts("x", ASSIGNMENTS, 0, 1, LpBinary)
use_vars = LpVariable.dicts("y", LOCATIONS, 0, 1, LpBinary)

prob += lpSum(use_vars[i] * FIXED_COST[i] for i in LOCATIONS)

for j in PRODUCTS:
    prob += lpSum(assign_vars[(i, j)] for i in LOCATIONS) == 1

for i in LOCATIONS:
    prob.relaxation[i] += lpSum(assign_vars[(i, j)] * REQUIREMENT[j] for j in PRODUCTS) <= CAPACITY * use_vars[i]

dippy.Solve(prob, {doPriceCut:1})

for i in LOCATIONS:
    if use_vars[i].varValue > 0:
        print "Location ", i, " is assigned: ",
        print [j for j in PRODUCTS if assign_vars[(i, j)].varValue > 0]
```
Updated upstream
def solve_subproblem(prob, index, redCosts, convexDual):
    ...
    return knapsack01(obj, weights, CAPACITY)
def knapsack01(obj, weights, capacity):
    ...
    return solution
def first_fit(prob):
    ...
    return bvs
prob.init_vars = first_fit
def choose_branch(prob, sol):
    ...
    return ([], down_branch_ub, up_branch_lb, [])
def generate_cuts(prob, sol):
    ...
    return new_cuts
def heuristics(prob, xhat, cost):
    ...
    return sols
dippy.Solve(prob, {'doPriceCut': '1'})
def solve_subproblem(prob, index, redCosts, convexDual):
    ...
    return knapsack01(obj, weights, CAPACITY)

def knapsack01(obj, weights, capacity):
    ...
    return solution

def first_fit(prob):
    ...
    return bvs

prob.init_vars = first_fit

def choose_branch(prob, sol):
    ...
    return ([], down_branch_ub, up_branch_lb, [])

def generate_cuts(prob, sol):
    ...
    return new_cuts

def heuristics(prob, xhat, cost):
    ...
    return sols

dippy.Solve(prob, {'doPriceCut': '1'})
In contrast to PuLP, Pyomo allows the creation of “abstract” models, like other AMLs.

Note, however, that it can also be used to create concrete models, like PuLP.

Like, it can read data from a wide range of sources.

It also allows constraints to involve more general functions.

As we will see, this power comes with some increased complexity.
model = ConcreteModel()

Bonds, Features, BondData, Liabilities = read_data('ded.dat')

Periods = range(len(Liabilities))

model.buy = Var(Bonds, within=NonNegativeReals)
model.cash = Var(Periods, within=NonNegativeReals)
model.obj = Objective(expr=model.cash[0] +
    sum(BondData[b, 'Price'] * model.buy[b] for b in Bonds),
    sense=minimize)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t] +
        sum(BondData[b, 'Coupon'] * model.buy[b] for b in Bonds if BondData[b, 'Maturity'] >= t) +
        sum(BondData[b, 'Principal'] * model.buy[b] for b in Bonds if BondData[b, 'Maturity'] == t) == Liabilities[t])

model.cash_balance = Constraint(Periods[1:], rule=cash_balance_rule)
from pulp import LpProblem, LpVariable, lpSum, LpMaximize, value

prob = LpProblem("Dedication Model", LpMaximize)

X1 = LpVariable("X1", 0, None)
X2 = LpVariable("X2", 0, None)

prob += 4*X1 + 3*X2
prob += X1 + X2 <= 100
prob += 2*X1 + X2 <= 150
prob += 3*X1 + 4*X2 <= 360

prob.solve()

print 'Optimal total cost is: ', value(prob.objective)

print "X1 ", X1.varValue
print "X2 ", X2.varValue
Like the simple AMPL model, we are not using indexing or any sort of abstraction here.

The syntax is very similar to AMPL.

To achieve separation of data and model, we use Python’s `import` mechanism.
from pulp import LpProblem, LpVariable, lpSum, LpMaximize, value
from bonds import bonds, max_rating, max_maturity, max_cash

prob = LpProblem("Bond Selection Model", LpMaximize)

buy = LpVariable.dicts('bonds', bonds.keys(), 0, None)

prob += lpSum(bonds[b]['yield'] * buy[b] for b in bonds)

prob += lpSum(buy[b] for b in bonds) <= max_cash, "cash"

prob += (lpSum(bonds[b]['rating'] * buy[b] for b in bonds) <= max_cash*max_rating, "ratings")

prob += (lpSum(bonds[b]['maturity'] * buy[b] for b in bonds) <= max_cash*max_maturity, "maturities")
We can use Python’s native `import` mechanism to get the data.

Note, however, that the data is read and stored before the model.

This means that we don’t need to declare sets and parameters.

Carriage returns are syntactic (parentheses imply line continuation).

Constraints

- Naming of constraints is optional and only necessary for certain kinds of post-solution analysis.
- Constraints are added to the model using a very intuitive syntax.
- Objectives are nothing more than expressions that are to be optimized rather than explicitly constrained.

Indexing

- Indexing in Python is done using the native dictionary data structure.
- Note the extensive use of comprehensions, which have a syntax very similar to quantifiers in a mathematical model.
prob.solve()

epsilon = .001

print 'Optimal purchases:'
for i in bonds:
    if buy[i].varValue > epsilon:
        print 'Bond', i, ':', buy[i].varValue
bonds = {'A': {'yield': 4, 'rating': 2, 'maturity': 3},
         'B': {'yield': 3, 'rating': 1, 'maturity': 4},
}

max_cash = 100
max_rating = 1.5
max_maturity = 3.6
Notes About the Data Import

- We are storing the data about the bonds in a “dictionary of dictionaries.”
- With this data structure, we don’t need to separately construct the list of bonds.
- We can access the list of bonds as \texttt{bonds.keys()}. 
- Note, however, that we still end up hard-coding the list of features and we must repeat this list of features for every bond.
- We can avoid this using some advanced Python programming techniques, but SolverStudio makes this easy.
Bond Portfolio Example: PuLP Model in SolverStudio

```
buy = LpVariable.dicts('bonds', bonds, 0, None)
for f in features:
    if limits[f] == "Opt":
        if sense[f] == '>' :
            prob += lpSum(bond_data[b, f] * buy[b] for b in bonds)
        else:
            prob += lpSum(-bond_data[b, f] * buy[b] for b in bonds)
    else:
        if sense[f] == '>' :
            prob += (lpSum(bond_data[b,f]*buy[b] for b in bonds) >= 
                      max_cash*limits[f], f)
        else:
            prob += (lpSum(bond_data[b,f]*buy[b] for b in bonds) <= 
                      max_cash*limits[f], f)
prob += lpSum(buy[b] for b in bonds) <= max_cash, "cash"
```

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Notes About the SolverStudio PuLP Model

- We’ve explicitly allowed the option of optimizing over one of the features, while constraining the others.
- Later, we’ll see how to create tradeoff curves showing the tradeoffs among the constraints imposed on various features.
**Portfolio Dedication**

**Definition**

*Dedication* or *cash flow matching* refers to the funding of known future liabilities through the purchase of a portfolio of risk-free non-callable bonds.

**Notes:**

- Dedication is used to eliminate interest rate risk.
- Dedicated portfolios do not have to be managed.
- The goal is to construct such portfolio at a minimal price from a set of available bonds.
- This is a multi-period model.
Example: Portfolio Dedication

- A pension fund faces liabilities totalling $\ell_j$ for years $j = 1, \ldots, T$.
- The fund wishes to dedicate these liabilities via a portfolio comprised of $n$ different types of bonds.
- Bond type $i$ costs $c_i$, matures in year $m_i$, and yields a yearly coupon payment of $d_i$ up to maturity.
- The principal paid out at maturity for bond $i$ is $p_i$. 

LP Formulation for Portfolio Dedication

- We assume that for each year \( j \) there is at least one type of bond \( i \) with maturity \( m_i = j \), and there are none with \( m_i > T \).
- Let \( x_i \) be the number of bonds of type \( i \) purchased, and let \( z_j \) be the cash on hand at the beginning of year \( j \) for \( j = 0, \ldots, T \). Then the dedication problem is the following LP.

\[
\begin{align*}
\min_{(x,z)} \quad & z_0 + \sum_i c_i x_i \\
\text{s.t.} \quad & z_{j-1} - z_j + \sum_{\{i:m_i \geq j\}} d_i x_i + \sum_{\{i:m_i = j\}} p_i x_i = \ell_j, \quad (j = 1, \ldots, T - 1) \\
& z_T + \sum_{\{i:m_i = T\}} (p_i + d_i) x_i = \ell_T. \\
& z_j \geq 0, j = 1, \ldots, T \\
& x_i \geq 0, i = 1, \ldots, n
\end{align*}
\]
Here is the model for the portfolio dedication example.

```AMPL
set Bonds;
param T > 0 integer;
param Liabilities {1..T};
param Price {Bonds};
param Maturity {Bonds};
param Coupon {Bonds};
param Principal {Bonds};

var buy {Bonds} >= 0;
var cash {0..T} >= 0;

minimize total_cost : cash[0] + sum {i in Bonds} Price[i] * buy[i]

subject to cash_balance {t in 1..T}: cash[t-1] - cash[t] +
    sum{i in Bonds : Maturity[i] >= t} Coupon[i] * buy[i] +
    sum{i in Bonds : Maturity[i] = t} Principal[i] * buy[i] =
    Liabilities[t];
```
In multi-period models, we have to somehow represent the set of periods. Such a set is different from a generic set because it involves ranged data. We must somehow do arithmetic with elements of this set in order to express the model.

In AMPL, a ranged set can be constructed using the syntax `1..T`. Both endpoints are included in the range.

Another important feature of the above model is the use of conditionals in the limits of the sum.

Conditionals can be used to choose a subset of the items in a given set satisfying some condition.
PuLP Model for Dedication (dedication-PuLP.py)

Bonds, Features, BondData, Liabilities = read_data('ded.dat')

prob = LpProblem("Dedication Model", LpMinimize)

buy = LpVariable.dicts("buy", Bonds, 0, None)
cash = LpVariable.dicts("cash", range(len(Liabilities)), 0, None)

prob += cash[0] + lpSum(BondData[b, 'Price'] * buy[b] for b in Bonds)

for t in range(1, len(Liabilities)):
    prob += (cash[t-1] - cash[t] + lpSum(BondData[b, 'Coupon'] * buy[b] for b in Bonds if BondData[b, 'Maturity'] >= t) + lpSum(BondData[b, 'Principal'] * buy[b] for b in Bonds if BondData[b, 'Maturity'] == t) == Liabilities[t], "cash_balance_%s"%t)
We are parsing the AMPL data file with a custom-written function `read_data` to obtain the data.

The data is stored in a two-dimensional table (dictionary with tuples as keys).

The `range` operator is used to create ranged sets in Python.

The upper endpoint is not included in the range and ranges start at 0 by default (`range(3) = [0, 1, 2]`).

The `len` operator gets the number of elements in a given data structure.

Python also supports conditions in comprehensions, so the model reads naturally in Python’s native syntax.

See also FinancialModels.xlsx:Dedication-PuLP.
Bonds, Features, BondData, Liabilities = read_data('ded.dat')

Periods = range(len(Liabilities))

model.buy = Var(Bonds, within=NonNegativeReals)
model.cash = Var(Periods, within=NonNegativeReals)
model.obj = Objective(expr=model.cash[0] +
    sum(BondData[b, 'Price'] * model.buy[b] for b in Bonds),
    sense=minimize)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t] +
        sum(BondData[b, 'Coupon'] * model.buy[b] for b in Bonds if BondData[b, 'Maturity'] >= t) +
        sum(BondData[b, 'Principal'] * model.buy[b] for b in Bonds if BondData[b, 'Maturity'] == t) == Liabilities[t])
model.cash_balance = Constraint(Periods[1:], rule=cash_balance_rule)
This model is almost identical to the PuLP model.

The only substantial difference is the way in which constraints are defined, using “rules.”

Indexing is implemented by specifying additional arguments to the rule functions.

When the rule function specifies an indexed set of constraints, the indices are passed through the arguments to the function.

The model is constructed by looping over the index set, constructing each associated constraint.

Note the use of the Python slice operator to extract a subset of a ranged set.
The easiest way to solve a Pyomo Model is from the command line.

```
pyomo -solver=cbc -summary dedication-PyomoConcrete.py
```

It is instructive, however, to see what is going on under the hood.

- Pyomo explicitly creates an “instance” in a solver-independent form.
- The instance is then translated into a format that can be understood by the chosen solver.
- After solution, the result is imported back into the instance class.

We can explicitly invoke these steps in a script.

This gives a bit more flexibility in post-solution analysis.
epsilon = .001

opt = SolverFactory("cbc")
instance = model.create()
results = opt.solve(instance)
instance.load(results)

print "Optimal strategy"
for b in instance.buy:
    if instance.buy[b].value > epsilon:
        print 'Buy %.2f of Bond %s' %(instance.buy[b].value, b)
Abstract Pyomo Model for Dedication

(dedication-PyomoAbstract.py)

model = AbstractModel()

model.Periods = Set()
model.Bonds = Set()
model.Price = Param(model.Bonds)
model.Maturity = Param(model.Bonds)
model.Coupon = Param(model.Bonds)
model.Principal = Param(model.Bonds)
model.Liabilities = Param(range(9))

model.buy = Var(model.Bonds, within=NonNegativeReals)
model.cash = Var(range(9), within=NonNegativeReals)
def objective_rule(model):
    return model.cash[0] + sum(model.Price[b] * model.buy[b]
        for b in model.Bonds)
model.objective = Objective(sense=minimize, rule=objective_rule)

def cash_balance_rule(model, t):
    return (model.cash[t-1] - model.cash[t]
        + sum(model.Coupon[b] * model.buy[b]
            for b in model.Bonds if model.Maturity[b] >= t)
        + sum(model.Principal[b] * model.buy[b]
            for b in model.Bonds if model.Maturity[b] == t)
        == model.Liabilities[t])

model.cash_balance = Constraint(range(1, 9), rule=cash_balance_rule)
In an abstract model, we declare sets and parameters abstractly. 
After declaration, they can be used without instantiation, as in AMPL. 
When creating the instance, we explicitly pass the name of an AMPL-style data file, which is used to instantiate the concrete model.

\[
\text{instance} = \text{model.create('dedication.dat')} 
\]

See also FinancialModels.xlsx:Dedication-Pyomo.
Example: Short Term Financing

A company needs to make provisions for the following cash flows over the coming five months: $-150K, -100K, 200K, -200K, 300K$.

- The following options for obtaining/using funds are available,
  - The company can borrow up to $100K$ at 1% interest per month,
  - The company can issue a 2-month zero-coupon bond yielding 2% interest over the two months,
  - Excess funds can be invested at 0.3% monthly interest.

How should the company finance these cash flows if no payment obligations are to remain at the end of the period?
All investments are risk-free, so there is no stochasticity.

What are the decision variables?
- $x_i$, the amount drawn from the line of credit in month $i$,
- $y_i$, the number of bonds issued in month $i$,
- $z_i$, the amount invested in month $i$,

What is the goal?
- To maximize the cash on hand at the end of the horizon.
The problem can then be modeled as the following linear program:

\[
\max_{(x, y, z, v) \in \mathbb{R}^{12}} f(x, y, z, v) = v \\
\text{s.t. } x_1 + y_1 - z_1 = 150 \\
x_2 - 1.01x_1 + y_2 - z_2 + 1.003z_1 = 100 \\
x_3 - 1.01x_2 + y_3 - 1.02y_1 - z_3 + 1.003z_2 = -200 \\
x_4 - 1.01x_3 - 1.02y_2 - z_4 + 1.003z_3 = 200 \\
-1.01x_4 - 1.02y_3 - v + 1.003z_4 = -300 \\
100 - x_i \geq 0 \quad (i = 1, \ldots, 4) \\
x_i \geq 0 \quad (i = 1, \ldots, 4) \\
y_i \geq 0 \quad (i = 1, \ldots, 3) \\
z_i \geq 0 \quad (i = 1, \ldots, 4) \\
v \geq 0.
\]
AMPL Model for Short Term Financing

(\texttt{short\_term\_financing.mod})

\begin{verbatim}
param T > 0 integer;
param cash\_flow \{0..T\};
param credit\_rate;
param bond\_yield;
param invest\_rate;

maximize wealth : invest[T];

subject to balance \{t in 0..T\} :
  credit[t] - (1 + credit\_rate) \times \text{credit}[t-1] +
  bonds[t] - (1 + bond\_yield) \times \text{bonds}[t-bond\_maturity] -
  invest[t] + (1 + invest\_rate) \times \text{invest}[t-1] = cash\_flow[t];

subject to initial\_credit : credit[-1] = 0;
subject to final\_credit : credit[T] = 0;
subject to initial\_invest : invest[-1] = 0;
subject to initial\_bonds \{t in 1..bond\_maturity\} : bonds[-t] = 0;
subject to final\_bonds \{t in T+1-bond\_maturity..T\} : bonds[t] = 0;
\end{verbatim}
These are the data for the example.

param T := 5;

param : cash_flow :=
0  150
1  100
2 -200
3  200
4 -50
5 -300;

param credit_rate := .01;
param bond_yield := .02;
param bond_maturity := 3;
param invest_rate := .003;
Note that we’ve created some “dummy” variables for use of bonds and credit and investment before time zero.

These are only for convenience to avoid edge cases when expressing the constraints.

Again, we see the use of the parameter $T$ to capture the number of periods.

See also FinancialModels.xlsx:Short-term-financing-AMPL.
from short_term_financing_data import cash, c_rate, b_yield
from short_term_financing_data import b_maturity, i_rate

T = len(cash)
credit = LpVariable.dicts("credit", range(-1, T), 0, None)
bonds = LpVariable.dicts("bonds", range(-b_maturity, T), 0, None)
invest = LpVariable.dicts("invest", range(-1, T), 0, None)

prob += invest[T-1]
for t in range(0, T):
    prob += (credit[t] - (1 + c_rate)* credit[t-1] +
             bonds[t] - (1 + b_yield) * bonds[t-int(b_maturity)] -
             invest[t] + (1 + i_rate) * invest[t-1] == cash[t])
prob += credit[-1] == 0
prob += credit[T-1] == 0
prob += invest[-1] == 0
for t in range(-int(b_maturity), 0): prob += bonds[t] == 0
for t in range(T-int(b_maturity), T): prob += bonds[t] == 0