AN ANALYTICAL SURVEY FOR THE QUADRATIC ASSIGNMENT PROBLEM

Eliane Maria Loiola[1]
Nair Maria Maia de Abreu[1]
Paulo Oswaldo Boaventura-Netto[1]
Peter Hahn[2]
Tania Querido[3]

Abstract

The quadratic assignment problem (QAP), one of the most difficult problems in the NP-hard class, models many applications in several areas such as operational research, parallel and distributed computing, and combinatorial data analysis. Other optimization combinatorial problems such as the traveling salesman problem, maximal clique, isomorphism and graph partitioning can be formulated as a QAP. In this paper, we survey some of the most important formulations available and classify them according to their mathematical sources. We also present a discussion on the theoretical resources used to define lower bounds for exact and heuristic algorithms, including those formulated according to metaheuristic strategies. Finally, we analyze the extension of the contributions brought about by the study of different approaches.

Keywords: quadratic assignment problem; formulations; combinatorial optimization.

0. Contents

A preliminary version of this survey, with 278 references, is Loiola *et al*. (2004).

1. Introduction

Let us consider the problem of assigning objects to positions in such a way that each object is designated to exactly one position and reciprocally. The distances between positions, the demand flows among the objects and, in the general case, the *object versus position* assignment costs are known. The international literature identifies the quadratic assignment problem (QAP) as the problem of finding a minimum cost allocation of objects into positions, taking the costs as the sum of all possible distance-flow products.

The main motivation for this survey is the continuous interest in QAP, shown by a number of researchers worldwide, for the theory, applications and solution techniques for this problem. Among the many references listed in this bibliography, we found over a hundred ones published since 1999. The last surveys, books and review articles in the literature are Burkard (1991), Pardalos *et al.* (1994), Burkard and Çela (1996), Çela (1998) and Burkard *et al.* (1998a). The article of Anstreicher (2003) reviews only the recent advances on algorithms. An article by Drezner *et al.* (2004) surveys the state-of-the-art in both heuristic and exact methods.

Koopmans and Beckmann (1957) first proposed the QAP as a mathematical model related to economic activities. Since then, it has appeared in several practical applications: Steinberg (1961) used the QAP to minimize the total amount of connections between components in a backboard wiring; Heffley (1972, 1980) applied it in economic problems; Francis and White (1974) developed a decision framework for assigning a new

[1] Federal University of Rio de Janeiro – eliane@cos.ufrj.br, nair@pep.ufrj.br, boaventu@pep.ufrj.br.
[2] University of Pennsylvania – hahn@seas.upenn.edu.
[3] Federal Center for Technological Education – tania.querido@att.net -Presently at University of Florida, Gainesville, supported by CNPq, Brazil.

facility (police posts, supermarkets, schools) in order to serve a given set of clients; Geoffrion and Graves (1976) focused on scheduling problems; Pollatschek *et al.* (1976) invoked it to define the best design for typewriter keyboards and control panels; Krarup and Pruzan (1978) applied it to archeology; Bokhari (1987), in parallel and distributed computing; Hubert (1987), in statistical analysis; Forsberg *et al.* (1994) used it in the analysis of reaction chemistry and Brusco and Stahl (2000), in numerical analysis. Nevertheless, the facilities layout problem is the most popular application for the QAP: Dickey and Hopkins (1972) applied the QAP for the assignment of buildings in a University campus, Elshafei (1977) in a hospital planning and Bos (1993) in a problem related to forest parks. Benjaafar (2002) introduced a formulation of the facility layout design problem in order to minimize work-in-process (WIP). In his work, he shows that layouts obtained using a WIP-based formulation can be very different from those obtained using the conventional QAP-formulation. For example, a QAP-optimal layout can be WIP-infeasible. The placement of electronic components was studied by Rabak and Sichman (2003) and Miranda et al (2004). Other applications can be found in McCormick (1970), Hubert and Schulz (1976), Heffley (1977), Los (1978), Khare *et al.* (1988a, 1988b), Krackhardt (1988), Eschermann and Wunderlich (1990), Bland and Dawson (1991), Balakrishnan *et al*. (1992) Lacksonen and Enscore (1993), Medova (1994), Philips and Rosen (1994), Gouveia and Voß (1995), Bozer and Suk-Chul (1996), Talbot and Cawley (1996), White (1996), Mason and Rönnqvist (1997), Ostrowski and. Ruoppila (1997), Ball *et al.* (1998), Haghani and Chen (1998), Kochhar *et al.* (1998), Martin (1998), Sarker *et al.* (1998), Spiliopoulos and Sofianopoulou (1998), Tansel and Bilen (1998), Tavakkoli-Moghaddain and Shayab (1998), Urban (1998), Gong *et al.* (1999), Rossin *et al.* (1999), Bartolomei-Suarez and Egbelu (2000), Ho and Moodie (2000), Urban *et al.* (2000), Hahn and Krarup (2001), Pitsoulis *et al.* (2001), Takagi (2001), Siu and Chang (2002), Wang and Sarker (2002), Niemi (2003), Youssef *et al.* (2003), Yu and Sarker (2003), Ciriani *et al.* (2004), and Solimanpur *et al.* (2004).

Since its first formulation, the QAP has been drawing researchers attention all over the world, not only because of its practical and theoretical importance, but also mainly because of its complexity. The QAP is one of the most difficult combinatorial optimization problems: in general, instances of order $n > 30$ cannot be solved in reasonable time. Sahni and Gonzales (1976) had shown that QAP is NP-hard and that, unless P = NP, it is not possible to find an f-approximation algorithm, for a constant f. Such results are valid even when flows and distances appear as symmetric coefficient matrices. Due to its high computation complexity, the QAP has been chosen as the first major test application for the GRIBB project (great international branch-and-bound search). This project is seeking to establish a software library for solving a large class of parallel search problems by the use of numerous computers around the world accessed by Internet. Preliminary results from test runs are presented in Moe (2003).

Several NP-hard combinatorial optimization problems, as the traveling salesman problem, the bin-packing problem, the max clique problem and the isomorphism of graphs, can be modeled as QAPs. The search for local optima in classical internet-available instances is a tendency that allows for the comparison of technique performances, even when the optimum is unknown, or when the use of exact algorithms in these instances is possible, Burkard *et al.* (1996a, 1998b) and Çela, (1998). In the QAP case, we can mention as examples, instances with recently proved optimal solutions: Bur26 (b to h), (2004) and Tai25a (2003) by Hahn; Ste36a (2001) by Brixius and Anstreicher; Bur26a (2001) by Hahn; Kra30a by Hahn; Kra30b, Kra32 and Tho30 (2000) by Anstreicher, Brixius, Goux and Linderoth; Nug30 (one of the most known and challenging instances) (2000) by Anstreicher, Brixius, Goux and Linderoth; Ste36b and Ste36c (1999) by Nystrõm. In 2003, Misevicius enhanced the best-known solution for Tai50a, Tai80a and Tai100a using a modified tabu search. Those results motivated the article of Anstreicher (2003) that registers the recent advances on QAP-solutions, exalting the new algorithms and computational structures used. Besides, the new instances are available for tests in Burkard *et al.* (1991, 1997), Li and Pardalos (1992) and QAPLIB (2004). Also, there are instance generators with known optimum values that are currently used for testing algorithms, Çela (1998). Finally, Palubeckis (1999, 2000), Drezner *et al.* (2004) and Stützle and Fernandes (2004) present new instance sets that are reported to be difficult for metaheuristics.

Other tendencies of combinatorial optimization experts concern the search for particular polynomial versions of NP problems and researches on mechanisms to measure the difficulty of instances. In the QAP cases: Christofides and Gerrard (1976) studied some special instances of QAP; Sylla and Babu (1987) developed a methodology for an orderly quadratic assignment problem; Chen (1995) presented other QAP-cases, followed by Çela (1998), who presented several polynomial instances; Herroeleven and Vangils (1985), Cyganski *et al.* (1994), Mautor and Roucairol (1994b) showed that Palubetskis instances are degenerate; Angel and Zissimopoulos (1998, 2000, 2001, 2002) discussed the difficulty of other instances based on the variance of their flow and distance sets; Abreu *et al.* (2002) derived a polynomial expression for the variance of the solution costs and defined a measure of the difficulty instances and Barvinok and Stephen (2003) built a distribution of QAP-values.

In the challenge of identifying new structural properties for QAP instances many formulations have appeared, based on different points of view. Here we propose to collect these formulations, highlighting their most important features to classify them according to used techniques, such as integer programming, positive semidefinite programming, discrete and combinatorial mathematics, graph and group theory, and linear algebra via spectral theory. Most of these formulations are equivalent (exception to those that characterize more general situations) and, considering the QAP challenging performance, they allow mathematical resources for the development of new solution techniques. By the end of this article, we discuss the contributions obtained from these formulations, building several tables and charts from the extensive bibliography concerning the elaboration of exact and heuristic algorithms, lower bound calculation, instance class characteristics and recording the development of QAP since 1957 to now.

The following surveys are essential references for those who want to have a more complete understanding of this problem: Hanan and Kurtzberg (1972), Burkard (1984, 1991, 2002), Pardalos *et al.* (1994) and Burkard *et al.* (1998a), as well as, the books by Pardalos and Wolkowicz (1994), Padberg and Rijal (1996), Dell'Amico *et al.* (1997) and Çela (1998).

2 Formulations of QAP and Related Problems

In this section, we present the QAP most important formulations, exploiting the type of approach adopted in each case.

2.1. Selected QAP Formulations

Integer Linear Programming Formulations (IP): Firstly we present the QAP as a Boolean program followed by a linear programming problem, where the binary constraints are relaxed. The Boolean formulation was initially proposed by Koopmans and Beckmann (1957) being used later in several works such as Steinberg (1961), Lawler (1963), Gavett and Plyter (1966), Elshafei (1977), Bazaraa and Sherali (1979), Bazaraa and Kirca (1983), Christofides and Benavent (1989), Bos (1993), Mans *et al.* (1995), Jünger and Kaibel (1996a, 1996b, 2000, 2001a, 2001b), Liang (1996), Torki *et al.* (1996), Tsuchiya *et al.* (1996, 2001), Maniezzo (1997), Ball *et al.* (1998), Ishii and Sato (1998), Kaibel (1998), Kochhar *et al.* (1998), Martin (1998), Spiliopoulos and Sofianopoulou (1998), and most recently, Siu and Chang (2002), Yu and Sarker (2003) and, finally, Fedjki and Duffuaa (2004).

We consider f_{ij} the flow between objects i and j, and d_{kp} the distance between positions k and p. It is our goal to calculate:

$$\min \quad \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} f_{ij} d_{kp} x_{ik} x_{jp} \quad \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} f_{ij} d_{kp} x_{ij} x_{kp} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ij} = 1 \qquad 1 \le j \le n, \tag{2.2}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad 1 \le i \le n, \tag{2.3}$$

$$x_{ij} \in \{0,1\} \qquad 1 \le i, j \le n. \tag{2.4}$$

If we consider the cost of assignment of activities to places, a general form for a QAP instance of order n is given by three matrices $F = [f_{ij}]$, $D = [d_{kp}]$ and $B = [b_{ik}]$, the two first ones defining the flows between objects and the distances between places, b_{ik} being the allocation costs of objects to positions. This problem can be defined as:

$$\min \quad \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} f_{ij} d_{kp} x_{ik} x_{jp} + \sum_{i,k=1}^{n} b_{ik} x_{ik} \quad \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} f_{ij} d_{kp} x_{ij} x_{kp} + \sum_{i,k=1}^{n} b_{ik} x_{ik} \tag{2.5}$$

s.t. (2.2), (2.3) and (2.4).

Since the linear term of (2.5) is easy to be solved, most authors discarded it.

A more general QAP version was proposed by Lawler (1963) and involves costs c_{ijkp} that do not necessarily correspond to products of flows by distances. The Lawler formulation is as follows:

$$\min \quad \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} c_{ijkp} x_{ik} x_{jp} \quad \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} c_{ijkp} x_{ij} x_{kp} \tag{2.6}$$

s.t.

(2.2), (2.3) and (2.4).

This model was also used in Bazaraa and Elshafei (1979), Drezner (1995), Sarket *et al.* (1995, 1998), Brüngger *et al.* (1997, 1998), Chiang and Chiang (1998), Hahn and Grant (1998), Hahn *et al.* (1998), Gong *et al.* (1999) and Rossin *et al.* (1999).

Mixed Integer Linear Programming Formulation (MILP): The QAP, as a mixed integer programming formulation, is found in the literature in different forms, all of them replacing the quadratic terms by linear ones. For example, Lawler (1963) used n^4 variables,

$c_{ijkp} = f_{ij} d_{kp}$ and $y_{ijkp} = x_{ik} x_{jp}$, ~~$y_{ijkp} = x_{ij} x_{kp}$,~~ $1 \le i,j,k,p \le n$.

Other formulations use relaxations of the original problem. In this category, one can find Love and Wong (1976a, 1976b), Kaufman and Broeckx (1978), Balas and Mazolla (1980), Bazaraa and Sherali (1980), Christofides *et al.* (1980), Burkard and Bonniger (1983), Frieze and Yadegar (1983), Assad and Xu (1985), Adams and Sherali (1986), Christofides and Benavent (1989) and the works of Adams and Johnson (1994), Drezner (1995), Gouveia and Voß (1995), Milis and Magirou (1995), Padberg and Rijal (1996), White (1996), Ramachandran and Pekny (1998), Karisch *et al.* (1999) and of Ramakrishnan *et al.* (2002). In general, QAP linearizations based on MILP models present a huge number of variables and constraints, a fact that makes this approach avoided in many cases. However, they allow exploiting properties that arise from the linearization of the objective function that, together with some constraint relaxations, lend to the achievement of lower bounds for the optimal solution. In this line we have the works of Kaufman and Broeckx (1978), Bazaraa and Sherali (1980), Frieze and Yadegar (1983), Adams and Sherali (1986), Adams and Johnson (1994) and Padberg and Rijal (1996). Çela (1998) mentions three QAP linearizations: Kaufman and Broeckx (1978), which has the advantage of a smaller number of restrictions; Frieze and Yadegar (1983) for achieving the best lower bounds via Lagrangean relaxation and Padberg and Rijal (1996) owing to its polytope description. The formulation presented by Frieze and Yadegar (1983) describes the QAP in a linear form, using n^4 real variables, n^2 Boolean variables and $n^4 + 4n^3 + n^2 + 2n$ constraints. The authors show that the formulation given in (2.7) to (2.16) below is equivalent to Equations (2.1) to (2.4), Çela (1998).

$$
\begin{array}{llll}
\min & \sum_{i,j=1}^{n} \sum_{k,p=1}^{n} f_{ij} d_{kp} \cdot y_{ijkp} & & \text{-(2.7)} \\
\text{s.t.} & \sum_{i=1}^{n} x_{ik} = 1 & 1 \le k \le n, & (2.8) \\
 & \sum_{k=1}^{n} x_{ik} = 1 & 1 \le i \le n, & (2.9) \\
 & \sum_{i=1}^{n} y_{ijkp} = x_{jp} & 1 \le j,k,p \le n, & (2.10) \\
 & \sum_{j=1}^{n} y_{ijkp} = x_{ik} & 1 \le i,k,p \le n, & (2.11) \\
 & \sum_{k=1}^{n} y_{ijkp} = x_{jp} & 1 \le i,j,p \le n, & (2.12) \\
 & \sum_{p=1}^{n} y_{ijkp} = x_{ik} & 1 \le i,j,k \le n, & (2.13) \\
 & y_{iikk} = x_{iik} & 1 \le i,k \le n, & (2.14) \\
 & x_{ik} \in \{0,1\} & 1 \le i,k \le n, & (2.15) \\
 & 0 \le y_{ijkp} \le 1 & 1 \le i,j,k,p \le n. & (2.16)
\end{array}
$$

Formulations by Permutations: Taking a simple approach, the pairwise allocation of object costs to adjacent positions is proportional to flows and to distances between them. The QAP formulation that arises from this proportionality and uses the permutation concept can be found in Hillier and Michael (1966), Graves and Whinston (1970), Pierce and Crowston (1971), Burkard and Stratman (1978), Roucairol (1979, 1987), Burkard (1984), Frenk *et al.* (1985), Brown *et al.* (1989), Bland and Dawson (1991, 1994), Battiti and Tecchiolli (1994a, 1994b), Bui and Moon (1994), Chakrapani and Skorin-Kapov (1994), Fleurent and Ferland (1994), Li *et al.* (1994b), Mautor and Roucairol (1994a, 1994b), Li and Smith (1995), Taillard (1995), Bozer and Suk-Chul (1996), Colorni *et al.* (1996), Huntley and Brown (1996), Peng *et al.* (1996), Tian *et al.* (1996, 1999), Cung *et al.* (1997), Mavridou and Pardalos (1997), Merz and Freisleben (1997), Nissen (1997), Pardalos *et al.* (1997), Angel and Zissimopoulos (1998), Deineko and Woeginger (1998), Talbi *et al.* (1998a, 1998b, 2001), Tansel and Bilen (1998), Abreu *et al.* (1999), Fleurent and Glover (1999), Gambardella *et al.* (1999) and Maniezzo and Colorni (1999). More recently, the following articles were released: Ahuja *et al.* (2000), Angel and Zissimopoulos (2000, 2001 e 2002), Stützle and Holger (2000), Arkin *et al.* (2001), Pitsoulis *et al.* (2001), Abreu *et al.* (2002), Gutin and Yeo (2002), Hasegawa *et al.* (2002), Boaventura-Netto (2003) and Rangel and Abreu (2003). Costa and Boaventura-Netto (1994) studied the non-symmetrical QAP through a directed graph formulation.

Let S_n be the set of all permutations with n elements, f_{ij} the flows between objects i and j and $d_{\partial(i)\partial(j)}$ the distances between positions $\pi(i)$ and $\pi(j)$. If each permutation π represents an allocation of objects to positions, the problem expression becomes:

$$\min_{\partial \in S_n} \sum_{i,j=1}^{n} f_{ij} d_{\pi(i)\pi(j)} . \tag{2.17}$$

This formulation is equivalent to the first one presented in (2.1) - (2.4), since the constraints (2.2) and (2.3) define permutation matrices $X = [x_{ij}]$ related to S_n elements, as in (2.17), where, for all $1 \le i, j \le n$,

$$x_{ij} = \begin{cases} 1, & \text{if } \partial(i) = j; \\ 0, & \text{if } \partial(i) \neq j. \end{cases} \tag{2.18}$$

Trace Formulation: This formulation is supported by linear algebra and exploits the trace function (the sum of the matrix main diagonal elements) in order to determine QAP lower bounds for the cost. This approach allows for the application of spectral theory, which makes possible the use of semidefinite programming to the QAP. The trace formulation, by Edwards (1977), can be stated as:

$$\min_{X \in S_n} \ tr(F.X.D.X^t) . \tag{2.19}$$

Afterwards, this approach was used in several works: Edwards (1980), Finke *et al.* (1987), Hadley *et al.* (1990, 1992a, 1992b, 1992c), Hadley (1994), Karisch *et al.* (1994), Karisch and Rendl (1995), Lin and Saigal (1997), Zhao *et al.* (1998), Anstreicher *et al.* (1999), Wolkowicz (2000a, 2000b) and Anstreicher and Brixius (2001).

Semidefinite Programming Relaxation (SDP): These formulations define QAP relaxations through the dual of the Lagrangean dual, as we find in Karisch *et al.* (1994), Zhao *et al.* (1998), Wolkowicz (2000a, 2000b). Let e be the vector such that each coordinate is equal to 1. If X is a permutation matrix and B is a cost matrix, then the SDP formulation is:

$$\min \ tr(F.X.D - 2B)X^t \tag{2.20}$$
$$\text{s.t.} \quad Xe = e , \tag{2.21}$$
$$X^t e = e , \tag{2.22}$$
$$X_{ij} \in \{0,1\} \qquad \forall\, i, j . \tag{2.23}$$

Another formulation that follows this approach, Zhao *et al.* (1998), is

$$\min \ tr\, F.X.D.X^t - 2BX^t \tag{2.24}$$
$$\text{s.t.} \quad XX^t = X^t X = I , \tag{2.25}$$
$$Xe = X^t e = e , \tag{2.26}$$
$$X_{ij}^2 - X_{ij} = 0 \qquad \forall\, i, j . \tag{2.27}$$

Graph Formulation: Let us consider two undirected weighted complete graphs, the first one having its edges associated to flows and the second one, to distances. The QAP can be thought as the problem of finding an optimal allocation of the vertices of one graph on those of the other. In this formulation the solution costs are given as the sum of products of corresponding edge weights. See Figure 2.1.

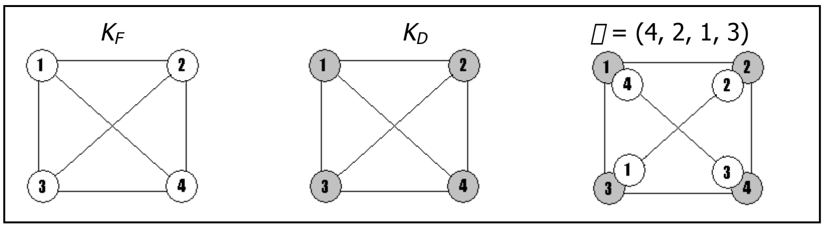

Figure 2.1 – Allocation of cliques K_F over K_D concerning permutation π = (4,2,1,3).

The algebraic and combinatorial approach adopted by Abreu *et al.* (1999) suggested Marins *et al.* (2004) to define a new algebraic graph-theoretical approach involving line-graph automorphisms. The line-graph of a given graph G, denoted $L(G)$, is determined by taking each edges of G as a vertex of $L(G)$, while an edge of $L(G)$ is defined as a pair of edges that are adjacent in G. A graph automorphism is a permutation of its vertices that preserves the edges. The set of all automorphisms of G together with the permutations composition is a group denoted by Aut($L(G)$) (Kreher and Stinson (1998)).

As a consequence of a theorem by Whitney (1932) we have that if $G = K_n$, $n \neq 2$ and 4, Aut(G) and Aut($L(G)$) constitute isomorphic groups. Based on this result, Marins *et al.* (2004) noticed that solving the QAP means either find a permutation $\partial \in S_n$ or find a $L(K_n)$ automorphism, which is a $C_{n,2}$ permutation, that minimizes the following expression:

$$\min_{\partial \in Aut(L(K_n))} \sum_{i=1}^{N} f_i d_{\partial(i)} \quad . \tag{2.28}$$

It is convenient to mention White (1995) where a number of QAP representations are discussed with respect to their convexity and concavity aspects and of Yamada (1992) where a formulation is presented for the QAP on an n-dimensional grid.

2.2 QAP related problems

The most classical QAP-related problem is, obviously, the Linear Assignment Problem (LAP), which is polynomial and easily solved by the Hungarian method. As several presentations of this problem can be found in the literature (for example, Burkard (2002)) we do not discuss it here, so we prefer to begin with another linear problem also used in QAP studies.

The three-index assignment problem (3-dimensional AP): was firstly suggested by Pierskalla (1967a, 1967b, 1968) searches for two permutations ∂ and $\psi \in S_n$, such that the following expression is minimized:

$$\min_{\partial, \psi \in S_n} \sum_{i=1}^{n} c_{i\partial(i)\psi(i)} \ . \tag{2.29}$$

Hansen and Kaufman (1973) presented later a primal-dual algorithm for this problem and Burkard and Fröhlich (1980) proposed a branch-and-bound algorithm to solve it. Emelichev *et al.* (1984) described transportation models with multiple indices, with details, based on this formulation. The 3-dimensional AP has been studied by several QAP experts: Vlach (1967), Frieze (1974, 1983), Frieze and Yadegar (1981), Burkard *et al.* (1986, 1996a, 1996b), Euler (1987), Balas and Saltzman (1989, 1991), Bandelt *et al.* (1991), Crama and Spieksma (1992), Balas and Qi (1993), Burkard and Rudolf (1993), Qi *et al.* (1994), Magos and Miliotis (1994), Poore (1994a, 1994b, 1995), Fortin and Rudolf (1995), Burkard and Çela (1996), Magos (1996), Poore and Robertson (1997), Aiex *et al*. (2000), Aiex *et al*. (to appear) and Burkard (2002).

The wide range of QAP theoretical studies involve several related quadratic problems, as the quadratic bottleneck assignment problem, the biquadratic assignment problem, the 3-dimensional QAP, the quadratic semiassignment problem and the multiobjective QAP. Almost all of these problems were reported in Burkard (2002).

Quadratic bottleneck assignment problem (QBAP): Steinberg (1961) considered QBAP a variation of QAP with applications to backboard wiring. In that work, a placement algorithm was presented for the optimal connection of n elements in individual positions such that the total wire needed to connect two elements should have minimum length. Its basic claim is: *the optimal weighted-wire-length equals the least among the maximum-wire-length norm*. This concept rises from the principle that it can be worthy to minimize the largest cost instead of the overall given cost.

The QBAP general program is obtained from the QAP formulation by substituting the maximum operation in the objective function for the sums, which suggests the term *bottleneck function*:

$$\min_{\partial \in S_n} \max \left\{ f_{ij} d_{\partial(i)\partial(j)} : 1 \le i,j \le n \right\}. \tag{2.30}$$

A general formulation related to (2.30) was studied in Burkard and Finke (1982), Burkard and Zimmermann (1982), Kellerer and Wirsching (1998) and Burkard (2002).

Biquadratic assignment problem (BiQAP): Proposed by Burkard *et al.* (1994), this problem can be found in other works, such as Burkard and Çela (1995), Mavridou *et al.* (1998) and Burkard (2002). The flow and distance matrices are order n^4 and the BiQAP-formulation is:

$$\min_{\partial \in S_n} \sum_{i,j,k,l=1}^{n} f_{ijkl} d_{\partial(i)\partial(j)\partial(k)\partial(l)} \ . \tag{2.31}$$

Quadratic 3-dimensional assignment problem (Q3AP): Pierskalla (1967b) introduces it in a technical memorandum. The work was never published in the open literature. Since then, nothing on the subject has been found in the publication databases. Hahn *et al*. (2004) re-discovered the Q3AP while working together with others on a problem arising in data transmission system design. The purpose is to jointly optimize pairs of mappings for multiple transmissions using higher order signal constellations. The resulting problem formulation is:

$$\min \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{p=1}^{N} b_{ijp} u_{ij} w_{ip} + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{p=1}^{N} \sum_{k=1}^{N} \sum_{n=1}^{N} \sum_{q=1}^{N} C_{ijpknq} u_{ij} u_{kn} w_{ip} w_{kq} : u \in U, w \in W; u, w \text{ binary} \right\}, \tag{2.32}$$

where $$x \in X \equiv \{x \geq 0 : \sum_{i=1}^{N} x_{ij} = 1 \text{ for } j = 1,\dots, N; \sum_{j=1}^{N} x_{ij} = 1 \text{ for } i = 1,\dots, N\}. \tag{2.33}$$

Quadratic semi-assignment problem (QSAP): this is a special case used to model clustering and partitioning problems by Hansen and Lih (1992), and scheduling problems by Malucelli and Pretolani (1993). It can be written as:

$$\min \sum_{k=1}^{m} \sum_{i,j=1}^{n} c_{ij} x_{ik} x_{jk} \tag{2.34}$$

$$\text{s.t.} \quad \sum_{k=1}^{m} x_{ik} = 1 \qquad 1 \leq i \leq n, \tag{2.35}$$

$$x_{ij} \in \{0,1\} \qquad 1 \leq i, j \leq n. \tag{2.36}$$

Other applications can be found in Simeone (1986a, 1986b) and Bullnheimer (1998). References for polynomial heuristics and lower bounds are Freeman *et al.* (1966), Magirou and Milis (1989), Malucelli and Pretolani (1993), Carraresi and Malucelli (1994) and Billionnet and Elloumi (2001).

Multiobjective QAP (mQAP): Knowles and Corne (2002a, 2002b) presented another QAP variation considering several flow and distance matrices. This problem is a *benchmark case* for multiobjective metaheuristics or multiobjective evolutionary algorithms. According to the authors, this model is more suitable for some layout problems, such as the allocation of facilities in hospitals, where it is desired to minimize the products of the flows by the distances between doctors and patients, and between nurses and medical equipment, simultaneously. The mathematical expression is then,

$$\min_{\pi \in S_n} \vec{C}(\pi) = \{C^1(\pi), C^2(\pi), \dots, C^m(\pi)\},$$

$$\text{where} \quad C^k(\pi) = \sum_{i,j=1}^{n} f_{ij}^k d_{\pi(i)\pi(j)}, \qquad 1 \leq k \leq m. \tag{2.37}$$

In this last constraint, f_{ij}^k denotes the *k*-the flow between *i*- and *j*-facilities. More recently, Knowles and Corne (2003) and Day *et al.* (2003) presented two instance generators for the multiobjective version of QAP and Paquete and Stützle (2004) developed a study of stochastic local search algorithms for the biobjective QAP with different degrees of correlation between the flow matrices.

3. Lower Bounds

The study of lower bounds is very important for the development of algorithms to solve mathematical programming and combinatorial optimization problems. Generally, the exact methods employ implicit enumeration, in an attempt to guarantee the optimum and, at the same time, to avoid the total enumeration of the feasible solutions. The performance of these methods depends on the computational quality and efficiency of the lower bounds. Thus, lower bounds are fundamental tools for branch-and-bound techniques and for the evaluation of the quality of the solutions obtained from some heuristic algorithms. One can measure the quality of a lower bound by the gap between its value and the optimal solution. So, good lower bounds should be closer to the optimum and lower bounds should be used within exact methods when they can be found quickly, while to be used in heuristics, their good quality is most important.

The QAP lower bound presented by Gilmore (1962) and Lawler (1963) is one of the best known. Its importance is due to its simplicity and its low computational cost. However, it shows an important drawback as its gap grows very quickly with the size of the problem, making it a weak bound for bigger instances. The most recent and promising trends of research are based on semidefinite programming, reformulation-linearization and lift-and-project techniques, although they usually need an extra computational effort. Anstreicher and Brixius (2001) reported a new QAP bound using semidefinite and convex quadratic programming with good relation between cost and quality. White (1994b) used a data decomposition method, linking the actual data to the data of a special class of assignment problems for which bounds are computationally tractable.

The Gilmore and Lawler lower bound (GLB) is given by the solution of the following linear assignment problem (LAP):

$$\min \quad \sum_{i,j=1}^{n} (b_{ij} + l_{ij}).x_{ij} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ij} = 1 \qquad 1 \le j \le n; \tag{3.2}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad 1 \le i \le n; \tag{3.3}$$

$$x_{ij} \in \{0,1\} \qquad 1 \le i, j \le n. \tag{3.4}$$

In order to solve (3.1) - (3.4), it is necessary to find the coefficients $l_{ij,}$ as below:

$$l_{ij} = \min \sum_{k,p=1}^{n} c_{ijkp} . y_{ijkp} \qquad k \neq i, p \neq j \tag{3.5}$$

$$\text{s.t.} \quad \sum_{k=1}^{n} y_{ijkp} = 1 \qquad 1 \le p \le n, \tag{3.6}$$

$$\sum_{p=1}^{n} y_{ijkp} = 1 \qquad 1 \le k \le n, \tag{3.7}$$

$$y_{ijkp} \in \{0,1\} \qquad 1 \le i, j, k, p \le n. \tag{3.8}$$

Roucairol (1979, 1987), Edwards (1980), Frieze and Yadegar (1983), Finke *et al.* (1987), White (1994a), Maniezzo (1997), Burkard (1991), Brüngger *et al.* (1997, 1998), and Spiliopoulos and Sofianopoulou (1998) present improvement methods for the GLB and its application to algorithms used to solve QAP.

Bounds based on MILP Relaxations: The optimal solution for a MILP-formulation is a lower bound for the corresponding QAP and each dual solution of the linear programming is also a lower bound for the QAP. Several researchers as in Frieze and Yadegar (1983), Assad and Xu (1985), Adams and Johnson (1994), Ramachandran and Pekny (1998) and Karisch *et al.* (1999) used this principle. Lagrangean relaxation has been applied to the QAP (Michelon and Maculan (1991)). Drezner (1995) also proved that the linear programming relaxation is equal or better than the GLB bound. Adams *et al.* (to appear) calculate bounds using a level-2 reformulation linearization technique (2-RLT) due to Hahn *et al.* (2001b). The RLT is a general theory for reformulating mixed 0-1 linear and polynomial programs in higher-variable spaces in such a manner that tight polyhedral outer-approximations of the convex hull of solutions are obtained (Adams and Sherali (1986, 1990)). In Sherali and Adams (1999a, 1999b) both first and second-level constructs for the QAP were presented as an illustration of the general methodology.

Bounds based on GLB reformulations: These bounds were adapted by several authors including Frieze and Yadegar (1983), Assad and Xu (1985), Carraresi and Malucelli (1992, 1994) and Adams and Johnson (1994). A bound based on a dual formulation was proposed in Hahn and Grant (1998) and Hahn *et al.* (1998). The bounds given by Assad and Xu (1985) and by Carraresi and Malucelli (1992, 1994) are comparable to the ones obtained by Frieze and Yadegar (1983) in terms of quality, with the advantage that they demand less computational time. However, there is no theoretical proof concerning its convergence. Those bounds characterize a finite sequence of problems related to the original one, producing a non-decreasing GLB sequence. The computational results in Hahn and Grant (1998) have shown that these bounds are competitive in terms of quality when compared to some of the best bounds and still better in computational time.

Bounds based on interior points methods: Resende *et al.* (1995) used Drezner (1995) theory and solved a MILP linear relaxation using an interior points algorithm (Karmarkar and Ramakrishnan (1991)). This technique gives better quality lower bounds than those obtained by Adams and Johnson (1994). However, these bounds require much computational effort, and they are not recommended for branch-and-bound algorithms. In this case, it is better to use the Hahn and Grant (1998) dual ascent lower bound.

Variance reduction bounds: Initially proposed by Li *et al.* (1994a), these bounds are based on reduction schemes and are defined from the variance of F and D matrices. These bounds, when used in a branch-and-bound algorithm, take less computational time and generally obtain better performance than GLB. They show more efficiency when the flow and distance matrices have high variances.

Bounds based on graph formulation: As we discussed previously, a pair of $n \times n$ matrices F and D associated to a given QAP instance can be seen as the adjacency matrices of two weighted complete graphs K_F and K_D. It is known that $\varphi \in S_n$ defines an isomorphism between K_F and K_D, then solving the QAP means to find an isomorphism $\varphi \in S_n$ such that Z_φ is minimum. Gavett and Plyter (1966) and Christofides and Gerrard (1981) used this concept, decomposing K_F and K_D in isomorphic spanning subgraphs to find lower bounds through an LAP relaxation.

Spectral bounds: We consider here the bounds derived from the trace formulation, using the calculation of data matrix eigenvalues. For some time, the quality of the results compensated the computational hardness of the calculations, but recently some of these bounds have been superseded by reformulation-linearization and semidefinite programming bounds. Some references on spectral bounds are Finke *et al.* (1984, 1987), Rendl (1985), Hadley *et al.* (1990, 1992a, 1992b), Rendl and Wolkowicz (1992) and Karisch *et al.* (1994).

Semidefinite programming and reformulation-linearization bounds: This new trend uses a number of theoretical tools to obtain linear programming representations of QAP. Zhao *et al*. (1998) study semidefinite programming (SDP) relaxations; Anstreicher (2001) compares SDP relaxations and eigenvalue bounds; Anstreicher and Brixius (2001) propose a SDP representation of a basic eigenvalue bound; Burer and Vandenbussche (2004) applied Lagrangean relaxation on a lift-and-project QAP relaxation, following the ideas in Lovász and Scrijver (1991), thus obtaining very tight SDP bounds.

Instance	Optimum	GLB	RRD95	HG98	KCCEB99	AB01	HH01	RRRP02	BV04
Had16	3720	3358		3558	3553	3595	3720*		3672
Had18	5358	4776		5083	5078	5143	5358*		5299
Had20	6922	6166		6571	6567	6677	6922*		6811
Kra30a	88900	68360		75853	75566	68572	86247		86678
Kra30b	91420	69065		76562	76235	69021	87107		87699
Nug12	578	493	523	523	521	498	578*	578*	568
Nug15	1150	963	1041	1039	1033	1001	1150*	---	1141
Nug20	2570	2057	2182	2179	2173	2290	2487	---	2506
Nug30	6124	4539		4793	4785	5365	5750		5934
Rou15	354210	298548		323943	323589	303777	354210*		350207
Rou20	725520	559948		642058	641425	607822	699390		695123
Tai20a	703482	580674		617206	616644	585139	675870		671685
Tai25a	1167256	962417		1006749	1005978	983456	1091618		1112862
Tai30a	1818146	1504688		1566309	1565313	1518059	1686290		1706875
Tho30	149936	90578		99995	99855	124684	136708		142814

* Problem solved exactly by lower bound calculation

Table 3.1 – Comparison of Lower Bounds.

Table 3.1 shows some results for different bounds, where GLB is the Gilmore-Lawler bound; RRD95 is the interior-point bound from Resende *et al*. (1995); HG98 is the interior-point bound from Hahn and Grant (1998); KCCEB99 is the dual-based bound from Karisch *et al*. (1999); RRRP02 is the 2-RLT interior point bound from Ramakrishnan *et al*. (2002); HH01 is the Hahn-Hightower 2-RLT bound from Adams *et al.* (to appear); AB01 is the Rendl-Wolkowicz eigenvalue bound (Anstreicher and Brixius (2001)); and BV04 is the lift-and-project SDP bound from Burer and Vandenbussche (2004). The best lower bounds are in the shaded cells of the table.

4 Resolution Methods

The methods used in combinatorial optimization problems can be either exact or heuristic. In the first case, the most frequent used strategies are branch-and-bound or dynamic programming general methods. On the other hand, there are a number of heuristic techniques using different conceptions. In what follows we discuss both approaches and we quote their most important references.

4.1 Exact Algorithms

The different methods used to achieve a global optimum for the QAP include branch-and-bound, cutting planes or combinations of these methods, like branch-and-cut, and dynamic programming. Branch-and-bound are the most known and used algorithms and are defined from allocation and cutting rules, which define lower bounds for the problem. We can find the first enumerative schemes that use lower bounds to eliminate undesired solutions: Gilmore (1962), Land (1963) and Lawler (1963). Several references concerning QAP branch-and-bound algorithms are available, as Gavett and Plyter (1966), Nugent *et al.* (1968), Graves and Whinston (1970), Pierce and Crowston (1971), Burkard and Stratman (1978), Bazaraa and Elshafei (1979), Mirchandani and Obata (1979), Roucairol (1979), Burkard and Derigs (1980), Edwards (1980), Bazaraa and Kirca (1983), Kaku and Thompson (1986), Pardalos and Crouse (1989), Burkard (1991), Laursen (1993), Mans *et al.* (1995), Bozer and Suk-Chul (1996), Pardalos *et al.* (1997), Brüngger *et al.* (1998), Ball *et al.* (1998), Spiliopoulos and Sofianopoulou (1998), Brixius and Anstreicher (2001) and Hahn *et al.* (2001a, 2001b). In recent years, procedures that combine branch-and-bound techniques with parallel implementation are being widely used. Due to them, the best results for the QAP are being achieved. Yet, it is important to observe that the success for the instances of bigger sizes is also related to the hardware technological improvements, Roucairol (1987), Pardalos and Crouse (1989), Mautor and Roucairol (1994a), Brüngger *et al.* (1997), Clausen and Perregaard (1997) and Maniezzo (1997).

Dynamic programming is a technique used for QAP special cases where the flow matrix is the adjacency matrix of a tree. Christofides and Benavent (1989) studied this case using a MILP approach to the relaxed problem. It was then solved with a dynamic programming algorithm, taking advantage of the polynomial complexity of the instances. This technique was also used by Urban (1998).

Cutting plane methods introduced by Bazaraa and Sherali (1980), initially, did not present satisfactory results. However, they contributed in the formulation of some heuristics that use MILP and Benders decomposition. The employed technique is not widely used so far, but good quality solutions for QAP cases are being presented. The slow convergence of this method makes it proper only for small instances (Kaufman and Broeckx (1978), Balas and Mazolla (1980), Bazaraa and Sherali (1980, 1982) and Burkard and Bonniger (1983). Recently, Miranda et al (2004) use Benders decomposition algorithm to deal with a motherboard design problem, including linear costs in the formulation. The branch-and-cut technique, a denomination proposed by Padberg and Rinaldi (1991), appears as an alternative strategy and exploits the polytope defined by the feasible solutions of the considered problem. Its main advantage over cutting planes is the use of cuttings that are valid to the polytope formed by all feasible solutions, defining facets. These cuts that are associated to the facets are more significant then the ones produced by cutting planes method, so the convergence to an optimal solution is accelerated. The little knowledge about the polytope characterized by QAP solutions is the reason why polyhedral cutting planes are not widely used in this case. Even in this scenario, some researches have been describing basic properties of this polytope, which can contribute for future algorithms development, Jünger and Kaibel (1996a, 1996b, 2000, 2001a, 2001b), Padberg and Rijal (1996), Kaibel (1998) and Blanchard *et al.* (2003).

The effects of methodology and computer speed improvements

Through Table 4.1 below, we look for presenting a quick landscape of which was achieved by QAP research in the last 35 years, Hahn (2000) and Brixius and Anstreicher (2001), through the work done on the classical Nugent instances, Nugent *et al*. (1968). The first result (for Nug08) was obtained by complete enumeration; all the others have been obtained by several branch-and-bound variations. Owing to lack of space, the references within the table are quoted by their number in the reference list. The column "Single CPU seconds" allows for some comparison of results obtained in different machines

Size	Bound	Year	Machine	Cpu speed	Single Cpu seconds	No. Nodes	Who [Ref]	Mins (Norm)
8	---	1968	GE 265*		3,374	40,320	(267)	
8	GL	1975	CDC CYBER-76		<1		(61)	
12	GL	1978	CDC CYBER-76		29		(62)	
15	GL	1980	CDC CYBER-76		2,947		(63)	
15	GL	1994	Cray 2		121		(233)	
16	GL	1994	Cray 2		969		(233)	
20	GL	1995	i860	40 MHz	811,440	360,148,026	(96)	845
20	RLT1	1995	SPARC10	75 MHz	159,900	724,289	(157)	333
20	QP	1999	HP-C3000	300 MHz	8,748	1,040,308	)17)	146
20	RLT1	1999	UltraSPARC10	360 MHz	5,087	181,073	(160)	42
22	C-M	1995	16 i860	40 MHz	48,308,400	48,538,844,413	(96)	50,321
22	RLT1	1995	DEC Alpha 500	300 MHz	1,812,420	10,768,366	(157)	10,270
22	QP	1999	HP-C3000	300 MHz	8,058	1,225,892	(17)	134
22	RLT1	1999	UltraSPARC10	360 MHz	48,917	1,354,837	(160)	408
24	GL	1997	32 Motorola 604		82,252,800	Unknown	(54)	466,099
24	RLT1	1997	DEC Alpha 500	300 MHz	4,859,940	49,542,338	(157)	27,540
24	QP	2000	HP-C3000	300 MHz	349,794	31,865,440	(17)	5,830
24	RLT2	2000	DEC Alpha 500	300 MHz	1,487,724	16,710,701	(160)	8,430
25	RLT1	1998	UltraSPARC10	360 MHz	5,698,818	108,738,131	(160)	64,207
25	QP	2000	HP-C3000 (1)*	300 MHz	715,020	71,770,751	(18)	11,917
25	RLT1	2000	HP-J5000	440 MHz	1,393,117	27,409,486	(160)	31,879
25	RLT2	2000	Dell 7150	733 MHz	254,179	11,796	(160)	5,816
27	QP	2000	HP-C3000 (2)*	300 MHz	5,676,480	~402,000,000	(18)	94,608
27	RLT2	2001	IBM S80	450 MHz	1,579,956.31	46,315	(160)	37,639
28	QP	2000	HP-C3000 (3)*	300 MHz	27,751,680	~2,230,000,000	(18)	462,528
28	RLT2	2001	IBM S80	450 MHz	8,682,044	202,295	(160)	206,922
30	QP	2000	HP-C3000 (4)*	300 MHz	218,859,840	11,892,208,412	(18)	3,647,664
30	RLT2	2004	Dell 7150	733 MHz	~40,000,000**	<500,000**	(6)	915,333**

* means equivalent single CPU seconds in HP-C3000, for time on computational pools with active machines (average): (1) 185; (2) 185; (3) 224; (4) 653.

** Estimated results from a 90% branch and bound enumeration.

Table 4.1 – The Nugent instances along time

A number of optimal solutions have been found in recent years. The details can be found in QAPLIB homepage (QAPLIB (2004)): Ste36b-c (Nyström, 1999); Bur26a (Hahn, 10/2001); Ste36a (Brixius and Anstreicher, 10/2001); Kra30b, Kra32, Tho30 (Anstreicher *et al*. 11/2000); Kra30a (Hahn, 12/2000); Tai25a (Hahn, 2003); Bur26b-h (Hahn, 2004).

4.2. Heuristic Algorithms

Heuristic algorithms do not give any guarantee of optimality for the best solution obtained. Approximate methods can be included in this category, having, in addition, the fact that properties with worst-case guarantee are known. As a matter of fact, it is usual to find approximate algorithms treated as heuristic algorithms in the Combinatorial Optimization literature, as in Osman and Laporte (1996). In this context, we are considering heuristic techniques only as a procedure dedicated to the search of good quality solutions. These procedures include the following categories: constructive, limited enumeration and improvement methods. The most recent techniques that can be adapted to a wide range of problems are the metaheuristics and will be the subject of the next section.

Gilmore (1962) introduced ***constructive methods*** that complete a permutation with each iteration of the algorithm. Sets A and L were considered, the first one concerning the allocated objects and the second, the occupied positions, both initially empty. In these methods, the construction of a permutation π is made by means of a heuristic and, in each step, a new allocation (i, j) is chosen, such that $i \in A, j \in L$ and making $\pi(i) = j$. For an instance of size n, the process is repeated until a complete permutation on the problem order is achieved. Those methods were used in Armour and Buffa (1963), Buffa *et al.* (1964), Sarker *et al.* (1995, 1998), Tansel and Bilen (1998), Burkard (1991), Arkin *et al.* (2001), Gutin and Yeo (2002) and Yu and Sarker (2003). At the end of 1990, multistart techniques are used to begin heuristic or metaheuristic methods. In this category, we cite Misevicius (1997), Fleurent and Glover (1999) and Misevicius and Riskus (1999).

Enumeration methods can guarantee that the obtained solution is optimum only if they can go to the end of the enumerative process. However, it is possible that a good solution, or even an optimal solution, is found by the beginning of the process. It can be observed that the best the information used to guide the enumeration, the bigger the chances to find prematurely good quality solutions. However, in general it may take long to guarantee optimality. In order to bound this enumeration, stopping conditions are defined: maximum number of loops for the whole execution, or between two successive improvements; a limit for the execution time and so on. It becomes clear that any one of these stopping criteria can eliminate the optimum solution, a fact that requires some attention when using bounded enumeration methods (Burkard and Bonniger (1983), West (1983)). Nissen and Paul (1995) applied the *threshold accepting* technique to the QAP.

Improvement methods correspond to local search algorithms. Most of the QAP heuristics are part of this category. An improvement method begins with a feasible solution and tries to improve it, searching for other solutions in its neighborhood. The process is repeated until no improvement can be found. The basic elements for this method are the neighborhood and the selection criterion that defines the order through which the neighbors are analyzed (Heider (1973), Mirchandani and Obata (1979), Bruijs (1984), Pardalos *et al.* (1993), Burkard and Çela (1995), Li and Smith (1995), Anderson (1996), Talbi *et al.* (1998a), Deineko and Woeginger (2000), Misevicius (2000a) and Mills *et al.* (2003)). Those category methods are frequently used by metaheuristics.

It is worthy to mention that up to this date, approximate algorithms with performance guaranteed for one constant were only obtained for special cases of QAP. Examples are the cases where the distance matrix satisfies the triangular inequality (Queyranne (1986)) or when the problem is treated as a maximal clique problem with given maximum bound (Arkin *et al.* (2001)).

White (1993) proposed a new approach, where the actual data is relaxed by embedding them in a data space that satisfies an extension of the metric triangle property. The computations become simpler and bounds are given for the loss of optimality.

4.3. Metaheuristics

Before the end of the 80's, most of the proposed heuristic methods for combinatorial optimization problems were specific and dedicated to a given problem. After that period, this paradigm has changed. More general techniques have appeared, known as metaheuristics. They are characterized by the definition of *a priori* strategies adapted to the problem structure. Several of these techniques are based on some form of simulation of a natural process studied within another field of knowledge (metaphors). With the advent of metaheuristics, QAP research received new and increased interest. Recall that the QAP is considered a classical challenge or "benchmark" as we mentioned earlier, Moe (2003).

4.3.1 The following metaheuristics are based on natural process metaphors.

Simulated annealing is a local search algorithm that exploits the analogy between combinatorial optimization algorithms and statistical mechanics (Kirkpatrick *et al.* (1983)). This analogy is made by associating the feasible solutions of the combinatorial optimization problem to states of the physical system, having costs associated to these states energies. Let E_i and E_{i+1} be two energy successive states, corresponding to two neighbor solutions and let $\Delta E = E_{i+1} - E_i$. The following situations can occur: if $\Delta E < 0$, there is an energy reduction and the process continues. In other words, there is a reduction on the problem cost function and the new allocation may be accepted; if $\Delta E = 0$, there is a stability situation and there is no change in the energy state. This means that the problem cost function was not changed; if $\Delta E > 0$, an increase on the energy is characterized and it is useful for the physical process to permit particle accommodation, i.e., the problem cost function is increased. Instead of eliminating this allocation, its use is subjected to the values of a probability function, to avoid convergence into poor local minima. Burkard and Rendl (1984) proposed one of the first applications of simulated annealing to the QAP. After that, Wilhelm and Ward (1987) presented the new equilibrium components for it. Connolly (1990) introduced an *optimal temperature* concept that gave valuable results. Later, Abreu *et al.* (1999) applied the technique by trying to reduce the number of inversions associated to the problem solution, together with the cost reduction. Other approaches for the simulated annealing applied to the QAP are Bos (1993), Yip and Pao (1994), Burkard and Çela (1995), Peng *et al.* (1996), Tian *et al.* (1996, 1999), Mavridou and Pardalos (1997), Chiang and Chiang (1998), Misevicius (2000b, 2003c), Tsuchiya *et al.* (2001), and Siu and Chang (2002).

Genetic algorithms are techniques that simulate the natural selection and adaptation of the species. These algorithms keep a *population* formed by a subset of *individuals* that correspond, in the QAP case, to the feasible permutations, with fitness values associated to these permutations costs. By means of the so-called *genetic operators*, and of *selection criteria*, the algorithm replaces a population by another with best fitness values. The basic idea consists in the believing that the best individuals survive and generate descendents with their genetic characteristics, in the same way as described by the biological species theory. The analogy is conducted by making the genetic algorithms begin with a population of randomly generated initial solutions, evaluate their costs, select a subset with the best solutions and apply genetic operations on them, generating a new solution set (a new population), Davis (1987) and Goldberg (1989). Some ideas for the use of genetic algorithms on the QAP can be found in Brown *et al.* (1989), Bui and Moon (1994), Tate and Smith (1995), Mavridou and Pardalos (1997), Kochhar *et al*. (1998), Tavakkoli-Moghaddain and Shayan (1998), Gong *et al.* (1999) and Drezner and Marcoulides (2003). The use of these algorithms in the *QAP* context presents some difficulties in getting the optimal solution, even for small instances. However, some hybrid ideas using genetic algorithms have shown to be more efficient, as discussed ahead.

Scatter search is a technique that was introduced by Glover (1977) in a heuristic study of integer linear programming problems. It is an evolutionary method that takes linear combinations of solution vectors in order to produce new solution vectors in successive generations. This metaheuristic is composed of initial and evolutionary phases. On the first one, a solution set is made with the better solutions that will be used as reference and, in the other; new solutions are generated using strategically selected combinations of the reference subsets. From that time on, a set of the best-generated solutions is included into the reference set. The evolutionary phase procedures are repeated until the moment that a stop criterion is satisfied. An application to the QAP can be found in Cung *et al.* (1997).

Ant colony is referred to a class of distributed algorithms that has as most important feature the definition of properties in the interaction of several simple agents. Its principle is the way through that the ants are able to find a path from the colony to a feeding source. Each simple agent is called an *ant* and the set of ants, cooperating in an ordinary activity to solve a problem, constitute the ant system. The main characteristic of this method is the fact that the interaction of these agents generates a synergetic effect, because the quality of the obtained solutions increases when these agents work together, interacting among themselves. Numerical results for the QAP are presented in Maniezzo and Colorni (1995, 1999), Colorni *et al.* (1996), Dorigo *et al.* (1996) and Maniezzo (1997). Gambardella *et al.* (1999) show ant colony as a competitive metaheuristic, mainly for instances that have few good solutions close to each other. Other references are in Stützle and Dorigo (1999), Stützle and Holger (2000), Talbi *et al.* (2001), Middendorf *et al.* (2002), Solimanpur *et al.* (2004) and Ying and Liao (2004).

Although neural networks and Markov chains are structurally different from metaheuristics, they are also based on a nature metaphor and they have been applied to the QAP, Bousonocalzon and Manning (1995), Liang (1996), Obuchi *et al.* (1996), Tsuchiya *et al.* (1996), Ishii and Sato (1998, 2001), Rossin *et al.* (1999), Niitisuma *et al.* (2001), Nishiyama *et al.* (2001), Hasegawa *et al.* (2002) and Uwate *et al*. (2004).

4.3.2 The following metaheuristics are based directly on theoretical and experimental considerations.

Tabu search is a local search algorithm that was introduced by Glover (1989a, 1989b) to find good quality solutions for integer programming problems. Its main feature is an updated list of the best solutions that were found in the search process. Each solution receives a priority value or an aspiration criterion. Their basic ingredients are: a *tabu list*, used to keep the history of the search process evolution; a mechanism that allows the acceptance or rejection of a new allocation in the neighborhood, based on the tabu list information and on their priorities; and a mechanism that allows the alternation between neighborhood diversification and intensification strategies. Adaptations for the QAP can be found in Skorin-Kapov (1990, 1994), Taillard (1991), Bland and Dawson (1991), Rogger *et al.* (1992), Chakrapani and Skorin-Kapov (1993), Misevicius (2003a) and Drezner (2005). Despite the inconvenience of depending on the size of the tabu list and the way this list is managed, the performances of those algorithms show them as being very efficient strategies for the QAP, as analyzed by Taillard (1991) and Battiti and Tecchiolli (1994a). Taillard (1995) presents a comparison between the uses of tabu search and genetic algorithm, when applied to the QAP.

Greedy randomized adaptive search procedure (GRASP) is a random and iterative technique where, at each step, an approximate solution for the problem is obtained. The final solution is the best resulting one among all iterations. At each step, the first solution is constructed through a random greedy function and the following solutions are obtained by applying on the previous solution a local search algorithm that gives a new best solution regarding to the previous one. At the end of all iterations, the resulting solution is the best generated one. It is not guaranteed that GRASP solutions do not stick to a local optimum, so it is important to apply the local search phase to try to improve them. The use of suitable data structures and careful implementations allow an efficient local search. This technique was applied to the QAP by several researchers, as follows: Li *et al.* (1994b), Feo and Resende (1995), Resende *et al.* (1996), Fleurent and Glover (1999), Ahuja *et al.* (2000), Pitsoulis *et al.* (2001) and Rangel *et al.* (2000). Oliveira *et al*. (2004) built a GRASP using the path-relinking strategy, which looks for improvements along the paths joining pairs of good solutions. GRASP was also applied to QAP variations: BiQAP*,* Mavridou *et al.* (1998) and 3-dimensional QAP*,* Aiex *et al.* (2000).

Variable neighborhood search (VNS) was introduced by Mladenovi (1995) and Mladenovi and Hansen (1997) and is based on systematic moving within a set of neighborhoods conveniently defined. A number of change rules can be utilized and a change is applied when the exploring on the current neighborhood does not give a better result. It has been applied on the solution of large combinatorial problem instances. In Taillard and Gambardella (1999), three strategies are proposed for the QAP. One of them is a search over variable neighborhood, according to the basic paradigm, and the other two are hybrid methods based on the combination of some of the previously described methods.

There are also several ideas of hybrid algorithms for the QAP. In Bölte and Thonemann (1996), a combination of simulated annealing with genetic is presented; Battiti and Tecchiolli (1994b), Bland and Dawson (1994), Chiang and Chiang (1998) and Misevicius (2001, 2004a) use tabu search with simulated annealing, while Talbi *et al.* (1998b) and Hasegawa *et al.* (2002) used tabu search with a neural network, and Youssef *et al.* (2003) use tabu search, simulated annealing and fuzzy logic. Some hybrid algorithms combine a genetic algorithm with tabu search, Fleurent and Ferland (1994), Drezner (2003), or with a greedy algorithm, Ahuja *et al.* (2000), were proved to be more promising than the genetic use. More recently, there are more procedures in this class as the algorithms of Lim *et al.* (2000, 2002) which work with hybrid genetic algorithms based on *k*-gene exchange local search, and Misevicius (2004b) whose introduced new results for the quadratic assignment problem used a improved hybrid genetic procedure. Dynamic programming was combined with evolutionary computation by Dunker et al (2004) for solving a dynamic facility layout problem. Balakrishnan *et al*. (2003) and Rodriguez *et al*. (2004) used GATS, a hybrid algorithm that considers a possible planning horizon, which combines genetic with tabu search and is designed to obtain all global optima. Some categories of hybrid genetic algorithms are known as memetic algorithms or evolutionary algorithms and some works can be found in this context: Brown *et al.* (1989), Brown and Huntley (1991), Carrizo *et al.* (1992), Nissen (1994), Huntley and Brown (1996), Merz and Freisleben (1997, 1999, 2000), Nissen (1997), Ostrowski and Ruoppila (1997). Misevicius *et al.* (2002) presented an algorithm based on reconstruct and improve principle. The main components of this meta-heuristics are a reconstruction (mutation) procedure and an improvement (local search) procedure. Misevicius (2003b, 2003d) presented a new heuristic, based on the run and recreate cause. The main components are ruin (mutation) and a recreate (improvement) procedure. There is still a technique introduced by Goldbarg and Goldbarg (2002) that uses a variation of the genetic algorithms, known as transgenetic heuristics. In the QAP case, the presented results are just compatible with other ones, without improvements on the computational time. The use of several metaheuristics and hybrid proposals on QAP is discussed and their results are compared and analyzed in Maniezzo and Colorni (1995) and Taillard *et al.* (2001). Kelly *et al.* (1994) studied diversification strategies for the QAP; Fedjki and Duffuaa (2004) developed a work using extreme points in a search algorithm to solve the QAP. Finally, several techniques use parallel and massive computation, Bokhari (1987), Roucairol (1987), Brown *et al.* (1989), Pardalos and Crouse (1989), Brown and Huntley (1991, 1996), Taillard (1991), Chakrapani and Skorin-Kapov (1993), Laursen (1993), Mans (1995), Mautor and Roucairol (1995), Obuchi *et al.* (1996), Brüngger *et al.* (1997, 1998), Clausen *et al.* (1998), Talbi *et al.* (1998a, 1998b, 2001), Aiex *et al.* (2000), Anstreicher *et al.* (2002) and Moe (2003). Almost these references were cited in other procedure classes.

5 The main research trends and tendencies

In this section, we seek to identify the behavior of the main research trends along the time, after almost 50 years of QAP appearance in the literature. This study raises a number of questions concerning researcher preferences and also the needs for formulations, techniques and theoretical developments. We also consider the influence of hardware development throughout different periods and the possibilities brought by the most recent conquests represented by parallel processing and metacomputing.

The bibliography presented in this work, listing **362** publications, from which **about 95% deal** directly with QAP, determines the considered universe. The curve displays in Figure 5.1 shows the consistency of interest in the problem along the more recent years.

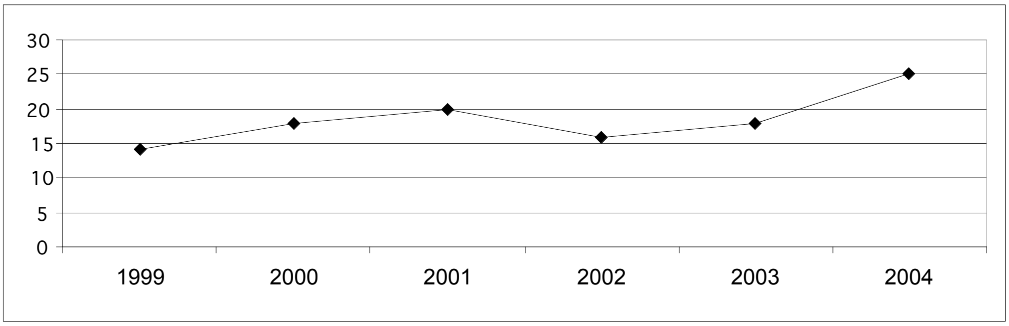

Figure 5.1 – Tendency of the Publications.

In the following figures, the references are grouped by the approach strategy, determined by the formulation classification given in Section 2; the kinds of lower bounds adopted according to classification of Section 3; the solution techniques or procedures given in Section 4; the reference distribution concerning algorithmic, theoretical or applied work along the time (periods of five years). To finish this section, we present a sketch pointing new research tendencies based on recent advances.

Figure 5.2 presents the number of publications related to the different QAP formulations, classified in this work as Permutations (PM), Integer Linear Programming (ILP), Mixed Integer Linear Programming (MILP), Trace (TR), Semidefinite Programming (SDP) and Graphs (GR). We observed that the QAP approach that identifies solutions with permutations is the most used, followed by ILP and MILP formulations. The formulations derived from semidefinite programming and the ones using exclusively graphs are less contemplated in the literature, perhaps because they are more recent.

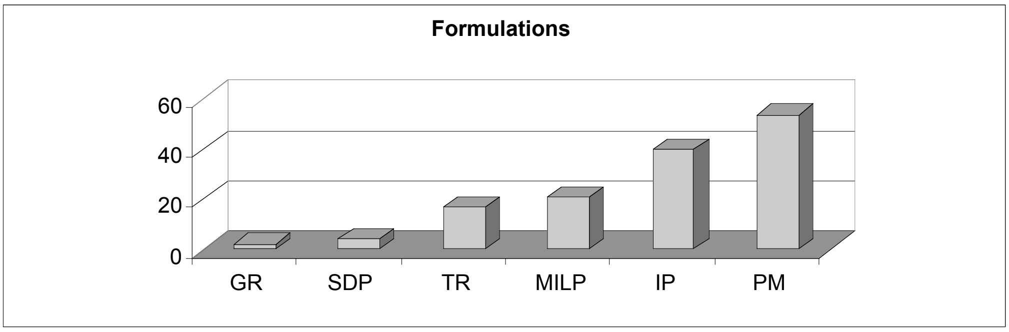

Figure 5.2 – Publications: QAP Formulations.

Figure 5.3 presents the number of publications related to lower bounds, following the classification that are adopted in this article: Gilmore and Lawler (GLB) bounds, MILP relaxation based bounds, trace formulation derived bounds (TRB), spectral bounds (SB), semidefinite programming (SDP), graph formulation based bounds (GRB), variance reduction bounds (VRB) and, finally, interior points based bounds (IPB).

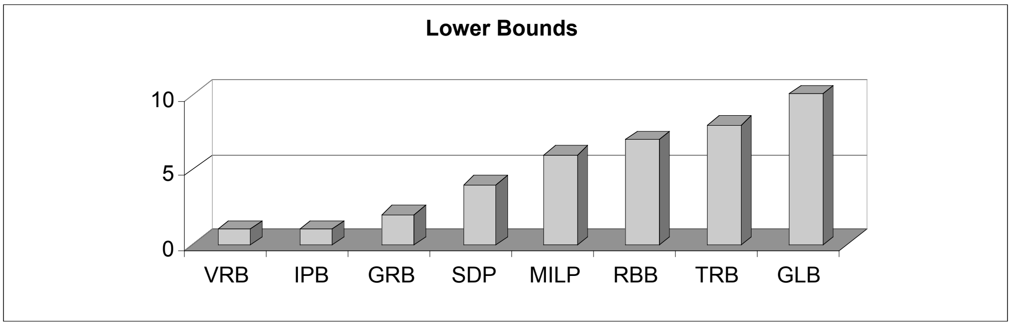

Figure 5.3 – Publications: lower bounds.

Observing Figure 5.3, we conclude that most works use lower bounds derived from the Gilmore and Lawler Bound (GLB), followed by MILP and TRB relaxation based bounds. However, GLB is the most traditional and frequently the quickest to produce results and this justifies the illustrated distribution in this figure.

Figure 5.4 registers the reference distribution by solution techniques that were classified in this work as Heuristic Methods, Exact Methods and Metaheuristics.

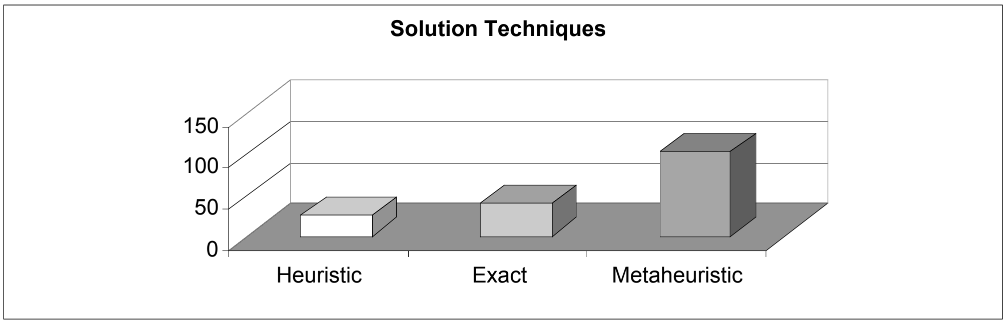

Figure 5.4 – Publications: solution techniques.

We can observe that about 30 papers deal with exact methods, while more that one hundred are dedicated to heuristic or metaheuristic methods, a natural consequence of the NP-hardness of the problem.

Figure 5.5 registers the reference distribution by metaheuristic resolution methods. In this arrangement we have: simulated annealing (SA), genetic algorithm (GA), scatter search (SS), ant colony (AC), neural networks and others (NNO), tabu search (TS), greedy randomized adaptive search procedure (GRASP), variable neighborhood search (VNS) and hybrid algorithms (HA).

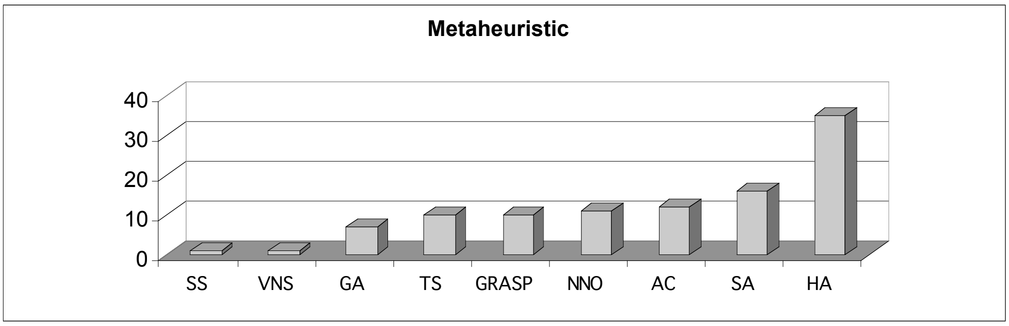

Figure 5.5 – Publications: Metaheuristics used to the QAP

From Figure 5.5 we can see that hybrid procedures that result from different metaheuristic compositions are the most used. However, when we look for comparison among pure metaheuristics, the procedures based on simulated annealing and genetic algorithms have been the most applied to the QAP.

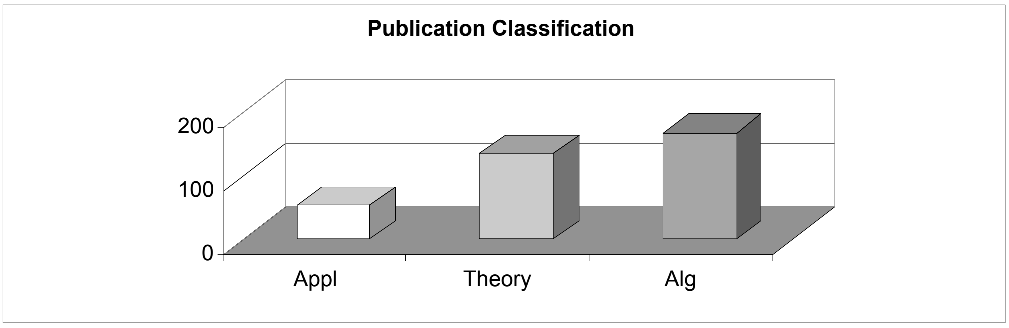

Figure 5.6 – Publications Classification according to their contents.

Figure 5.6 shows the reference distribution with relation to QAP applications, theoretical works involving formulations, complexity studies and lower bound techniques, and those dedicated to algorithms.

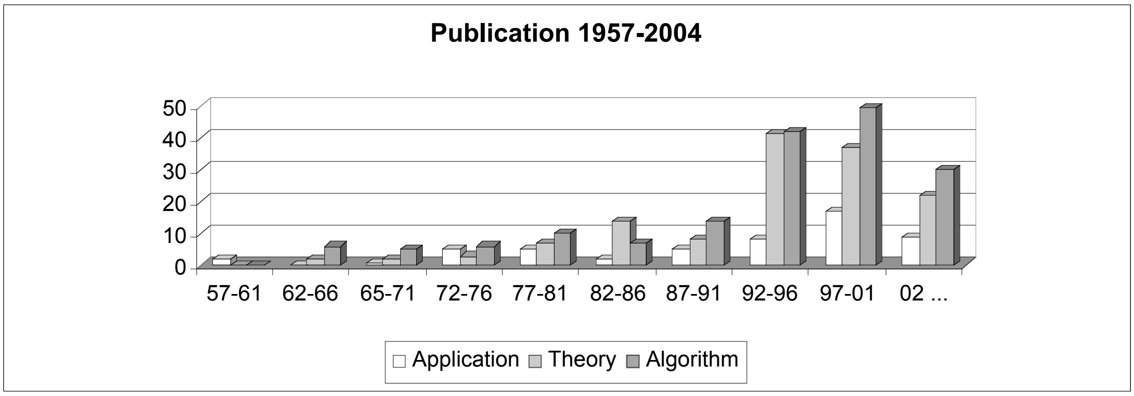

Figure 5.7 – Number of publications in a five-year basis.

Figure 5.7 distributes the number of articles by 5-year periods since 1957, when the problem was proposed. For each period, the work is also classified according to the same categories of Figure 5.6. We can observe that an explosion of interest in theory and algorithm development occurred in the 1992-2001 period. The last period is only half passed but it seems to keep the same level and trends.

The problem seems to have attracted little interest until the middle of the 70's. The 80's have seen a number of theoretical developments followed, near their end, by a growing interest in algorithms to which the theory naturally conducted. By the end of the 80's, with the emerging of metaheuristics, the problem received more attention, partly as a benchmark: a metaheuristic would be considered competitive if, when applied to the QAP, could achieve better results than the known ones. The end of the 90's profited from the development of computer technology, both in hardware and in capacity management (parallel computing and metacomputing). This, combined with the available exact techniques, made possible to find optimal solutions for larger instances (over $n = 30$) and also to obtain better ones for some bigger instances, QAPLIB (2004).

Figure 5.8, where the more recent work is indicated, shows that the interest in algorithms continues very strong, even after 2001, ~~when the interest~~while we observe a periodical trend on theoretical developments ~~decreases (perhaps because few novelties were achieved~~). Applications continue to be presented but in a lesser extent.

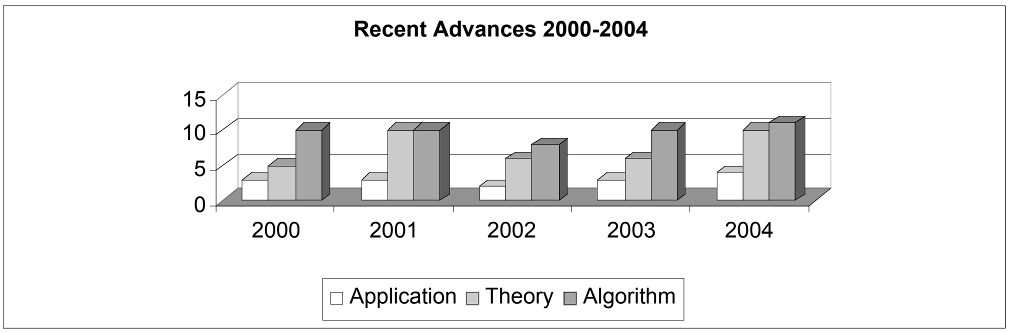

Figure 5.8 – Recent Advances.

6 Acknowledgements

We are indebted to CNPq (Council for the Scientific and Technological Development, of the Brazilian government) for the support we received both with scholarships and financial support for research. We are also indebted to Kurt Anstreicher and Zvi Drezner for their criticism and suggestions. The reporting of results in Tables 3.1 and 4.1 was partially supported by the U.S. National Science Foundation Grant Number 9900183.

7 References

(1) Abreu, N.M.M., Querido, T.M. and Boaventura-Netto, P.O. (1999). RedInv-SA: A simulated annealing for the quadratic assignment problem. *RAIRO Operations Research*, **33(3)**, 249-273.

(2) Abreu, N.M.M., Boaventura-Netto, P.O., Querido, T.M. and Gouvêa, E.F. (2002). Classes of quadratic assignment problem instances: isomorphism and difficulty measure using a statistical approach. *Discrete Applied Mathematics*, **124(1-3)**, 103-116.

(3) Adams, W.P. and Sherali, H.D. (1986). A tight linearization and an algorithm for zero-one quadratic programming problems. *Management Science*, **32(10)**, 1274-1290.

(4) Adams, W.P. and Sherali, H.D. (1990). Linearization strategies for a class of zero-one mixed integer programming problems. *Operations Research*, **38(2)**, 217-226.

(5) Adams, W.P. and Johnson, T.A. (1994). Improved linear programming-based lower bounds for the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 43-75, AMS, Rhode Island.

(6) Adams, W.P., Guignard, M., Hahn, P.M., and Hightower, W.L. (to appear). A level-2 reformulation-linearization technique bound for the quadratic assignment problem. To appear in *European Journal of Operational Research*.

(7) Ahuja, R., Orlin, J.B. and Tiwari, A. (2000). A greedy genetic algorithm for the quadratic assignment problem. *Computers and Operations Research*, **27(10)**, 917-934.

(8) Aiex, R.M., Pardalos, P.M., Pitsoulis, L.S. and Resende, M.G.C. (2000). A GRASP for computing approximate solutions for the three-index assignment problem. **In:** *Proceedings of the 15 IPDPS 2000 Workshops on Parallel and Distributed Processing, Lecture Notes in Computer Science*, **1800**, 504-547, Springer-Verlag.

(9) Aiex, R.M., Resende, M.G.C., Pardalos, P.M. and Toraldo, G. GRASP with path relinking for the three-index assignment problem. To appear in *Informs Journal on Computing*.

(10) Anderson, E.J. (1996). Theory and methodology: mechanisms for local search. *European Journal of Operational Research*, **88**, 139-151.

(11) Angel, E. and Zissimopoulos, V. (1998). On the quality of local search for the quadratic assignment problem. *Discrete Applied Mathematics*, **82(1-3)**, 15-25.

(12) Angel, E. and Zissimopoulos, V. (2000). On the classification of NP-complete problems in terms of their correlation coefficient. *DAMATH: Discrete Applied Mathematics and Combinatorial Operations Research and Computer Science*, **99(1-3)**, 261-277.

(13) Angel, E. and Zissimopoulos, V. (2001). On the landscape ruggedness of the quadratic assignment problem. *Theoretical Computer Science*, **263(1-2)**, 159-172.

(14) Angel, E. and Zissimopoulos, V. (2002). On the hardness of the quadratic assignment problem with metaheuristics. *Journal of Heuristics*, **8(4)**, 399-414.

(15) Anstreicher, K.M., Chen, X., Wolkowicz, H. and Yuan, Y. (1999). Strong duality for a trust-region type relaxation of the quadratic assignment problem. *Linear Algebra and its Applications*, **301(1-3)**, 121-136.

(16) Anstreicher, K.M. (2001). Eigenvalue bounds versus semidefinite relaxations for the quadratic assignment problem. *SIAM Journal on Optimization*, **11(1)**, 254-265.

(17) Anstreicher, K.M. and Brixius, N.W. (2001). A new bound for the quadratic assignment problem based on convex quadratic programming. *Mathematical Programming*, **89(3)**, 341-357.

(18) Anstreicher, K.M., Brixius, N.W., Goux, J.P. and Linderoth, J. (2002). Solving large quadratic assignment problems on computational grids. *Mathematical Programming*, **91(3)**, 563-588.

(19) Anstreicher, K.M. (2003). Recent advances in the solution of quadratic assignment problems. *Mathematical Programming*, **97(1-2)**, 27-42.

(20) Arkin, E.M., Hassin, R. and Sviridenko, M. (2001). Approximating the maximum quadratic assignment problem. *Information Processing Letters*, **77(1)**, 13-16.

(21) Armour, G.C. and Buffa, E.S. (1963). Heuristic algorithm and simulation approach to relative location of facilities. *Management Science*, **9(2)**, 294-309.

(22) Assad, A.A. and Xu, W. (1985). On lower bounds for a class of quadratic {0,1} programs. *Operations Research Letters*, **4(4)**, 175-180.

(23) Balakrishnan, J., Cheng, C.H., Conway, D.G. and Lau, C.M. (2003). A hybrid genetic algorithm for the dynamic plant layout problem. *International Journal of Production Economics*, **86(2)**, 107-120.

(24) Balakrishnan, J., Jacobs, F.R. and Venkataramanan, M.A. (1992). Solutions for the constrained dynamic facility layout problem, *European Journal of Operational Research* **15**, 280-286.

(25) Balas, E. and Mazolla, J.B. (1980). Quadratic 0-1 programming by a new linearization. *Presented at the Join ORSA/TIMS National Meeting*, Washington DC.

(26) Balas, E. and Saltzman, M.J. (1989). Facets of the three-index assignment polytope. *Discrete Applied Mathematics*, **23**, 201-229.

(27) Balas, E. and Saltzman, M.J. (1991). An algorithm for the three-index assignment problem. *Operations Research*, **39(1)**, 150-161.

(28) Balas, E. and Qi, L. (1993). Linear-time separation algorithms for the three-index assignment polytope. *Discrete Applied Mathematics*, **43**, 1-12.

(29) Ball, M.O., Kaku, B.K. and Vakhutinsky, A. (1998). Network-based formulations of the quadratic assignment problem. *European Journal of Operational Research*, **104(1)**, 241-249.

(30) Bandelt, H–J., Crama, Y. and Spieksma, F.C.R. (1991). Approximation algorithms for multi-dimensional assignment problems with decomposable costs. *Discrete Applied Mathematics*, **49**, 25-50.

(31) Bartolomei-Suarez S.M. and Egbelu P.J. (2000). Quadratic assignment problem QAP with adaptable material handling devices. *International Journal of Production Research*, **38(4)**, 855-873.

(32) Barvinok, A. and Stephen, T. (2003). The distribution of values in the quadratic assignment problem. *Mathematics of Operations Research*, **28**, 64-91.

(33) Battiti, R. and Tecchiolli, G. (1994a). The reactive tabu search. *ORSA Journal on Computing*, **6(2)**, 126-140.

(34) Battiti, R. and Tecchiolli, G. (1994b). Simulated annealing and tabu search in the long run: a comparison on QAP tasks. *Computer and Mathematics with Applications*, **28(6)**, 1-8.

(35) Bazaraa, M.S. and Elshafei, A.N. (1979). An exact branch-and-bound procedure for the quadratic assignment problem. *Naval Research Logistics Quarterly*, **26**, 109-121.

(36) Bazaraa, M.S. and Sherali, H.D. (1979). New approaches for solving the quadratic assignment problem. *Operations Research Verfahren*, **32**, 29-46.

(37) Bazaraa, M.S. and Sherali, H.D. (1980). Benders' partitioning scheme applied to a new formulation of the quadratic assignment problem. *Naval Research Logistics Quarterly*, **27**, 29-41.

(38) Bazaraa, M.S. and Sherali, H.D. (1982). On the use of exact and heuristic cutting plane methods for the quadratic assignment problem. *Journal of the Operational Research Society*, **33**, 991-1003.

(39) Bazaraa, M. S. and Kirca, O. (1983). A branch-and-bound based heuristic for solving the quadratic assignment problem. *Naval Research Logistics Quarterly*, **30**, 287-304.

(40) Benjaafar, S. (2002). Modeling and analysis of congestion in the design of facility layouts. *Management Science*, **48(5)**, 679-704.

(41) Billionnet, A. and Elloumi, S. (2001). Best reduction of the quadratic semi-assignment problem. *Discrete Applied Mathematics*, **109(3)**, 197-213.

(42) Blanchard, A., Elloumi, S., Faye, A. and Wicker, N. (2003). A cutting algorithm for the quadratic assignment problem. *INFOR*, **41(1)**, 35-49.

(43) Bland, J.A. and Dawson, G.P. (1991). Tabu search and design optimization. *Computer Aided Design*, **23(3)**, 195-201.

(44) Bland, J.A. and Dawson, G.P. (1994). Large-scale layout of facilities using a heuristic hybrid algorithm. *Applied Mathematical Modeling*, **18(9)**, 500-503.

(45) Boaventura-Netto, P.O. (2003). Combinatorial instruments in the design of a heuristic for the quadratic assignment problems. *Pesquisa Operacional*, **23(03)**, 383-402.

(46) Bokhari, S.H. (1987). *Assignment Problems in Parallel and Distributed Computing*. Kluwer Academic Publisher, Boston USA.

(47) Bölte, A. and Thonemann, U.W. (1996). Optimizing simulated annealing schedules with genetic programming. *European Journal of Operational Research*, **92(2)**, 402-416.

(48) Bos, J. (1993). A quadratic assignment problem solved by simulated annealing. *Journal of Environmental Management*, **37(2)**, 127-145.

(49) Bousonocalzon, C. and Manning, M.R.W. (1995). The Hopfield neural-network applied to the quadratic assignment problem. *Neural Computing and Applications*, **3(2)**, 64-72.

(50) Bozer, Y.A. and Suk-Chul, R. (1996). A branch and bound method for solving the bidirectional circular layout problem. *Applied Mathematical Modeling*, **20(5)**, 342-351.

(51) Brixius, N.W. and Anstreicher, K.M. (2001). Solving quadratic assignment problems using convex quadratic programming relaxations. *Optimization Methods and Software*, **16**, 49-68.

(52) Brown, D.E., Huntley, C.L. and Spillane (1989). A parallel genetic heuristic for the quadratic assignment problem. **In:** *Proceedings of the third International Conference on Genetic Algorithms*, 406-415, Morgan Kaufmann Publishers.

(53) Brown, D.E. and Huntley, C.L. (1991). A parallel heuristic for the quadratic assignment problem. *Computers and Operations Research*, **18**, 275-289.

(54) Bruijs, P.A. (1984). On the quality of heuristic solutions to a 19 x 19 quadratic assignment problem. *European Journal of Operational Research*, **17**, 21-30.
(55) Brüngger, A., Marzetta, A., Clausen, J. and Perregaard, M. (1997). Joining forces in solving large-scale quadratic assignment problems. **In:** *Proceedings of the 11th International Parallel Processing Symposium IPPS*, IEEE Computer Society Press, 418-427.
(56) Brüngger, A., Marzetta, A., Clausen, J. and Perregaard, M. (1998). Solving large-scale QAP problems in parallel with the search library ZRAM. *Journal of Parallel and Distributed Computing*, **50(1-2)**, 157-169.
(57) Brusco, M.J. and Stahl, S. (2000). Using quadratic assignment methods to generate initial permutations for least-squares unidimensional scaling of symmetric proximity matrices. *Journal of Classification*, **17(2)**, 197-223.
(58) Buffa, E.S., Armour, G.C. and Vollmann, T.E. (1964). Allocating facilities with CRAFT. *Harvard Business Review*, **42(2)**, 136-158.
(59) Bui, T.N. and Moon, B.R. (1994). A genetic algorithm for a special class of the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 99-116, AMS, Rhode Island.
(60) Bullnheimer, B. (1998). An examination-scheduling model to maximize students' study time. *Lecture Notes in Computer Science*, **1408**, 78-91.
(61) Burer, S. and Vandenbussche, D. (2004). Solving lift-and-project relaxations of binary integer programs. Available in *http://www.optimization-online.org/DB_HTML/2004/06/890.html*.
(62) Burkard, R.E. (1975). Numerische Erfahungen mit Summen und Bottleneck Zuordnungsproblemen. In: *Numerische Methoden bei graphentheoretischen und kombinatorischen Problemen*, Eds. L. Collatz and H. Werner, Birkhauser Verlag, Basel.
(63) Burkard, R.E. and Stratman, R.H. (1978). Numerical investigations on quadratic assignment problem. *Naval Research Logistics Quarterly*, **25**, 129-140.
(64) Burkard, R.E. and Derigs, U. (1980). Assignment and matching problems: solutions methods with Fortran programs. **In:** *Lectures Notes in Economics and Mathematical Systems*, **184**, Springer-Verlag New York, Secaucus, NJ.
(65) Burkard, R.E. and Fröhlich, K. (1980). Some remarks on 3-dimensional assignment problems. *Methods of Operations Research*, **36**, 31-36.
(66) Burkard, R.E. and Finke, G. (1982). On random quadratic bottleneck assignment problems. *Mathematical Programming*, **23**, 227-232.
(67) Burkard, R.E. and Zimmermann, U. (1982). Combinatorial optimization in linearly ordered semimodules: a survey. **In:** *Modern Applied Mathematics: Optimization and Operations Research* [edited by B. Korte], North Holland, Amsterdam, 392-436.
(68) Burkard, R.E and Bonniger, T. (1983). A heuristic for quadratic Boolean programs with applications to quadratic assignment problems. *European Journal of Operation Research*, **13**, 374-386.
(69) Burkard, R.E. (1984). Quadratic assignment problems. *European Journal of Operational Research*, **15**, 283-289.
(70) Burkard, R.E. and Rendl, F. (1984). A thermodynamically motivated simulation procedure for combinatorial optimization problems. *European Journal of Operational Research*, **17(2)**, 169-174.
(71) Burkard, R.E., Euler, R. and Grommes, R. (1986). On Latin squares and the facial structure of related polytopes. *Discrete Mathematics*, **62**, 155-181.
(72) Burkard, R.E. (1991). Locations with spatial interactions: the quadratic assignment problem. **In:** *Discrete Location Theory* [edited by P.B. Mirchandani and R.L. Francis], John Wiley and Sons, 387-437.
(73) Burkard, R.E., Karisch, S. and Rendl, F. (1991). QAPLIB - A quadratic assignment problem library. *European Journal of Operational Research*, **55**, 115-119.
(74) Burkard, R.E. and Rudolf, R. (1993). Computational investigations on 3-dimensional axial assignment problems. *Belgian Journal of Operations Research, Statist. Comput. Sci.*, **32**, 85-98.
(75) Burkard, R.E., Çela, E. and Klinz, B. (1994). On the biquadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 117-146, AMS, Rhode Island.
(76) Burkard, R.E. and Çela, E. (1995). Heuristics for biquadratic assignment problems and their computational comparison. *European Journal of Operational Research*, **83(2)**, 283-300.
(77) Burkard R.E. and Çela, E. (1996). Quadratic and three-dimensional assignment problems: an annotated bibliography. **In:** *Annotated Bibliographies in Combinatorial Optimization* [edited by M. Dell'Amico, F. Maffioli, and S. Martello], Wiley, Chichester, 373-392.
(78) Burkard, R.E., Çela, E., Rote, G. and Woeginger, G.J. (1996a). The quadratic assignment problem with a monotone Anti-Monge and a symmetric Toeplitz matrix: easy and hard cases. **In:** *Integer Programming and Combinatorial Optimization* [edited by W.H. Cunningham, S.T. McCormick, and M. Queyranne],

Proceedings of the 5th International IPCO Conference, Vancouver, British Columbia, Canada, *Lecture Notes in Computer Science*, **1084**, 204-218.
(79) Burkard, R.E., Rudolf, R. and Woeginger, G.J. (1996b). Three-dimensional axial assignment problems with decomposable cost coefficients. *Discrete Applied Mathematics*, **65**, 123-139.
(80) Burkard, R.E., Karisch, S. and Rendl, F. (1997). QAPLIB - A quadratic assignment problem library. *Journal of Global Optimization*, **10**, 391-403.
(81) Burkard, R.E., Çela, E., Pardalos, P.M. and Pitsoulis, L. (1998a). The quadratic assignment problem. **In:** *Handbook of Combinatorial Optimization* (edited by P.M. Pardalos and D.-Z. Du), Kluwer Academic Publishers, 241-338.
(82) Burkard, R.E., Çela, E., Rote, G. and Woeginger, G.J. (1998b). The quadratic assignment problem with a monotone anti-Monge and a symmetric Toeplitz matrix: Easy and hard cases. *Mathematical Programming*, **82(1-2)**, 125-158.
(83) Burkard, R.E. (2002). Selected topics on assignment problems. *Discrete Applied Mathematics*, **123(1-3)**, 257-302.
(84) Carraresi, P. and Malucelli, F. (1992). A new lower bound for the quadratic assignment problem. *Operations Research*, **40(1)**, S22-S27.
(85) Carraresi, P. and Malucelli, F. (1994). A reformulation scheme and new lower bounds for the QAP. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 147-160, AMS, Rhode Island.
(86) Carrizo, J., Tinetti, F.G. and Moscato, P. (1992). A computational ecology for the quadratic assignment problem. **In:** *Proceedings of the 21st Meeting on Informatics and Operations Research*, Buenos Aires, Argentina.
(87) Çela, E. (1998). The quadratic assignment problem: Theory and algorithms. **In:** *Combinatorial Optimization* [edited by D.Z. Du and P. Pardalos], Kluwer Academic Publishers, Dordrecht.
(88) Chakrapani, J. and Skorin-Kapov, J. (1993). Massively parallel tabu search for the quadratic assignment problem. *Annals of Operations Research*, **41(1-4)**, 327-342.
(89) Chakrapani, J. and Skorin-Kapov, J. (1994). A constructive method to improve lower bounds for the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 161-171, AMS, Providence, Rhode Island.
(90) Chen, B. (1995). Special cases of the quadratic assignment problem. *European Journal of Operational Research*, **81(2)**, 410-419.
(91) Chiang, W.C. and Chiang, C. (1998). Intelligent local search strategies for solving facility layout problems with the quadratic assignment problem formulation. *European Journal of Operational Research*, **106(2-3)**, 457-488.
(92) Christofides, N. and Gerrard, M. (1976). Special cases of the quadratic assignment problem. *Management Science Research Report*, **391**, Carnegie-Mellon University, Pittsburgh, PA.
(93) Christofides, N., Mingozzi, A. and Toth, P. (1980). Contributions to the quadratic assignment problem. *European Journal of Operations Research*, **4**, 243-247.
(94) Christofides, N. and Gerrard, M. (1981). A graph theoretic analysis of bounds for the quadratic assignment problem. **In:** *Studies on graphs and discrete programming* [edited by P. Hansen], North-Holland, 61-68.
(95) Christofides, N. and Benavent, E. (1989). An exact algorithm for the quadratic assignment problem. *Operation Research*, **37(5)**, 760-768.
(96) Ciriani, V., Pisanti, N. and Bernasconi, A. (2004). Room allocation: a polynomial subcase of the quadratic assignment problem. *Discrete Applied Mathematics*~~, (accepted for publication).~~ 144, (3), 263-269.
(97) Clausen, J. and Perregaard, M. (1997). Solving large quadratic assignment problems in parallel. *Computational Optimization and Applications*, **8**, 111-127.
(98) Clausen J., Karisch S.E., Perregaard M. and Rendl F. (1998). On the applicability of lower bounds for solving rectilinear quadratic assignment problems in parallel, *Computational Optimization and Applications*, **10(2)**, 127-147.
(99) Colorni, A., Dorigo, M., Maffioli, F., Maniezzo, V., Righini, G. and Trubian, M. (1996). Heuristics from nature for hard combinatorial optimization problems. *International Transactions in Operational Research*, **3(1)**, 1-21.
(100) Connolly, D. T. (1990). An improved annealing scheme for the QAP. *European Journal of Operational Research*, **46**, 93-100.
(101) Costa, C.S. and Boaventura-Netto, P.O. (1994). An algebraic-combinatorial description for the asymmetric quadratic assignment problem. Adv. Mod. Analysis A, **22(2)**, 1-11.
(102) Crama, Y. and Spieksma, F.C.R. (1992). Approximation algorithms for three-dimensional assignment problems with triangle inequalities. *European Journal Operational Research*, **60**, 273-279.

(103) Cung, V-D., Mautor, T., Michelon, P. and Tavares, A. (1997). A scatter search based approach for the quadratic assignment problem. **In:** *Proceedings of IEEE International Conference on Evolutionary Computation*, 165-169.

(104) Cyganski D., Vaz R.F. and Virball V.G. (1994). Quadratic assignment problems generated with the Palubetskis algorithm are degenerate. *IEEE Transactions on Circuits and Systems I-Fundamental Theory and Applications*, **41(7)**, 481-484

(105) Davis, L. (1987). *Genetic algorithms and simulated annealing*. Morgan Kaufmann Publishers.

(106) Day, R.O., Kleeman, M.P. and Lamont, G.B., (2003). Solving the multi-objective quadratic assignment problem using a fast messy genetic algorithm. **In:** *Proceedings of Conference on Evolutionary Computation 2003 (CEC'03)*, **4,** 2277- 2283.

(107) Deineko, V.G. and Woeginger, G.J. (1998). A solvable case of the quadratic assignment problem. *Operations Research Letters*, **22(1)**, 13-17.

(108) Deineko, V.G. and Woeginger, G.J. (2000). A study of exponential neighborhoods for the traveling salesman problem and for the quadratic assignment problem. *Mathematical Programming*, **Ser. A, 78**, 519-542.

(109) Dell'Amico, M., Maffioli, F. and Martello, S. (1997). *Annotated bibliographies in combinatorial optimization*. Wiley, Chichester.

(110) Dickey, J.W. and Hopkins, J.W. (1972). Campus building arrangement using topaz. *Transportation Research*, **6**, 59-68.

(111) Dorigo, M., Maniezzo, V. and Colorni, A. (1996). The ant system: optimization by a colony of cooperating agents. *IEEE Transaction on Systems, Man, and Cybernetics – Part B*, **26(2)**, 29-41.

(112) Drezner, Z. (1995). Lower bounds based on linear programming for the quadratic assignment problem. *Computational Optimization and Applications*, **4(2)**, 159-165.

(113) Drezner, Z. (2003). A new genetic algorithm for the quadratic assignment problem. *Informs Journal on Computing*, **15**(**3**), 320-330.

(114) Drezner, Z. and Marcoulides, G.A. (2003). A distance-based selection of parents in genetic algorithms. **In:** *Metaheuristics: Computer Decision-Making* [edited by M.G.C. Resende and J.P. de Sousa], *Combinatorial Optimization Book Series*, Kluwer Academic Publishers, 257-278.

(115) Drezner, Z., Hahn, P. and Taillard, E. (2004). A study of quadratic assignment problem instances that are difficult for meta-heuristic methods. *Annals of Operations Research,* special issue devoted to the *State-of-the-Art in Integer Programming* [edited by M. Guignard-Spielberg and K. Spielberg], (accepted for publication).

(116) Drezner, Z. (2005). The extended concentric tabu for the quadratic assignment problem. To appear in *European Journal of Operational Research*, **160**, 416-422.

(117) Dunker, T., Radons, G., Westkämper, E. (2004). Combining evolutionary computation and dynamic programming for solving a dynamic facility layout problem, *European Journal of Operational Research*, article in Press, Corrected proof, available March 2004, http://www.sciencedirect.com.

~~(117)~~(118) Edwards, C.S. (1977). The derivation of a greedy approximator for the Koopmans-Beckmann quadratic assignment problem. **In:** *Proceedings of the 77-th Combinatorial Programming Conference (CP77)*, 55-86.

~~(118)~~(119) Edwards, C.S. (1980). A branch and bound algorithm for the Koopmans-Beckmann quadratic assignment problem. *Mathematical Programming Study*, **13**, 35-52.

~~(119)~~(120) Elshafei, A.N. (1977). Hospital layout as a quadratic assignment problem. *Operations Research Quarterly*, **28(1)**, 167-179.

~~(120)~~(121) Emelichev, V.A., Kovalev, M.N. and Kravtsov, M. K. (1984). *Polytopes, graphs and optimization*, Cambridge University Press.

~~(121)~~(122) Eschermann, B. and Wunderlich, H.J. (1990). Optimized synthesis of self-testable finite state machines. **In:** *Proceedings of the 20th International Symposium on Fault-Tolerant Computing (FFTCS 20)*, Newcastle upon Tyne.

~~(122)~~(123) Euler, R. (1987). Odd cycles and a class of facets of the axial 3-index assignment polytope. *Applicationes Mathematicae (Zastosowania Matematyki)*, **19**, 375-386.

~~(123)~~(124) Fedjki, C.A. and Duffuaa, S.O. (2004). An extreme point algorithm for a local minimum solution to the quadratic assignment problem**.** *European Journal of Operational Research*, **156**(3), 566-578.

~~(124)~~(125) Feo, T.A. and Resende, M.G.C. (1995). Greedy randomized adaptive search procedures. *Journal of Global Optimization*, **6**, 109-133.

~~(125)~~(126) Finke, G., Burkard, R.E. and Rendl, F. (1984). Eigenvalue approach to quadratic assignment problems. **In:** *Presented at 5th Symposium on Operations Research*, Osnabrück.

~~(126)~~(127) Finke, G., Burkard, R.E. and Rendl, F. (1987). Quadratic assignment problems. *Annals of Discrete Mathematics*, **31**, 61-82.

~~(127)~~(128) Fleurent, C. and Ferland, J.A. (1994). Genetic hybrids for the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 173-187, AMS, Rhode Island.

~~(128)~~(129) Fleurent, C. and Glover, F. (1999). Improved constructive multistart strategies for the quadratic assignment problem using adaptive memory. *INFORMS Journal on Computing*, **11**, 189-203.

~~(129)~~(130) Forsberg, J.H., Delaney, R.M., Zhao, Q., Harakas, G. and Chandran, R. (1994). Analyzing lanthanide-included shifts in the NMR spectra of lanthanide (III) complexes derived from 1, 4, 7, 10-tetrakis (N, N-diethylacetamido)-1, 4, 7, 10-tetraazacyclododecane. *Inorganic Chemistry*, **34**, 3705-3715.

~~(130)~~(131) Fortin, D. and Rudolf, R. (1995). Weak algebraic Monge arrays. SFB Report 33, Institute of mathematics, Graz University of Technology.

~~(131)~~(132) Francis, R.L. and White, J.A. (1974). *Facility layout and location: An analytical approach*. Prentice-Hall, Englewood Cliffs, New Jersey.

~~(132)~~(133) Freeman, R.J., Gogerty, D.C., Graves, G.W. and Brooks, R.B.S. (1966). A mathematical model of supply for space operations. *Operations Research*, **14**, 1-15.

~~(133)~~(134) Frenk, J.B.G., Houweninge, M.V. and Kan, A.H.G.R. (1985). Asymptotic properties of the quadratic assignment problem. *Mathematics of Operations Research*, **10(1)**, 100-116.

~~(134)~~(135) Frieze, A. M. (1974). A bilinear programming formulation of the 3-dimensional assignment problems. *Mathematical Programming*, **7**, 376-379.

~~(135)~~(136) Frieze, A.M. and Yadegar, J. (1981). An algorithm for solving 3-dimensional assignment problems with applications to scheduling a teaching practice. *Operations Research*, **32**, 989-995.

~~(136)~~(137) Frieze, A. M. (1983). Complexity of a 3-dimensional assignment problem. *European Journal of Operational Research*, **13**, 161-164.

~~(137)~~(138) Frieze, A.M. and Yadegar, J. (1983). On the quadratic assignment problem. *Discrete Applied Mathematics*, **5**, 89-98.

~~(138)~~(139) Gambardella, L.M., Taillard, D. and Dorigo, M. (1999). Ant colonies for the QAP. *Journal of Operational Research. Society*, **50**, 167-176.

~~(139)~~(140) Gavett, J.W. and Plyter, N.V. (1966). The optimal assignment of facilities to locations by branch-and-bound. *Operations Research*, **14**, 210-232.

~~(140)~~(141) Geoffrion, A.M. and Graves, G.W. (1976). Scheduling parallel production lines with changeover costs: practical applications of a quadratic assignment/LP approach. *Operations Research*, **24**, 595-610.

~~(141)~~(142) Gilmore, P.C. (1962). Optimal and suboptimal algorithms for the quadratic assignment problem. *SIAM Journal on Applied Mathematics*, **10**, 305-313.

~~(142)~~(143) Glover, F. (1977). Heuristics for integer programming using surrogate constraints. *Decision Science*, **8**, 156-166.

~~(143)~~(144) Glover, F. (1989a). Tabu search – Part I. *ORSA Journal on Computing*, **1**, 190-206.

~~(144)~~(145) Glover, F. (1989b). Tabu search – Part II. *ORSA Journal on Computing*, **2**, 4-32.

~~(145)~~(146) Goldbarg, M. C. and Goldbarg E.F.G. (2002). Transgenética computacional: uma aplicação ao problema quadrático de alocação. *Pesquisa Operacional*, **22(3)**, 359-386

~~(146)~~(147) Goldberg, D.E. (1989). *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley, Wokingham, England.

~~(147)~~(148) Gong, D., Yamazaki, G., Gen, M. and Xu, W. (1999). A genetic algorithm method for one-dimensional machine location problems. *International Journal of Production Economics*, **60-1**, 337-342.

~~(148)~~(149) Gouveia, L. and Voß, S. (1995). A classification of formulations for the (time-dependent) traveling salesman problem. *European Journal of Operational Research*, **83(1)**, 69-82.

~~(149)~~(150) Graves, G.W. and Whinston, A.B. (1970). An algorithm for the quadratic assignment problem. *Management Science*, **17(7)**, 453-471.

~~(150)~~(151) Gutin, G. and Yeo, A. (2002). Polynomial approximation algorithms for TSP and QAP with a factorial domination number. *Discrete Applied Mathematics*, **119(1-2)**, 107-116.

~~(151)~~(152) Hadley, S.W., Rendl, F. and Wolkowicz, H. (1990). Bounds for the quadratic assignment problem using continuous optimization techniques. **In:** *Integer Programming and Combinatorial Optimization,* 237-248, University of Waterloo Press.

~~(152)~~(153) Hadley, S.W., Rendl, F. and Wolkowicz, H. (1992a). Nonsymmetric quadratic assignment problems and the Hoffman-Wielandt inequality. *Linear Algebra and its Applications*, **58**, 109-124.

~~(153)~~(154) Hadley, S.W., Rendl, F. and Wolkowicz, H. (1992b). A new lower bound via projection for the quadratic assignment problem. *Mathematics of Operations Research*, **17**, 727-739.

~~(154)~~(155) Hadley, S.W., Rendl, F. and Wolkowicz, H. (1992c). Symmetrization of nonsymmetric quadratic assignment problems and the Hoffman-Wielandt inequality. *Linear Algebra and its Applications*, **167**, 53-64.

~~(155)~~(156) Hadley, S.W. (1994). Domination and separation applied to the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 189-196, AMS, Rhode Island.

~~(156)~~(157) Haghani, A. and Chen, M.-C. (1998). Optimizing gate assignments at airport terminals. *Transportation. Research A*, **32(6)**, 437-454.

~~(157)~~(158) Hahn, P. and Grant, T. (1998). Lower bounds for the quadratic assignment problem based upon a dual formulation. *Operations Research*, **46**, 912-922.

~~(158)~~(159) Hahn, P., Grant, T. and Hall, N. (1998). A branch-and-bound algorithm for the quadratic assignment problem based on the Hungarian method. *European Journal of Operational Research*, **108**, 629-640.

~~(159)~~(160) Hahn, P. (2000). Progress in solving the Nugent instances of the quadratic assignment problem. In http:// www-unix.mcs.anl.gov/metaneos/nug30/nug30.pdf

~~(160)~~(161) Hahn, P.M., Hightower, W.L., Johnson, T.A., Guignard-Spielberg, M. and Roucairol, C. (2001a). Tree elaboration strategies in branch and bound algorithms for solving the quadratic assignment problem. *Yugoslavian Journal of Operational Research*, **11(1)**, 41-60.

~~(161)~~(162) Hahn, P.M., Hightower, W.L., Johnson, T.A., Guignard-Spielberg, M. and Roucairol, C. (2001b). A level-2 reformulation-linearization technique bound for the quadratic assignment problem. Working Paper 01-04, Systems Engineering Department, University of Pennsylvania.

~~(162)~~(163) Hahn, P.M. and Krarup, J. (2001). A hospital facility layout problem finally solved. *Journal of Intelligent Manufacturing*, **12**, 487-496.

~~(163)~~(164) Hahn, P.M, Kim, B-J, Hightower, W.L., Stützle, T, Kanthak, S, Samra, H, Ding, Z and Guignard, M., (2004). The Quadratic Three-dimensional Assignment Problem: Exact and Heuristic Solution Methods, OPIM Working Report No. 04-08-02, The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania, USA (Aug 2004).

~~(164)~~(165) Hanan, M. and Kurtzberg, J.M. (1972). A review of the placement and quadratic assignment problem. *SIAM Review*, **14**, 324-342.

~~(165)~~(166) Hansen, P. and Kaufman, L. (1973). A primal-dual algorithm for the three-dimensional assignment problem. *Cahiers du CERO*, **15**, 327-336.

~~(166)~~(167) Hansen, P. and Lih, K-W (1992). Improved Algorithms for Partitioning Problems in Parallel, Pipelined, and Distributed Computing. *IEEE Trans. Computers* 41(6), 769-771.

~~(167)~~(168) Hasegawa, M., Ikeguchi, T., Aihara, K. and Itoh, K. (2002). A novel chaotic search for quadratic assignment problems. *European Journal of Operational Research*, **139(3)**, 543-556.

~~(168)~~(169) Heffley, D.R. (1972). The quadratic assignment problem: A note. *Econometrica*, **40(6)**, 1155-1163.

~~(169)~~(170) Heffley, D.R. (1977). Assigning runners to a relay team. **In:** *Optimal Strategies in Sports* [edited by S.P. Ladany and R.E. Machol], 169-171, North Holland, Amsterdam.

~~(170)~~(171) Heffley, D.R. (1980). Decomposition of the Koopmans-Beckmann problem. *Regional Science and Urban Economics*, **10(4)**, 571-580.

~~(171)~~(172) Heider, C.H. (1973). An N-step, 2-variable search algorithm for the component placement problem, *Naval Research Logistics Quarterly* 20, 699-724.

~~(172)~~(173) Herroeleven, W. and Vangils, A. (1985). On the use of flow dominance in complexity measures for facility layout problems. *International Journal of Production Research*, **23,** 97-108.

~~(173)~~(174) Hillier, F.S. and Michael, M.C. (1966). Quadratic assignment problem algorithms and the location of indivisible facilities. *Management Science*, **13**, 44-57.

~~(174)~~(175) Ho, Y.C. and Moodie, C.L. (2000). A hybrid approach for concurrent layout design of cells and their flow paths in a tree configuration. *International Journal of Production Research*, **38(4)**, 895-928.

~~(175)~~(176) Hubert, L. and Schulz, J. (1976). Quadratic assignment as a general data analysis strategy. *British Journal of Mathematical Psychology*, **29**, 190-241.

~~(176)~~(177) Hubert, L. (1987). *Assignment methods in combinatorial data analysis*. **In:** *Statistics: Textbooks and Monographs Series*, **73**, Marcel Dekker.

~~(177)~~(178) Huntley, C.L. and Brown, D.E. (1996). Parallel genetic algorithms with local search. *Computers and Operations Research*, **23(6)**, 559-571.

~~(178)~~(179) Ishii, S. and Sato, M. (1998). Constrained neural approaches to quadratic assignment problems. *Neural Networks*, **11(6)**, 1073-1082.

~~(179)~~(180) Ishii, S. and Sato, M. (2001). Doubly constrained network for combinatorial optimization. *Neurocomputing*, **43(1-4)**, 239-257.

~~(180)~~(181) Jünger, M. and Kaibel, V. (1996a). On the QAP polytope. Technical Report 96.241, Angerwandte Mathematik und Informatik, Universität zu Köln.

~~(181)~~(182) Jünger, M. and Kaibel, V. (1996b). A basic study of the QAP polytope. Technical Report 96.215, Angerwandte Mathematik und Informatik, Universität zu Köln.

~~(182)~~(183) Jünger, M. and Kaibel, V. (2000). On the SQAP-polytope. *SIAM Journal on Optimization*, **11(2)**, 444-463.

~~(183)~~(184) Jünger, M. and Kaibel, V. (2001a). The QAP-polytope and the star transformation. *Discrete Applied Mathematics*, **111(3)**, 283-306.

~~(184)~~(185) Jünger, M. and Kaibel, V. (2001b). Box-inequalities for quadratic assignment polytopes. *Mathematical Programming*, **91(1),** 175-197.
~~(185)~~(186) Kaibel, V. (1998). Polyhedral combinatorics of quadratic assignment problems with less objects than locations. *Integer Programming and Combinatorial Optimization*, *Lecture Notes in Computer Science*, **1412**, 409-422.
~~(186)~~(187) Kaku, B.K. and Thompson, G.L. (1986). An exact algorithm for the general quadratic assignment problem. *European Journal of Operational Research*, **2**, 382-390.
~~(187)~~(188) Karisch, S.E., Rendl, F. and Wolkowicz, H. (1994). Trust regions and relaxations for the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 199-219, AMS, Rhode Island.
~~(188)~~(189) Karisch, S.E. and Rendl, F. (1995). Lower bounds for the quadratic assignment problem via triangle decompositions. *Mathematical Programming*, **71**, 137-152.
~~(189)~~(190) Karisch, S.E., Çela, E., Clausen, J. and Espersen, T. (1999). A dual framework for lower bounds of the quadratic assignment problem based on linearization. *Computing*, **63**, 351-403.
~~(190)~~(191) Karmarkar, N. K. and Ramakrishnan, K. G. (1991). Computational results of an interior point algorithm for large-scale linear programming. *Mathematical Programming*, **52**, 555-586.
~~(191)~~(192) Kaufman, L. and Broeckx, F. (1978). An algorithm for the quadratic assignment problem using Bender's decomposition. *European Journal of Operation Research*, **2**, 204-211.
~~(192)~~(193) Kellerer, H. and Wirsching, G. (1998). Bottleneck quadratic assignment problems and the bandwidth problem. *Asia-Pacific Journal of Operational Research*, **15**, 169-177.
~~(193)~~(194) Kelly, J.P., Laguna, M. and Glover, F. (1994). A study of diversification strategies for the quadratic assignment problem. *Computers and Operations Research*, **21(8)**, 885-893.
~~(194)~~(195) Khare, V.K., Khare, M.K. and Neema, M.L. (1988a). Estimation of distribution parameters associated with facilities design problems involving forward and backtracking of materials. *Computers and Industrial Engineering*, **14(1)**, 63-75.
~~(195)~~(196) Khare, V.K., Khare, M.K. and Neema, M.L. (1988b). Combined computer-aided approach for the facilities design problem and estimation of the distribution parameter in the case of multigoal optimization. *Computers and Industrial Engineering*, **14(4)**, 465-476.
~~(196)~~(197) Kirkpatrick, S., Gelatt, C. D. and Vecchi, M.P. (1983). Optimization by simulated annealing. *Science*, **220**, 671-680.
~~(197)~~(198) Knowles, J.D. and Corne, D.W. (2002a). Towards landscape analyses to inform the design of a hybrid local search for the multiobjective quadratic assignment problem. **In:** *Soft Computing Systems: Design, Management and Applications* [edited by A. Abraham, J. Ruiz-del-Solar and M. Koppen], 271-279, IOS Press, Amsterdam.
~~(198)~~(199) Knowles, J.D. and Corne, D.W. (2002b). Instance generators and test suites for the multiobjective quadratic assignment problem. IRIDIA technical report TR/IRIDIA/2002-25.
~~(199)~~(200) Knowles, J. and Corne, D. (2003). Instance generators and test suites for the multiobjective quadratic assignment problem. **In:** *Proceedings on Evolutionary Multi-Criterion Optimization, Lecture Notes in Computer Science*, **2632**, 295-310.
~~(200)~~(201) Kochhar, J.S., Foster, B.T. and Heragu, S.S. (1998). Hope: A genetic algorithm for the unequal area facility layout problem. *Computers and Operations Research*, **25(7-8)**, 583-594.
~~(201)~~(202) Koopmans, T.C. and Beckmann, M.J. (1957). Assignment problems and the location of economic activities. *Econometrica*, **25**, 53-76.
~~(202)~~(203) Krackhardt, D. (1988). Predicting with networks: Nonparametric multiple regression analysis of dyadic data. *Social Networks*, **10(4)**, 359-381.
~~(203)~~(204) Krarup, J. and Pruzan, P.M. (1978). Computer-aided layout design. *Mathematical Programming Study*, **9**, 75-94.
~~(204)~~(205) Kreher, D.L. and Stinson, D.R. (1998). *Combinatorial algorithms: generation, enumeration, and search*. **In:** *DIMACS Discrete Mathematics and its Applications*, CRC Press.
~~(205)~~(206) Lacksonen, T.A. and Enscore, Jr. E.E. (1993). Quadratic assignment algorithms for the dynamic layout. *International Journal of Production Research*, **31(3)**, 503-517.
~~(206)~~(207) Land, A.M. (1963). A problem of assignment with interrelated costs. *Operational Research Quarterly*, **14**, 185-198.
~~(207)~~(208) Laursen, P. S. (1993). Simple approaches to parallel branch-and-bound. *Parallel Computing*, **19**, 143-152.
~~(208)~~(209) Lawler, E.L. (1963). The quadratic assignment problem. *Management Science*, **9**, 586-599.
~~(209)~~(210) Li, Y. and Pardalos, P.M. (1992). Generating quadratic assignment test problems with known optimal permutations. *Computational Optimization and Applications*, **1**, 163-184.
~~(210)~~(211) Li, Y., Pardalos, P.M., Ramakrishnan, K.G. and Resende, M.G.C. (1994a). Lower bounds for the quadratic assignment problem. *Operations Research*, **50**, 387-410.

~~(211)~~(212) Li, Y., Pardalos, P. M. and Resende, M.G.C. (1994b). A greedy randomized adaptive search procedure for the quadratic assignment problem. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 237-261, AMS, Rhode Island.

~~(212)~~(213) Li, W.-J. and Smith, M. (1995). An algorithm for quadratic assignment problems. *European Journal of Operational Research*, **81**, 205-216.

~~(213)~~(214) Liang, Y. (1996). Combinatorial optimization by Hopfield networks using adjusting neurons. *Information Sciences*, **94(1-4)**, 261-276.

~~(214)~~(215) Lim, M.H., Yuan, Y. and Omatu, S. (2000). Efficient genetic algorithms using simple genes exchange local search policy for the quadratic assignment problem. *Computational Optimization and Applications*, **15(3)**, 248-268.

~~(215)~~(216) Lim, M.H., Yuan, Y. and Omatu, S. (2002). Extensive testing of a hybrid genetic algorithm for solving quadratic assignment problems. *Computational Optimization and Applications*, **23(1)**, 47-64.

~~(216)~~(217) Lin, C.-J. and Saigal, R. (1997). On solving large-scale semidefinite programming problems - a case study of quadratic assignment problem. Technical Report, Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor, MI.

~~(217)~~(218) Loiola, E.M., Abreu, N.M.M. and Boaventura-Netto, P.O. (2004). Uma revisão comentada das abordagens do problema quadrático de alocação (in Portuguese). *Pesquisa Operacional*, **24(1)**, 73-110.

~~(218)~~(219) Los, M. (1978). Simultaneous optimization of land use and transportation: A synthesis of the quadratic assignment problem and the optimal network problem. *Regional Science and Urban Economics*, **8(1)**, 21-42.

~~(219)~~(220) Lovász, L. and Schrijver, A. (1991). Cones of matrices and set-functions, and 0-1 optimization. *SIAM Journal on Optimization,* **1**, 166-190.

~~(220)~~(221) Love, R.F. and Wong, J.Y. (1976a). Solving quadratic assignment problems with rectangular distances and integer programming. *Naval Research Logistics Quarterly*, **23**, 623-627.

~~(221)~~(222) Love, R.F. and Wong, J.Y. (1976b). On solving a one-dimensional space allocation problem with integer programming. *INFOR*, **14**, 139-143.

~~(222)~~(223) Magirou, V.F. and Milis, J.Z. (1989). An algorithm for the multiprocessor assignment problem. *Operations Research Letters*, **8**, 351-356.

~~(223)~~(224) Magos, D. and Miliotis, P. (1994). An algorithm for the planar three-index assignment problem. *European Journal of Operational Research*, **77**, 141-153.

~~(224)~~(225) Magos, D. (1996). Tabu search for the planar three-index assignment problem. *Journal of Global Optimization*, **8**, 35-48.

~~(225)~~(226) Malucelli, F. and Pretolani, D. (1993). Lower bounds for the quadratic semi-assignment problem. Technical Report 955, Centre des Recherches sur les Transports, Université de Montréal.

~~(226)~~(227) Maniezzo, V. and Colorni, A. (1995). Algodesk: An experimental comparison of eight evolutionary heuristics applied to the quadratic assignment problem. *European Journal of Operational Research*, **81(1)**, 188-204.

~~(227)~~(228) Maniezzo, V. (1997). Exact and approximate nondeterministic tree-search procedures for the quadratic assignment problem. Research Report CSR 97-1, Scienze dell'Informazione, Cesena site, University of Bologna.

~~(228)~~(229) Maniezzo, V. and Colorni, A. (1999). The ant system applied to the quadratic assignment problem. *Knowledge and Data Engineering*, **11(5)**, 769-778.

~~(229)~~(230) Mans, B., Mautor, T. and Roucairol, C. (1995). A parallel depth first search branch and bound algorithm for the quadratic assignment problem. *European Journal of Operational Research*, **81**, 617-628.

~~(230)~~(231) Marins, M.T.A, Abreu, N. M. M. and Jurkiewicz, S. (2004). Automorphism of groups and quadratic assignment problem. To appear in *Annals of XII Congreso Latino-Iberoamericano de Investigación de Operaciones y Sistemas* (*CLAIO 2004*), Habana, Cuba.

~~(231)~~(232) Martin, W. (1998). Fast equi-partitioning of rectangular domains using stripe decomposition. *Discrete Applied Mathematics*, **82(1-3)**, 193-207.

~~(232)~~(233) Mason, A. and Rönnqvist, M. (1997). Solution methods for the balancing of jet turbines. *Computers and Operations Research*, **24(2)**, 153-167.

~~(233)~~(234) Mautor, T. and Roucairol, C. (1994a). A new exact algorithm for the solution of quadratic assignment problems. *Discrete Applied Mathematics*, **55**, 281-293.

~~(234)~~(235) Mautor, T. and Roucairol C. (1994b). Difficulties of exact methods for solving the QAP. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 263-274, AMS, Rhode Island.

~~(235)~~(236) Mavridou, T. and Pardalos, P.M. (1997). Simulated annealing and genetic algorithms for the facility layout problem: A survey. *Computational Optimization and Applications*, **7**, 111-126.

~~(236)~~(237) Mavridou, T., Pardalos, P.M., Pitsoulis, L.S. and Resende, M.G.C. (1998). A GRASP for the biquadratic assignment problem. *European Journal of Operations Research*, **105(3)**, 613-621.

~~(237)~~(238) McCormick, E.J. (1970). *Human Factors Engineering*, McGraw-Hill, New York.

~~(238)~~(239) Medova, E. (1994). Using QAP bounds for the circulant TSP to design reconfigurable networks. **In**: *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 275-292, AMS, Rhode Island.

~~(239)~~(240) Merz, P. and Freisleben, B. (1997). A genetic local search approach to the quadratic assignment problem. **In:** *Proceedings of the 7th International Conference on Genetic Algorithms* [edited by T. Back], Morgan Kaufmann, 465-472.

~~(240)~~(241) Merz, P. and Freisleben, B. (1999). A comparison of memetic algorithms, tabu search, and ant colonies for the quadratic assignment problem. **In:** *Proceedings of the 1999 International Congress of Evolutionary Computation* (*CEC'99*), *IEEE Press*, 2063-2070.

~~(241)~~(242) Merz, P. and Freisleben, B. (2000). Fitness landscape analysis and memetic algorithms for the quadratic assignment problem. *IEEE Transactions on Evolutionary Computation*, **4(4)**, 337-352.

~~(242)~~(243) Michelon, P. and Maculan, N. (1991). Lagrangean decomposition for integer nonlinear programming with linear constraints. *Mathematical Programming*, **52(2)**, 303-313.

~~(243)~~(244) Middendorf, M., Reischle, F. and Schmeck, H. (2002). Multi colony ant algorithms. *Journal of Heuristics*, **8(3)**, 305-320.

~~(244)~~(245) Milis, I.Z. and Magirou, V.F. (1995). A Lagrangian-relaxation algorithm for sparse quadratic assignment problems. *Operations Research Letters*, **17(2)**, 69-76.

~~(245)~~(246) Mills, P., Tsang, E. and Ford, J. (2003). Applying an extended guided local search to the quadratic assignment problem. *Annals of Operations Research*, **118(1-4)**, 121-135.

(247) Miranda, G., Luna, H.P.L., Mateus, G.R. and Ferreira, R.P.M. (2004). A performance guarantee heuristic for eletronic componentes placement problems incluiding thermal effects. *Computers & Operational Research*, Article in Press, Corrected Proof. Available in http://www.sciencedirect.com, 19 july 2004.

~~(246)~~(248) Mirchandani, P. B. and Obata, T. (1979). Algorithms for a class of quadratic assignment problems. *Presented at the Joint ORSA/TIMS National Meeting*, New Orleans.

~~(247)~~(249) Misevicius, A. (1997). A new algorithm for the quadratic assignment problem. *Information Technology and Control*, **5**, 39–44.

~~(248)~~(250) Misevicius, A. and Riskus, A. (1999). Multistart threshold accepting: experiments with the quadratic assignment problem. *Information Technology and Control*, **12**, 31–39.

~~(249)~~(251) Misevicius, A. (2000a). An intensive search algorithm for the quadratic assignment problem. *Informatica*, **11**, 145–162.

~~(250)~~(252) Misevicius, A. (2000b). A new improved simulated annealing algorithm for the quadratic assignment problem. *Information Technology and Control*, **17**, 29–38.

~~(251)~~(253) Misevicius, A. (2001). Combining simulated annealing and tabu search for the quadratic assignment problem. *Information Technology and Control*, **20**, 37-50.

~~(252)~~(254) Misevicius, A., Rubliauskas, D. and Smolinskas, J. (2002). Reconstruct and improve principle-based algorithm for the quadratic assignment problem. *Information Technology and Control*, **23**, 7–17.

~~(253)~~(255) Misevicius, A. (2003a). A modification of tabu search and its applications to the quadratic assignment problem. *Information Technology and Control*, **27**, 12-20.

~~(254)~~(256) Misevicius, A. (2003b). Genetic algorithm hybridized with ruin and recreate procedure: application to the quadratic assignment problem. *Knowledge-Based Systems*, **16(5-6)**, 261-268.

~~(255)~~(257) Misevicius, A. (2003c). A modified simulated annealing algorithm for the quadratic assignment problem. *Informatica*, **14(4)**, 497-514.

~~(256)~~(258) Misevicius A. (2003d). Ruin and recreate principle-based approach for the quadratic assignment problem. **In:** *Proceedings on Genetic and Evolutionary Computation (GECCO 2003), PT I, Lecture Notes in Computer Science*, **2723**, 598-609.

~~(257)~~(259) Misevicius, A. (2004a). An improved hybrid optimization algorithm for the quadratic assignment problem. *Mathematical Modelling and Analysis*, **9(2)**, 149-168.

~~(258)~~(260) Misevicius, A. (2004b). An improved hybrid genetic algorithm: new results for the quadratic assignment problem. *Knowledge-Based Systems*, ~~(accepted for publication).~~17, (2-4), 65-73.

~~(259)~~(261) Mladenovi, N. (1995). A variable neighborhood algorithm – a new metaheuristic for combinatorial optimization. **In:** *Abstracts of Papers at Optimization Days*, **112**.

~~(260)~~(262) Mladenovi, N. and Hansen P. (1997). Variable neighborhood search. *Computers and Operations Research,* **24**, 1097-1100.

~~(261)~~(263) Moe, R. (2004). GRIBB - Branch-and-bound methods on the Internet. *Parallel Processing and Applied Mathematics 2003, Lecture Notes in Computer Science*, **3019**, 1020-1027.

~~(262)~~(264) Niemi, E. (2003). Optimal tool magazine operation. Part 1: Rotating magazines with unbuffered tool change. *International Journal of Advanced Manufacturing Technology*, **22(9-10)**, 720-726.

~~(263)~~(265) Niitsuma, H., Ishii, S. and Ito, M. (2001). Chaotic optimization method combined with coordinate transformation. *Electronics and Communications in Japan Part III-Fundamental Electronic Science*, **84(2)**, 12-20.

~~(264)~~(266) Nishiyama, T., Tsuchiya, K. and Tsujita, K. (2001). A Markov chain Monte Carlo algorithm for the quadratic assignment problem based on replicator equations. **In:** *Proceedings of the Artificial Neural Networks (ICANN 2001), Lecture Notes in Computer Science*, 2130, 148-155.

~~(265)~~(267) Nissen, V. (1994). Solving the quadratic assignment problem with clues from nature. *IEEE Transactions on Neural Networks*, **5(1)**, 66-72.

~~(266)~~(268) Nissen, V. and Paul, H. (1995). A modification of threshold accepting and its application to the quadratic assignment problem. *OR Spektrum*, **17(2-3)**, 205-210.

~~(267)~~(269) Nissen, V. (1997). Quadratic assignment. **In:** *Handbook of Evolutionary Computation* [edited by T. Back, D.B. Fogel, Z. Michalewicz, and T. Baeck], **G9.10**, 1-8, Institute of Physics Publishing and Oxford University Press.

~~(268)~~(270) Nugent, C.E., Vollmann, T.E. and Ruml, J. (1968). An experimental comparison of techniques for the assignment of facilities to locations. *Operations Research*, **16**, 150-173.

~~(269)~~(271) Obuchi, Y., Masui, H. and Ohki, M. (1996). Weighted parallel problem solving by optimization networks. *Neural Networks*, **9(2)**, 357-366.

~~(270)~~(272) Oliveira, C.A.S., Pardalos, M. P. and Resende, M.G.G. (2004). GRASP with path relinking for the quadratic assignment problem. *Experimental and Efficient Algorithms*, *Third International Workshop* (*WEA 2004*), Brazil, LNCS 3059, Springer, 356-368.

~~(271)~~(273) Osman, I.H. and Laporte, G. (1996). Metaheuristics: A bibliography. *Annals of Operations Research*, **63**, 513-623.

~~(272)~~(274) Ostrowski, T. and Ruoppila, V.T. (1997). Genetic annealing search for index assignment in vector quantization. *Pattern Recognition Letters*, **18(4)**, 311-318.

~~(273)~~(275) Padberg, M.W. and Rinaldi, G. (1991). A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems. *SIAM Review*, **33**, 60-100.

~~(274)~~(276) Padberg, W. and Rijal, P. (1996). *Location, Scheduling, Design and Integer Programming.* Kluwer Academic Publishers, Boston.

~~(275)~~(277) Palubeckis, G. (1999). Generating hard test instances with known optimal solution for the rectilinear quadratic assignment problem. *Journal of Global Optimization*, **15(2)**, 127-156.

~~(276)~~(278) Palubeckis, G. (2000). An algorithm for construction of test cases for the quadratic assignment problem. *Informatica*, **11(3)**, 281-296.

(279) Paquete, L. and Stützle, T. (2004). A study of stochastic local search algorithms for biobjective QAP with correlated flow matrices, *European Journal of Operational Research*, article in Press, Corrected Proof, available online 26 Oct 2004, http://www.sciencedirect.com

~~(277)~~(280) Pardalos, P. and Crouse, J. (1989). A parallel algorithm for the quadratic assignment problem. **In:** *Proceedings of the Supercomputing Conference 1989*, 351-360, ACM Press.

~~(278)~~(281) Pardalos, P. M., Murthy, K. A. and Harrison, T. P. (1993). A computational comparison of local search heuristics for solving quadratic assignment problems. *Informatica*, **4(1-2)**, 172-187.

~~(279)~~(282) Pardalos, P.M. and Wolkowicz, H. (Editors), (1994). *Quadratic Assignment and Related Problems.* **In:** *Proceedings DIMACS Workshop on Quadratic Assignment Problem. DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, AMS, Rhode Island.

~~(280)~~(283) Pardalos, P.M., Rendl, F. and Wolkowicz, H. (1994). The quadratic assignment problem: A survey of recent developments. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 1-42, AMS, Rhode Island.

~~(281)~~(284) Pardalos, P.M., Ramakrishnan, K.G., Resende, M.G.C. and Li, Y. (1997). Implementation of a variance reduction-based lower bound in a branch-and-bound algorithm for the quadratic assignment problem. *SIAM Journal on Optimization*, **7(1)**, 280-294.

~~(282)~~(285) Peng, T., Huanchen, W. and Dongme, Z. (1996). Simulated annealing for the quadratic assignment problem: A further study. *Computers and Industrial Engineering*, **31(3-4)**, 925-928.

~~(283)~~(286) Phillips, A.T. and Rosen, J.B. (1994). A quadratic assignment formulation of the molecular-conformation problem. *Journal of Global Optimization*, **4(2)**, 229-241.

~~(284)~~(287) Pierce, J.F. and Crowston, W.B. (1971). Tree-search algorithms for quadratic assignment problems. *Naval Research Logistics Quarterly*, **18**, 136.

~~(285)~~(288) Pierskalla, W.P. (1967a). The tri-substitution method for the three-multidimensional assignment problem. *Canadian Operational Research Society Journal*, **5(2)**, 71-81.

~~(286)~~(289) Pierskalla, W.F. (1967b). The Multi-Dimensional Assignment Problem. Technical Memorandum No. 93, Operations Research Department, CASE Institute of Technology, (September 1967). See http://www.anderson.ucla.edu/faculty/william.pierskalla/Chronological_Bank/Research_and_Publication_Chro.html#Mathematical.

~~(287)~~(290) Pierskalla, W.P. (1968). The multidimensional assignment problem. *Operations Research*, **16**, 422-431.
~~(288)~~(291) Pitsoulis, L.S., Pardalos, P.M. and Hearn, D.W. (2001). Approximate solutions to the turbine balancing problem. *European Journal of Operational Research*, **130(1)**, 147-155.
~~(289)~~(292) Pollatschek, M.A., Gershoni, N. and Radday, Y.T. (1976). Optimization of the Typewriter Keyboard by Simulation. *Angewandte Informatik*, **17(0)**, 438-439.
~~(290)~~(293) Poore, A. (1994a). Multidimensional assignment formulation of data association problems arising from multitarget and multisensor tracking. *Computational Optimization and Applications*, **3**, 27-57.
~~(291)~~(294) Poore, A. (1994b). Partitioning multiple data sets: multidimensional assignment and Lagrangean relaxation. **In:** *Quadratic Assignment and Related Problems* [edited by P.M. Pardalos and H. Wolkowicz], *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **16**, 317-341, AMS, Rhode Island.
~~(292)~~(295) Poore, A. (1995). Multidimensional assignment and multitarget tracking. **In:** *DIMACS Series DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, **19**, 169-196.
~~(293)~~(296) Poore, A. and Robertson, A. (1997). A new Lagrangean relaxation based algorithm for a class of multidimensional assignment problems. *Computational Optimization and Applications*, **8**, 129-150.
~~(294)~~(297) QAPLIB (2004). QAPLIB Home Page (http://www.seas.upenn.edu/qaplib/).
~~(295)~~(298) Qi, L., Balas, E. and Gwan, G. (1994). A new facet class and a polyhedral method for the three-index assignment problem. **In:** *Advances in Optimization and Approximation* [edited by D.-Z. Du and J. Sun], Kluwer Academic, 256-274.
~~(296)~~(299) Queyranne, M. (1986). Performance ration of polynomial heuristics for triangle inequality quadratic assignment problems. *Operations Research Letters*, **4**, 231-234.
(300) Rabak, C.S., Sichman, J.S. (2003). Using A-Teams to optimize automatic insertion of electronic components, *Advanced Engineering Informatics*, **17. (2),** 95-106.
~~(297)~~(301) Ramachandran, B. and Pekny, J.F. (1998). Lower bounds for nonlinear assignment problems using many body interactions. *European Journal of Operational Research*, **105(1)**, 202-215.
~~(298)~~(302) Ramakrishnan, K.G., Resende, M.G.C., Ramachandran, B. and Pekny, J.F. (2002). Tight QAP bounds via linear programming. **In:** *Combinatorial and Global Optimization* [edited by P.M. Pardalos, A. Migdalas, and R.E. Burkard], 297-303, World Scientific Publishing, Singapore.
~~(299)~~(303) Rangel, M.C., Abreu, N.M.M. and Boaventura-Netto, P.O. (2000). GRASP para o PQA: Um limite de aceitação para soluções iniciais (in Portuguese). *Pesquisa Operacional*, **20(1)**, 45-58.
~~(300)~~(304) Rangel, M.C. and Abreu, N.M.M. (2003). Ordenações parciais nos conjuntos das soluções dos problemas de alocação linear e quadrático (in Portuguese). *Pesquisa Operacional*, **23(2)**, 265-284.
~~(301)~~(305) Rendl, F. (1985). Ranking scalar products to improve bounds for the quadratic assignment problem. *European Journal of Operational Research*, **20**, 363-372.
~~(302)~~(306) Rendl, F. and Wolkowicz, H. (1992). Applications of parametric programming and eigenvalue maximization to the quadratic assignment problem. *Mathematical Programming*, **53**, 63-78.
~~(303)~~(307) Resende, M.G.C., Ramakrishnan, K.G. and Drezner, Z. (1995). Computing lower bounds for the quadratic assignment with an interior point algorithm for linear programming. *Operations Research*, **43**, 781-791.
~~(304)~~(308) Resende, M.G.C., Pardalos, P.M. and Li, Y. (1996). Algorithm 754: Fortran subroutines for approximate solution of dense quadratic assignment problems using GRASP. *ACM Transactions on Mathematical Software*, **22(1)**, 104-118.
~~(305)~~(309) Rodriguez, J.M., Macphee, F.C., Bonham, D.J. and Bhavsar, V.C. (2004). Solving the quadratic assignment and dynamic plant layout problems using a new hybrid meta-heuristic approach, **In:** *Proceedings of the 18th Annual International Symposium on High Performance Computing Systems and Applications (HPCS)*, [edited by M.R. Eskicioglu], 9-16 (GATS homepage: http://acrl.cs.unb.ca/research/gats/)
~~(306)~~(310) Rogger, A., Fiechter, C.N. and Werra, D. (1992). Basic ideas of tabu search with an application to traveling salesman and quadratic assignment. *Ricerca Operativa*, **62**, 5-28.
~~(307)~~(311) Rossin, D.F., Springer, M.C. and Klein, B.D. (1999). New complexity measures for the facility layout problem: an empirical study using traditional and neural network analysis. *Computers and Industrial Engineering*, **36(3)**, 585-602.
~~(308)~~(312) Roucairol, C. (1979). A reduction method for quadratic assignment problem. *Methods of Operations Research*, **32**, 185-187.
~~(309)~~(313) Roucairol, C. (1987). A parallel branch and bound algorithm for the quadratic assignment problem. *Discrete Applied Mathematics*, **18**, 211-225.
~~(310)~~(314) Sahni, S. and Gonzales, T. (1976). P-complete approximation problems. *Journal of the Association for Computing Machinery*, **23**, 555-565.
~~(311)~~(315) Sarker, B.R., Wilhelm, W.E., Hogg, G.L. and Han, M.-H. (1995). Backtracking of jobs in one-dimensional machine location problems. *European Journal of Operational Research*, **85(3)**, 593-609.

~~(312)~~(316) Sarker, B.R., Wilhelm, W.E. and Hogg, G.L. (1998). One-dimensional machine location problems in a multi-product flowline with equidistant locations. *European Journal of Operational Research*, **105(3)**, 401-426.

~~(313)~~(317) Scriabin, M. and Vergin, R.C. (1975). Comparison of computer algorithms and visual based methods for plant layout. *Management Science*, **22**, 172-187.

~~(314)~~(318) Sherali, H.D. and Adams, W.P. (1999a). Reformulation-linearization techniques for discrete optimization problems, in *Handbook of combinatorial optimization,* Eds. Du, D.Z. and Pardalos, M., Kluwer Academic Publishers, **1**, 479-532.

(319) Sherali, H.D. and W. P. Adams (1999b). *A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Optimization Problems*, Kluwer Academic Publishers, Dordrecht, The Netherlands,.

~~(315)~~(320) Simeone, B. (1986a). An asymptotically exact polynomial time algorithm for equipartition problems. *Discrete Applied Mathematics*, **14**, 283-293.

~~(316)~~(321) Simeone, B. (1986b). Topological network synthesis. **In:** *Combinatorial Optimization* [edited by B. Simeone], *Lecture Notes in Mathematics*, **1403**, 282-303, Springer-Verlag.

~~(317)~~(322) Siu, F. and Chang, R.K.C. (2002). Effectiveness of optimal node assignments in wavelength division multiplexing networks with fixed regular virtual topologies. *Computer Networks*, **38(1)**, 61-74.

~~(318)~~(323) Skorin-Kapov, J. (1990). Tabu search applied to the quadratic assignment problem. *ORSA Journal on Computing*, **2(1)**, 33-45.

~~(319)~~(324) Skorin-Kapov, J. (1994). Extensions of a tabu search adaptation to the quadratic assignment problem. *Journal of Computers and Operations Research*, **21(8)**, 855-865.

~~(320)~~(325) Solimanpur, M., Vrat P. and Shankar, R. (2004). Ant colony optimization algorithm to the inter-cell layout problem in cellular manufacturing. *European Journal of Operational Research*, **157(3)**, 592-606.

~~(321)~~(326) Spiliopoulos, K. and Sofianopoulou, S. (1998). An optimal tree search method for the manufacturing systems cell formation problem. *European Journal of Operational Research*, **105(3)**, 537-551.

~~(322)~~(327) Steinberg, L. (1961). The backboard wiring problem: A placement algorithm. *SIAM Review*, **3**, 37-50.

~~(323)~~(328) Stützle, T. and Dorigo, M. (1999). ACO Algorithms for the Quadratic Assignment Problem. **In:** *New Ideas in Optimization* [edited by D. Corne, M. Dorigo and F. Glover], 33-55, McGraw-Hill.

~~(324)~~(329) Stützle, T. and Holger, H. (2000). MAX-MIN ant system. *Future Generation Computer Systems*, **16(8)**, 889-914.

~~(325)~~(330) Stützle, T. and Fernandes, S. (2004). New benchmark instances for the QAP and the experimental analysis of algorithms. **In:** *Proceedings of the Evolutionary Computation in Combinatorial Optimization, Lecture Notes in Computer Science*, **3004**, 199-209.

~~(326)~~(331) Sylla, C. and Babu, A.J.G. (1987). Methodology for an orderly quadratic assignment problem. *Computers and Industrial Engineering*, **13(1-4)**, 281-284.

~~(327)~~(332) Taillard, E. (1991). Robust taboo search for the quadratic assignment problem. *Parallel Computing*, **17**, 443-455.

~~(328)~~(333) Taillard, E. (1995). Comparison of iterative searches for the quadratic assignment problem. *Location Science*, **3(2)**, 87-105.

~~(329)~~(334) Taillard, E. and Gambardella, L. (1999). Adaptive Memories for the Quadratic Assignment Problem. Technical Report I-87-97, IDSIA, Lugano.

~~(330)~~(335) Taillard, E., Gambardella, L.M., Gendreau, M. and Potvin, J.Y. (2001). Adaptive memory programming: A unified view of metaheuristics. *European Journal of Operational Research*, **135(1)**, 1-16.

~~(331)~~(336) Takagi, M. (2001). Optimization of placement by candidate sieving. *IEEE Transactions on Electronics Packaging Manufacturing*, **24(3)**, 178-187.

~~(332)~~(337) Talbi, E.-G., Hafidi, Z., Kebbal, D. and Geib, J.-M. (1998a). A fault-tolerant parallel heuristic for assignment problems. *Future Generation Computer Systems*, **14(5-6)**, 425-438.

~~(333)~~(338) Talbi, E.-G., Hafidi, Z. and Geib, J.-M. (1998b). A parallel adaptive tabu search approach. *Parallel Computing*, **24(14)**, 2003-2019.

~~(334)~~(339) Talbi, E.-G., Roux, O., Fonlupt, C. and Robillard, D. (2001). Parallel ant colonies for the quadratic assignment problem. *Future Generation Computer Systems*, **17(4)**, 441-449.

~~(335)~~(340) Talbot, N. L. C. and Cawley, G. C. (1996). A quadratic index assignment algorithm for vector quantization over noisy transmission channels. **In:** *Proceedings of the Institute of Acoustics Autumn Conference (Speech and Hearing 96)*, **18**, 195-199.

~~(336)~~(341) Tansel, B.C. and Bilen, C. (1998). Move based heuristics for the unidirectional loop network layout problem. *European Journal of Operational Research*, **108(1)**, 36-48.

~~(337)~~(342) Tate, D.E. and Smith, A.E. (1995). A genetic approach to the quadratic assignment problem. *Computers and Operations Research*, **22**, 73-83.

~~(338)~~(343) Tavakkoli-Moghaddain, R. and Shayan, E. (1998). Facilities layout design by genetic algorithms. *Computers and Industrial Engineering*, **35(3-4)**, 527-530.

~~(339)~~(344) Tian, P., Wang, H.C. and Zhang, D.M. (1996). Simulated annealing for the quadratic assignment problem: A further study. *Computers and Industrial Engineering*, **31(3-4)**, 925-928.

~~(340)~~(345) Tian, P., Ma, J. and Zhang, D.-M. (1999). Application of the simulated annealing algorithm to the combinatorial optimization problem with permutation property: An investigation of generation mechanism. *European Journal of Operational Research*, **118(1)**, 81-94.

~~(341)~~(346) Torki, A., Yajima, Y. and Enkawa, T. (1996). A low-rank bilinear programming approach for sub-optimal solution of the quadratic assignment problem. *European Journal of Operational Research*, **94(2)**, 384-391.

~~(342)~~(347) Tsuchiya, K., Bharitkar, S. and Takefuji, Y. (1996). A neural network approach to facility layout problems. *European Journal of Operational Research*, **89(3)**, 556-563.

~~(343)~~(348) Tsuchiya, K., Nishiyama, T. and Tsujita, K. (2001). A deterministic annealing algorithm for a combinatorial optimization problem using replicator equations. *Physica D: Nonlinear Phenomena*, **149(3)**, 161-173.

~~(344)~~(349) Urban, T., L. (1998). Solution procedures for the dynamic facility layout problem. *Annals of Operations Research*, **76**, 323-342.

~~(345)~~(350) Urban, T.L., Chiang W.C. and Russell R.A. (2000). The integrated machine allocation and layout problem. *International Journal of Production Research*, **38(13)**, 2911-2930.

~~(346)~~(351) Uwate, Y., Nishio, Y. and Ushida, A. (2004). Markov chain modeling of intermittency chaos and its application to Hopfield NN. *IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences*, **E87A(4),** 774-779.

~~(347)~~(352) Vlach, M. (1967). A branch-and-bound method for the three-index assignment problem. *Ekonomicko-Matematicky Obzor*, **3**, 181-191.

~~(348)~~(353) Wang, S.J. and Sarker, B.R. (2002). Locating cells with bottleneck machines in cellular manufacturing systems. *International Journal of Production Research*, **40(2)**, 403-424.

~~(349)~~(354) West, D.H. (1983). Algorithm 608: Approximate solution of the quadratic assignment problem. *ACM Transactions on Mathematical Software*, **9**, 461-466.

~~(350)~~(355) White, D.J. (1993). A parametric-based heuristic program for the quadratic assignment problem. *Naval Research Logistics*, **40(4)**, 553-568.

~~(351)~~(356) White, D.J. (1994a). Strengthening Gilmore's bound for the quadratic assignment problem. *European Journal of Operational Research*, **77(1)**, 126-140.

~~(352)~~(357) White, D.J. (1994b). The use of specially structured models for obtaining bounds in the quadratic assignment problem. *Journal of the Operational Research Society*, **45(4)**, 451-462.

~~(353)~~(358) White, D.J. (1995). Some concave-convex representations of the quadratic assignment problem. *European Journal of Operational Research*, **80(2)**, 418-424.

~~(354)~~(359) White, D.J. (1996). A Lagrangean relaxation approach for a turbine design quadratic assignment problem. *Journal of the Operational Research Society*, **47(6)**, 766-775.

~~(355)~~(360) Whitney, H. (1932). Congruent graphs and the connectivity of graphs. *American Journal Mathematics*, **54**, 150-168.

~~(356)~~(361) Wilhelm, M.R. and Ward, T.L. (1987). Solving quadratic assignment problems by simulated annealing. *IEEE Transactions*, **19**, 107-119.

~~(357)~~(362) Wolkowicz, H. (2000a). *Handbook of semidefinite programming: Theory, algorithms, and applications* [edited by H. Wolkowicz, R. Saigal, and L. Vandenberghe]. *International Series in Operations Research*, **27**, Kluwer.

~~(358)~~(363) Wolkowicz, H. (2000b). Semidefinite programming approaches to the quadratic assignment problem. **In:** *Nonlinear Assignment Problems: Algorithms and Applications* [edited by P.M. Pardalos and L. Pitsoulis], *Combinatorial Optimization Series*, **7**, 143-174, Kluwer Academic Publishers.

~~(359)~~(364) Yamada, S. (1992). A new formulation of the quadratic assignment problem on r-dimensional grid. *IEEE Transactions on Circuits and Systems I-Fundamental Theory and Applications*, **39(10)**, 791-797.

~~(360)~~(365) Ying, K.C. and Liao, C.J. (2004). An ant colony system for permutation flow-shop sequencing. *Computers and Operations Research*, **31(5),** 791-801.

~~(361)~~(366) Yip, P.P.C. and Pao, Y.H. (1994). A guided evolutionary simulated annealing approach to the quadratic assignment problem. *IEEE Transactions on Systems Man and Cybernetics*, **24(9)**, 1383-1387.

~~(362)~~(367) Youssef, H., Sait, S.M. and Ali, H. (2003). Fuzzy simulated evolution algorithm for VLSI cell placement. *Computers and Industrial Engineering*, **44(2)**, 227-247.

~~(363)~~(368) Yu, J. and Sarker, B.R. (2003). Directional decomposition heuristic for a linear machine-cell location problem. *European Journal of Operational Research*, **149(1)**, 142-184.

~~(364)~~(369) Zhao, Q., Karisch, S. E., Rendl, F. and Wolkowicz, H. (1998). Semidefinite programming relaxations for the quadratic assignment problem. *Journal Combinatorial Optimization*, **2**, 71-109.