Multi-Echelon Inventory Management

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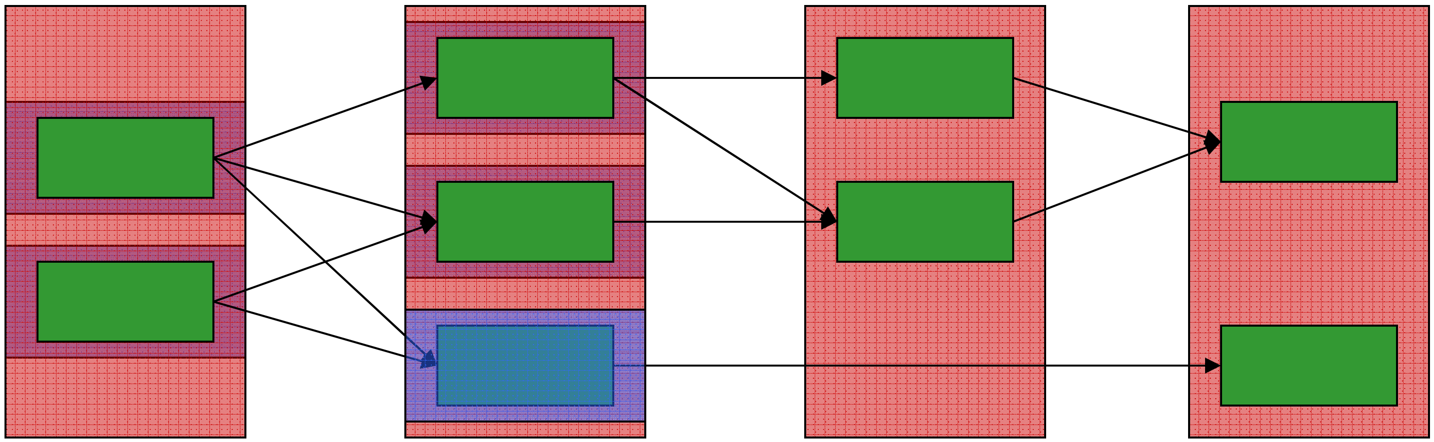
Outline

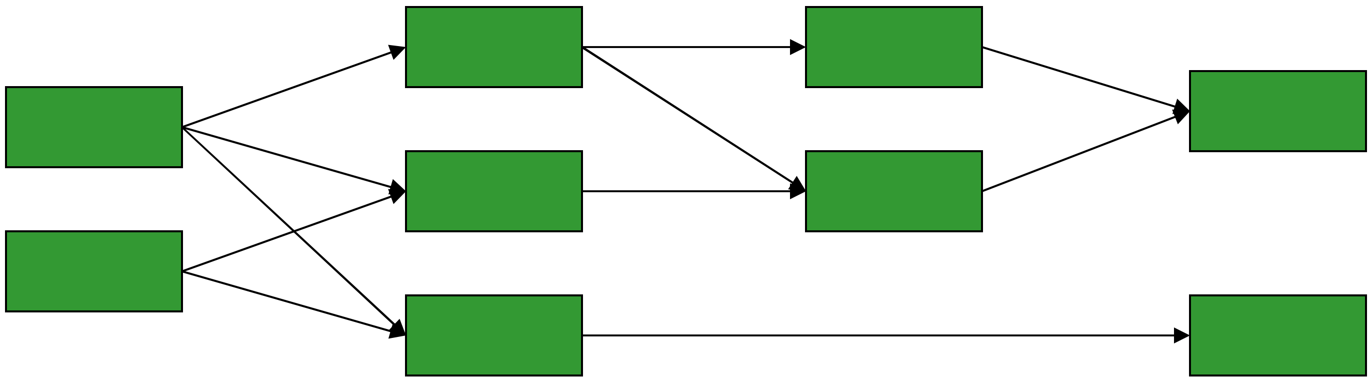
- Introduction
  - Overview
  - Network topology
  - Assumptions
  - Deterministic models
- Stochastic models
- Decentralized systems

Overview



- System is composed of stages (nodes, sites, …)
- Stages are grouped into echelons
- Stages can represent
  - Physical locations
  - BOM
  - Processing activities

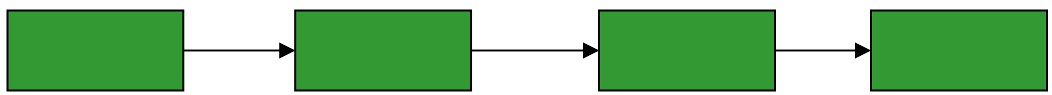
Overview



- Stages to the left are upstream
- Those to the right are downstream
- Downstream stages face customer demand

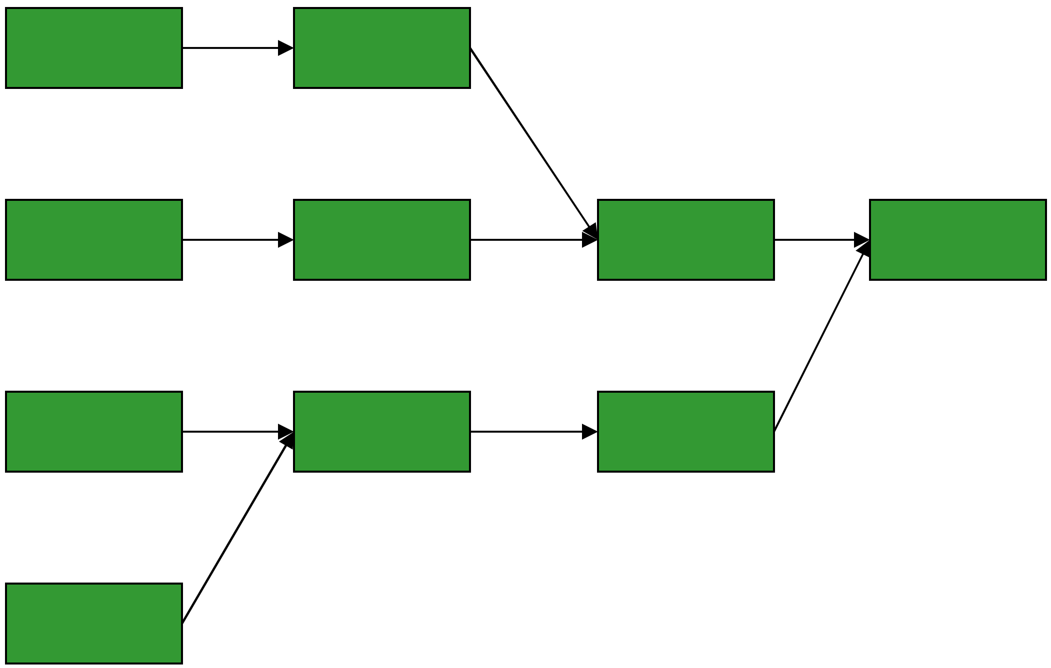
Network Topology

- Serial system:



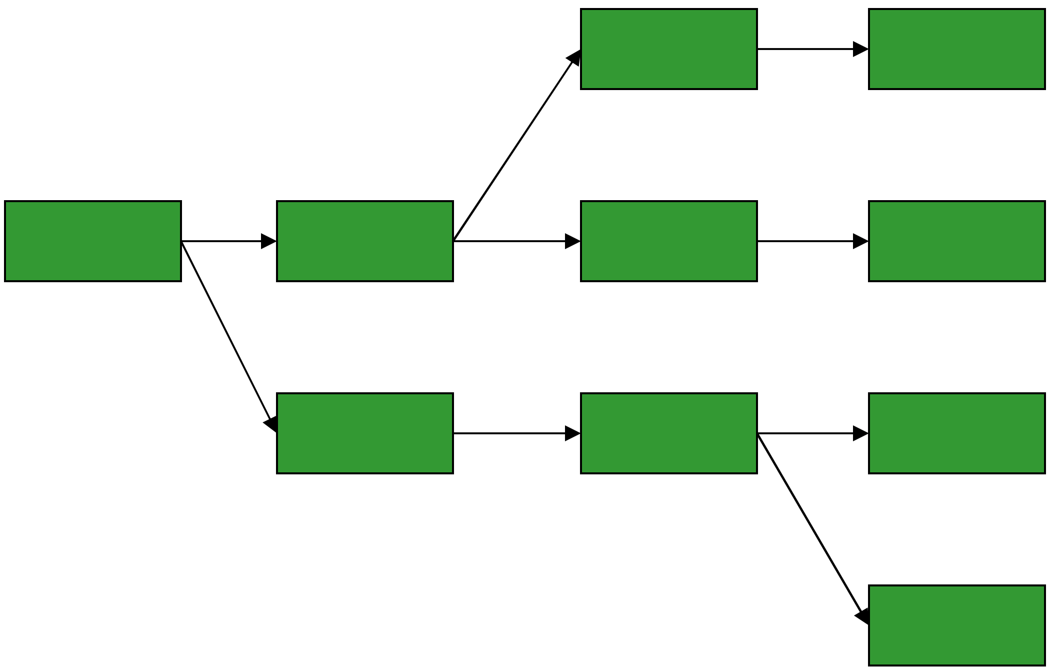
Network Topology

- Assembly system:



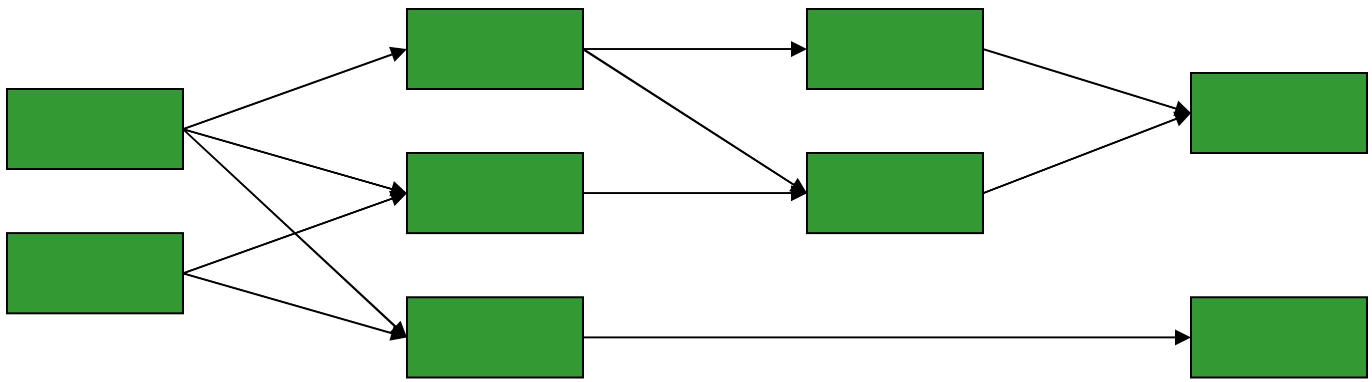
Network Topology

- Distribution system:



Network Topology

- Mixed system:



Assumptions

- Periodic review
  - Period = week, month, …
- Centralized decision making
  - Can optimize system globally
  - Later, I will talk about decentralized systems
- Costs
  - Holding cost
  - Fixed order cost
  - Stockout cost (vs. service level)

Deterministic Models

- Suppose everything in the system is deterministic (not random)
  - Demands, lead times, …
  - Possible to achieve 100% service
- If no fixed costs, explode BOM every period
- If fixed costs are non-negligible, key tradeoff is between fixed and holding costs
  - Multi-echelon version of EOQ
  - MRP systems (optimization component)

Outline

- Introduction
- Stochastic models
  - Base-stock model
  - Stochastic multi-echelon systems
  - Strategic safety stock placement
  - Supply uncertainty
- Decentralized systems

Stochastic Models

- Suppose now that demand is stochastic (random)
  - Still assume supply is deterministic
  - Including lead time, yield, …
- I'll assume:
  - No fixed cost
  - Normally distributed demand: $N(\mu,\sigma^2)$
- Key tradeoff is between holding and stockout costs

The Base-Stock Model

- Single stage (and echelon)
- Excess inventory incurs holding cost of h per unit per period
- Unmet demand is backordered at a cost of p per unit per period
- Stage follows base-stock policy
  - Each period, "order up to" base-stock level, y
  - aka order-up-to policy
  - Similar to days-of-supply policy: y / μ DOS

The Base-Stock Model

- Optimal base-stock level:

$$y^* = \mu + z_\alpha \sigma$$

where z_α comes from normal distribution and

$$\alpha = \frac{p}{p+h}$$

- α is sometimes called the "newsboy ratio"

Interpretation

$$y^* = \mu + z_\alpha \sigma$$

- In other words, base-stock level = mean demand + some # of SD's worth of demand
- # of SD's depends on relationship between h and p
- As $h \uparrow \Rightarrow z_\alpha \downarrow \Rightarrow y^* \downarrow$
- As $p \uparrow \Rightarrow z_\alpha \uparrow \Rightarrow y^* \uparrow$
- If lead time = L:

$$y^* = \mu L + z_\alpha \sigma \sqrt{L}$$

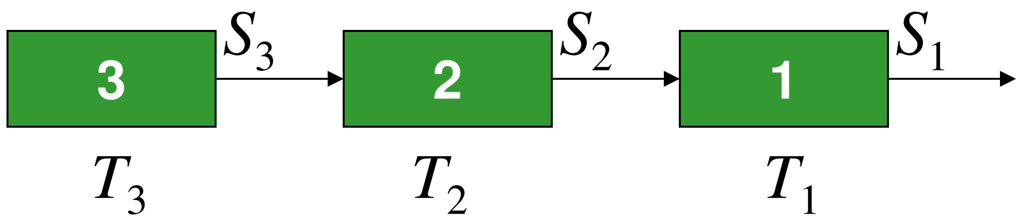
Stochastic Multi-Echelon Systems

- Need to set y at each stage
- Could use base-stock formula
  - But how to quantify lead time?
  - Lead time is stochastic
  - Depends on upstream base-stock level and stochastic demand
- For serial systems, exact algorithms exist
  - Clark-Scarf (1960)
  - But they are cumbersome

An Approximate Method

- Assume that each stage carries sufficient inventory to deliver product within S periods "most of the time"
  - Definition of "most" depends on service level constant, z_α
  - S is called the committed service time (CST)
- We simply ignore the times that the stage does not meet its CST
  - For the purposes of the optimization
  - Allows us to pretend LT is deterministic

Net Lead Time



- Each stage has a processing time T and a CST S
- Net lead time at stage i = $\boxed{S_{i+1} + T_i - S_i}$

$\underbrace{S_{i+1} + T_i}_{\text{"bad" LT}} \; \underbrace{- S_i}_{\text{"good" LT}}$

Net Lead Time vs. Inventory

- Suppose $S_i = S_{i+1} + T_i$
  - e.g., inbound CST = 4, proc time = 2, outbound CST = 6
  - Don't need to hold any inventory
  - Operate entirely as pull (make-to-order, JIT) system
- Suppose $S_i = 0$
  - Promise immediate order fulfillment
  - Make-to-stock system

Net Lead Time vs. Inventory

- Precise relationship between NLT and inventory:

$$y^* = \mu \times NLT + z_\alpha \sigma \sqrt{NLT}$$

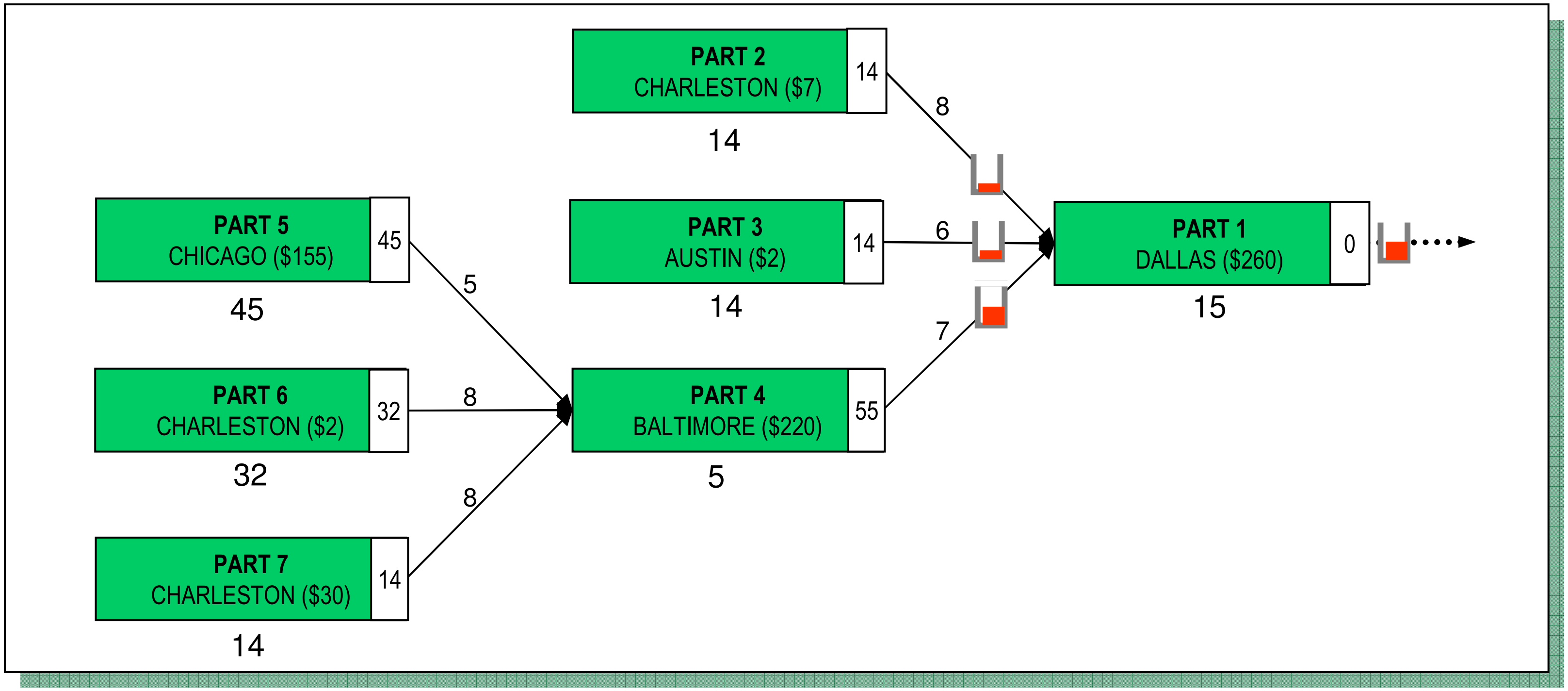
- NLT replaces LT in earlier formula
- So, choosing inventory levels is equivalent to choosing NLTs, i.e., choosing S at each stage
- Efficient algorithms exist for finding optimal S values to minimize expected holding cost while meeting end-customer service requirement

Key Insight

- It is usually optimal for only a few stages to hold inventory
  - Other stages operate as pull systems
- In a serial system, every stage either:
  - holds zero inventory (and quotes maximum CST)
  - or quotes CST of zero (and holds maximum inventory)

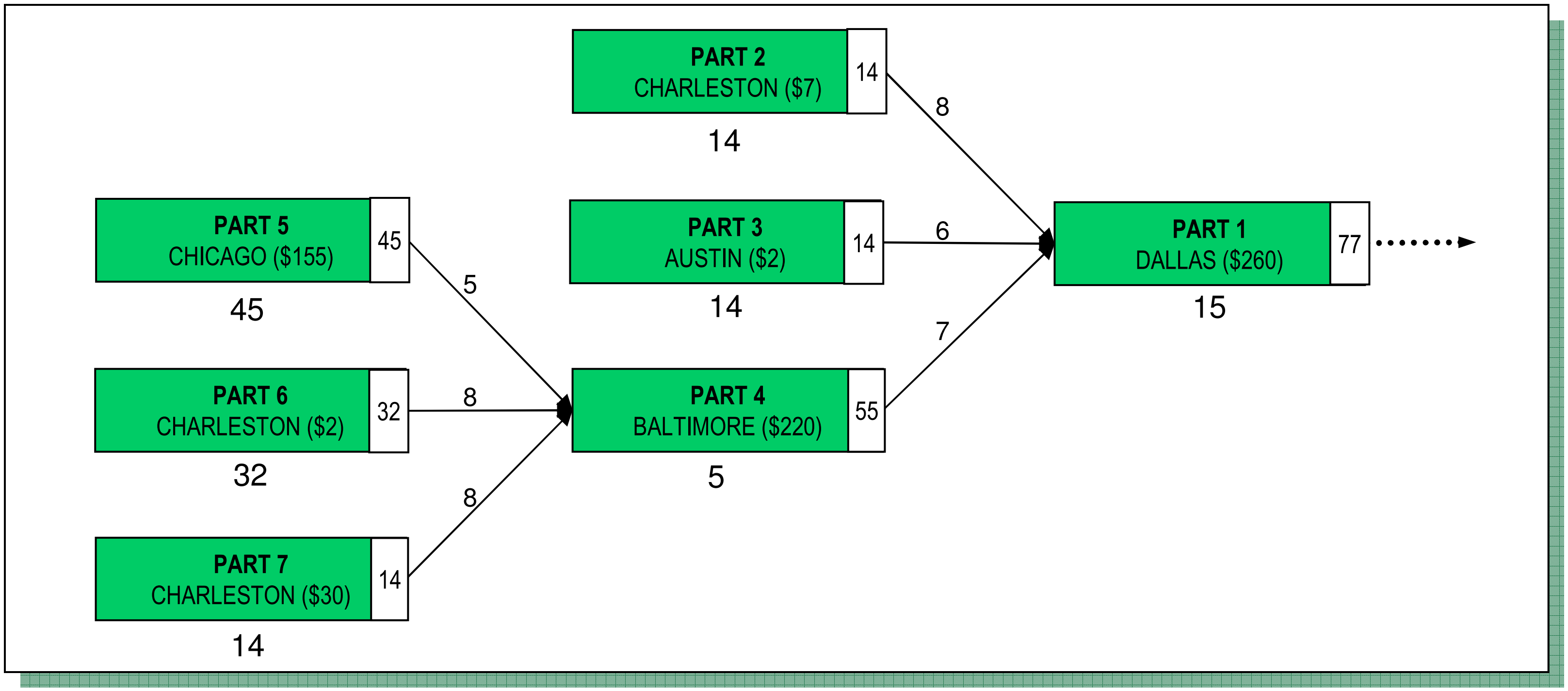
Case Study

(Adapted from Simchi-Levi, Chen, and Bramel, *The Logic of Logistics*, Springer, 2004)



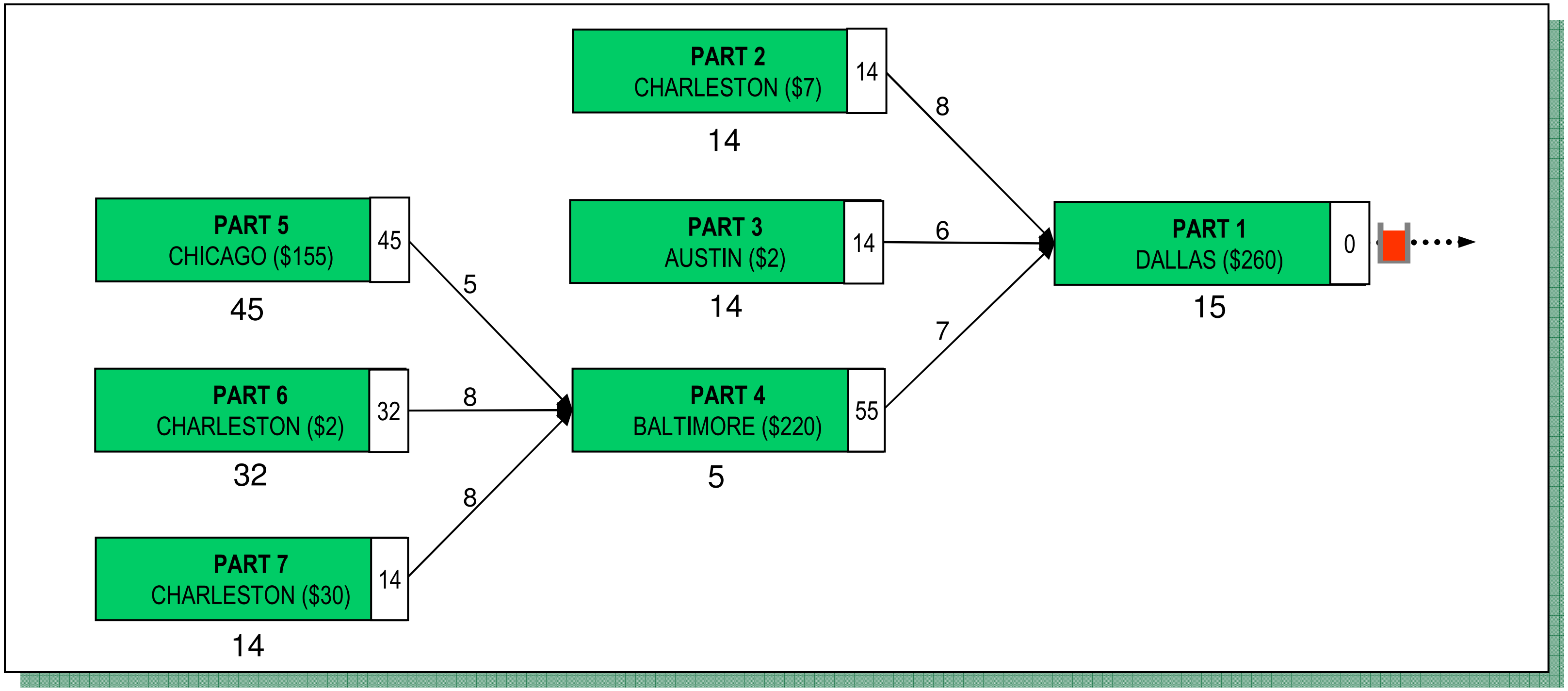
- # below stage = processing time
- # in white box = CST
- In this solution, inventory is held of finished product and its raw materials

A Pure Pull System



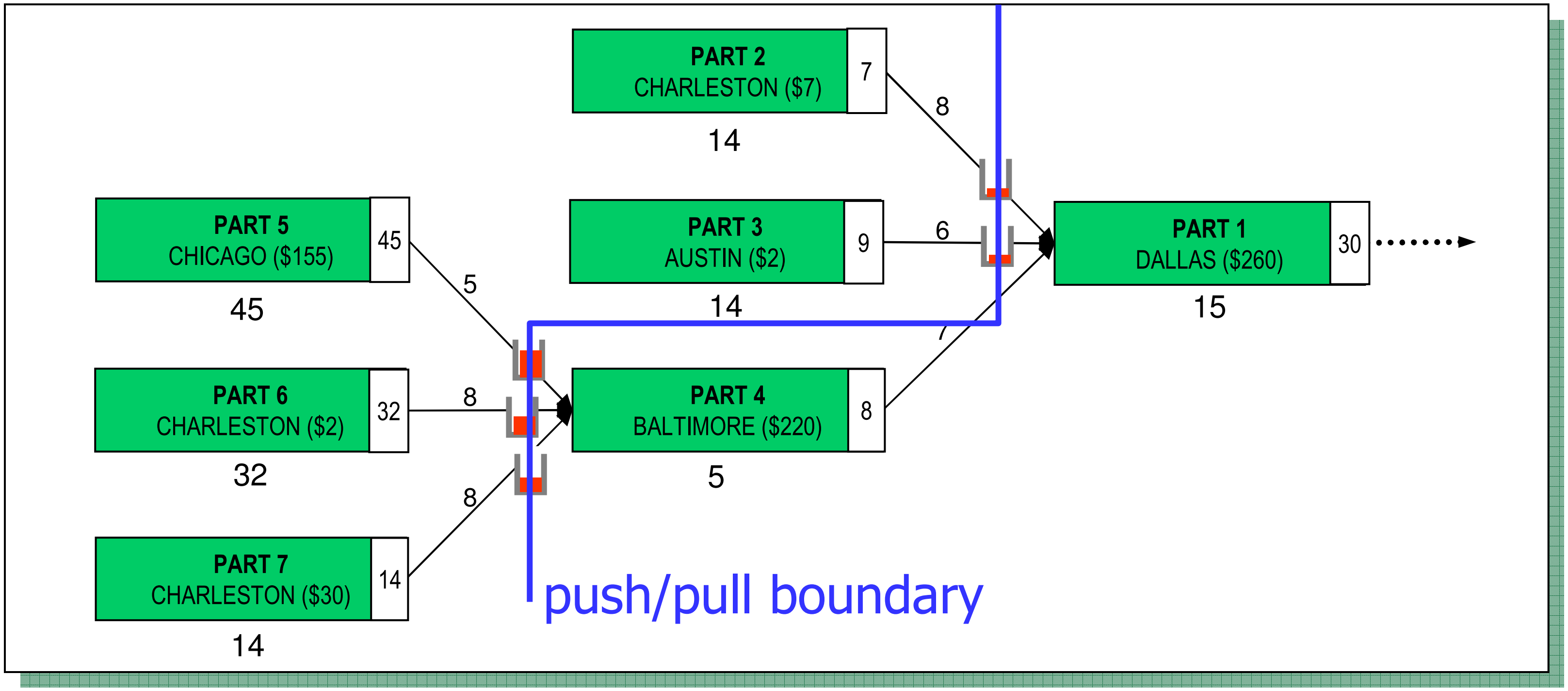
- Produce to order
- Long CST to customer
- No inventory held in system

A Pure Push System



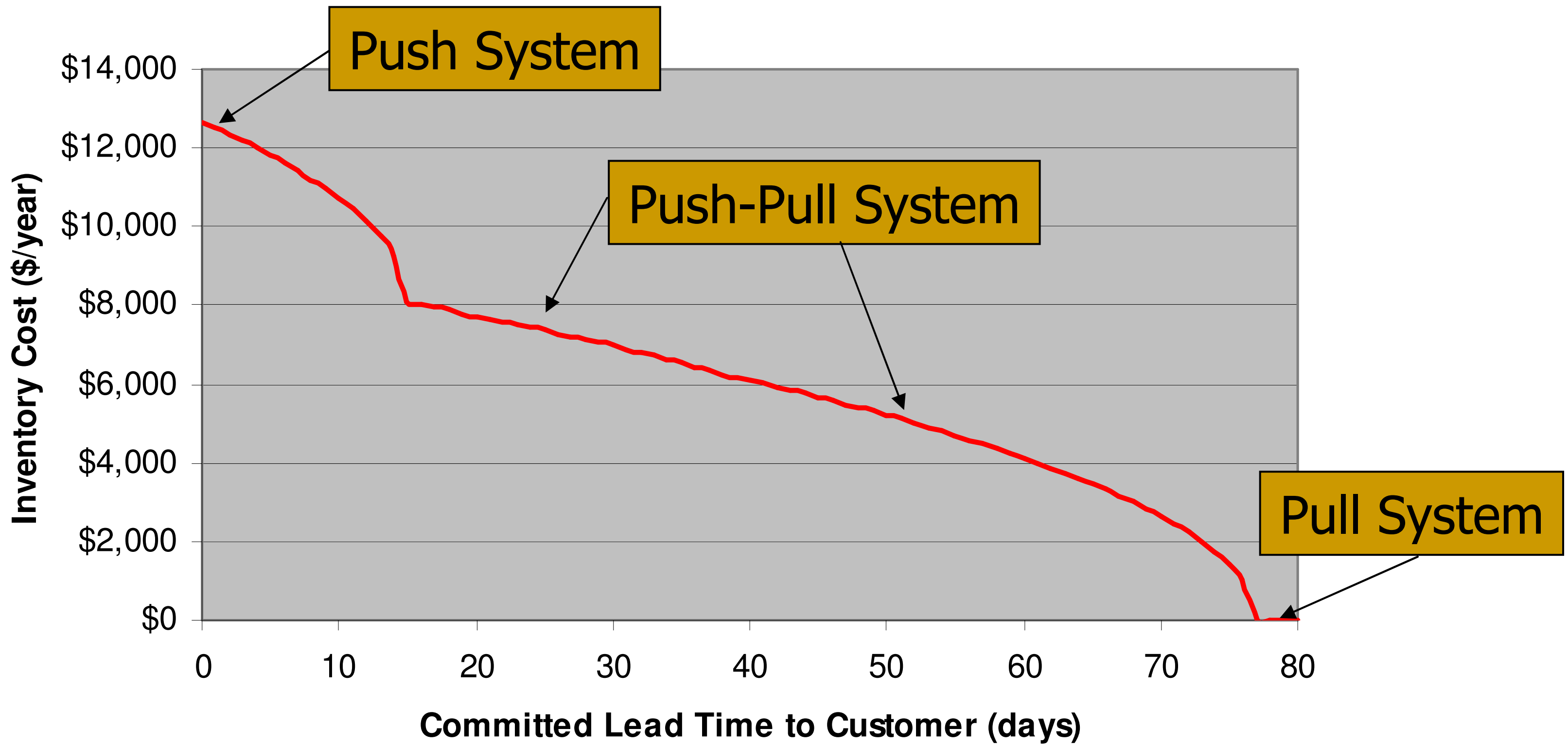
- Produce to forecast
- Zero CST to customer
- Hold lots of finished goods inventory

A Hybrid Push-Pull System

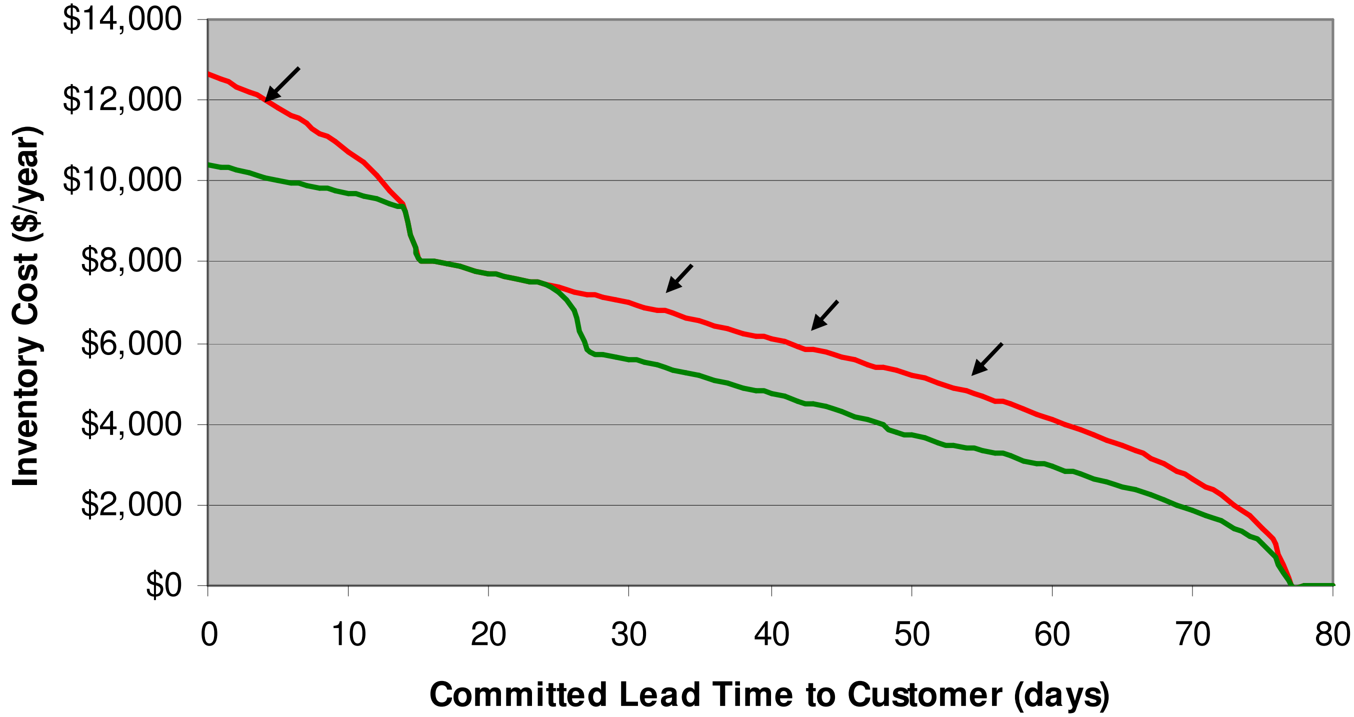


- Part of system operated produce-to-stock, part produce-to-order
- Moderate lead time to customer

CST vs. Inventory Cost



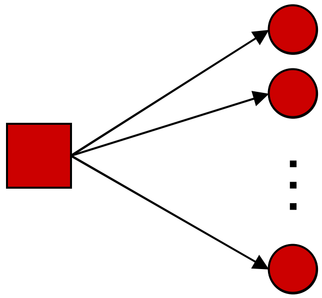
Optimization Shifts the Tradeoff Curve



Supply Uncertainty

- Types of supply uncertainty:
  - Lead-time uncertainty
  - Yield uncertainty
  - Disruptions
- Strategies for dealing with demand and supply uncertainty are similar
  - Safety stock inventory
  - Dual sourcing
  - Improved forecasts
- But the two are not the same

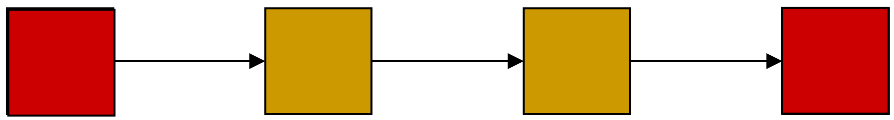
Risk Pooling



- One warehouse, several retailers
  - Who should hold inventory?
- If demand is uncertain:
  - Smaller inventory req't if warehouse holds inv.
  - Consolidation is better
- If supply is uncertain (but demand is not):
  - Disruption risk is minimized if retailers hold inv.
  - Diversificaiton is better

Inventory Placement

- Hold inventory upstream or downstream?



- Conventional wisdom:
  - Hold inventory upstream
  - Holding cost is smaller
- Under supply uncertainty:
  - Hold inventory downstream
  - Protects against stockouts anywhere in system

Outline

- Introduction
- Stochastic models
- Decentralized systems
  - Suboptimality
  - Contracting
  - The bullwhip effect

Decentralized Systems

- So far, we have assumed the system is centralized
  - Can optimize at all stages globally
  - One stage may incur higher costs to benefit the system as a whole
- What if each stage acts independently to minimize its own cost / maximize its own profit?

Suboptimality

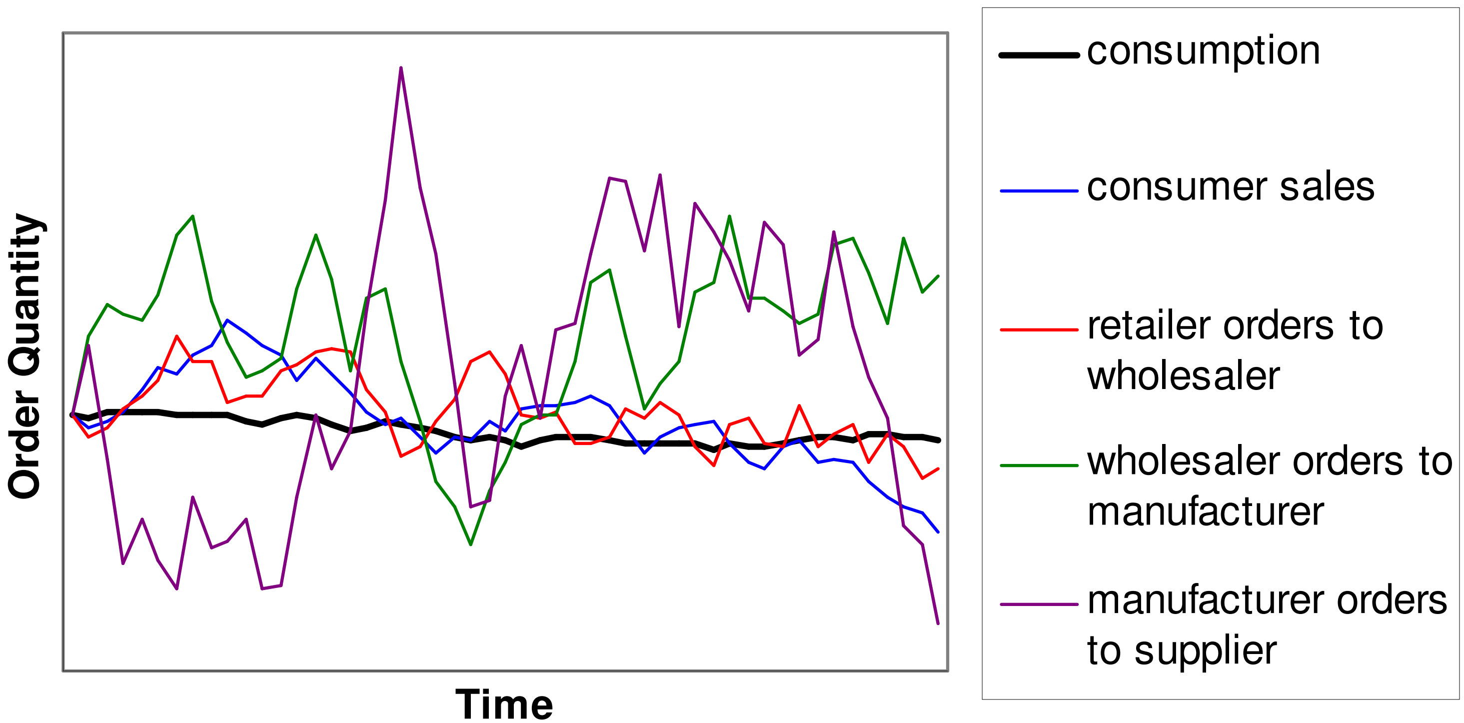
- Optimizing locally is likely to result in some degree of suboptimality
- Example: upstream stages want to operate make-to-order
  - Results in too much inventory downstream
- Another example:
  - Wholesaler chooses wholesale price
  - Retailer chooses order quantity
  - Optimizing independently, the two parties will always leave money on the table

Contracting

- One solution is for the parties to impose a contracting mechanism
  - Splits the costs / profits / risks / rewards
  - Still allows each party to act in its own best interest
  - If structured correctly, system achieves optimal cost / profit, even with parties acting selfishly
- There is a large body of literature on contracting
  - In practice, idea is commonly used
  - Actual OR models rarely implemented

Bullwhip Effect (BWE)

- Demand for diapers:



Irrational Behavior Causes BWE

- Firms over-react to demand signals
  - Order too much when they perceive an upward demand trend
  - Then back off when they accumulate too much inventory
- Firms under-weight the supply line
- Both are irrational behaviors
- Demonstrated by "beer game"

Rational Behavior Causes BWE

- Famous paper by Hau Lee, et al. (1997)
- BWE can be cause by rational behavior
  - i.e., by acting in "optimal" ways according to OR inventory models
- Four causes:
  - Demand forecast updating
  - Batch ordering
  - Rationing game
  - Price variations

Questions?