6th AIMMS-MOPTA Optimization Modeling Competition (submit your solution by June 13, 2014 23:59 EDT)

Time-ahead pricing of Energy Supply

The management of the electricity system is fundamentally changing and becoming increasingly challenging. This is mainly due to the transition to low carbon, sustainable, and renewable energy sources; and the need to reliably satisfy increasing energy demands in a scarce resourced, highly competitive, and interconnected economy. As a result, the development of novel and improved quantitative tools is required to support decision-making in all areas of the energy market (e.g., pricing of energy, optimal power flow, unit commitment problems, etc.). These tools will play a keystone role in supporting the continued evolution towards a smart grid for the generation, transmission, distribution, and consumption of electric energy.

In this case the aim is to consider one key decision-making process within the management of the distribution electricity system; namely, how to price the energy for the end consumers. This problem alone has been the subject of very active developments. Here, we will focus on one of the key aspects (among many others) that plays a role in deciding the price of energy for end consumers. That is, we will focus on taking into account the fact that consumers have the ability to change their energy consumption behavior based on the energy price at a given moment in time. The goal of your team is to develop effective, quantitative, user-friendly tools to support the desired energy pricing decisions by using the AIMMS modeling environment. AIMMS is an optimization modeling technology that is used by leading companies for supporting and improving decision-making in a wide range of industries. Further information about AIMMS is given below, and at `http://www.aimms.com`.

1 Problem description

The particular problem that your team is tasked to address is to recommend how to **set the price** of energy for end consumers for the next day (i.e., 24 hour period). To set-up the problem statement, suppose the government of a small Borough (or County) in the US is in charge of procuring (buying) the energy that is needed for all the energy consumers in the Borough. Also, the Borough's government (from now on we will loosely say the Major to refer to the Borough's government) decides at which price it will provide the electricity to the Borough's consumers. That is, the Major serves as an intermediary between the generators in the market, and the consumers in the Borough (there are actually a number of Boroughs or Counties in the US that operate in this manner).

In what follows, Sections 1.1 and 1.2 will describe the related problems in abstract form, and in Section 2.2 all the actual data related to the problems will be described. **Your team is encouraged to address both problems, but partial solutions will also be accepted and judged.**

1.1 A First Approach

More formally, let:

$$\begin{array}{ll} T & = \text{Total time-ahead period for which the price rates are going to be set} \\ \mathcal{I} & = \{1,\dots,N\} \text{ be the set of customers} \\ p_i(t) & = \text{Set price of energy (e.g. dollar per Kilowatt-hour, \$/kWh) at time } 0 \le t \le T \\ & \quad \text{for consumer } i \in \mathcal{I} \\ C_i(t, p_i(t)) & = \text{Consumption of energy (e.g. kWhs) by customer } i \in I \text{ at time } 0 \le t \le T \\ & \quad \text{when the energy price is } p_i(t) \end{array} \tag{1}$$

As mentioned before, and formally stated in the definitions above, here the aim is to center on taking into account potential changes in energy consumption based on the energy price. Currently, in the Borough the energy price for consumers is set to a constant:

$$\begin{array}{ll} p^a & = \text{Current price of energy at any time} \\ C_i(t, p^a) & = \text{Energy consumption by customer } i \text{ at time } t \text{ with the current energy price } p^a \end{array} \tag{2}$$

With the definitions in equations (1) and (2), the Borough's management would like to find the best way to set the energy prices for its customer with the following approach:

- The Major would like to set the energy price so as to maximize profit.
- Changes in both price and consumption will be assumed to happen only at specific times $\mathcal{T} = \{t_1, t_2, \dots, t_m = T\}$. It is assumed $t_0 = 0$.
- The Major would like to include equity principles amongst different types of consumers. More specifically, the average rates faced by the subset of consumers $\mathcal{I}_1 \subseteq \mathcal{I}$ are bounded by the average rates of the subset of consumers $\mathcal{I}_2 \subseteq \mathcal{I}$ multiplied by a given percentage γ (see second constraint in (3) below).
- The consumption of energy of a particular customer cannot deviate too much from its current consumption level. Specifically, the i-th consumer's total consumption should belong to the interval $\mu^- W_i, \mu^+ W_i$ for some given values of μ^-, μ^+, where W_i denotes the total consumption of the i-th customer during the period T at the current energy price p^a (see first constraint in (3) and equation (4) below).

Thus, the overall problem could be formulated as follows:

$$\begin{array}{lll} \max & \displaystyle\sum_{i=1}^{N}\sum_{j=1}^{m}(t_j - t_{j-1})C_i(t_j, p_i(t_j))p_i(t_j) & \\ \text{st.} & \mu^- W_i \le \displaystyle\sum_{j=1}^{m}(t_j - t_{j-1})C_i(t_j, p_i(t_j)) \le \mu^+ W_i & \forall i \in \mathcal{I} \\ & \displaystyle\sum_{i\in\mathcal{I}_1}\frac{1}{|\mathcal{I}_1|}p_i(t_j) \le \gamma \sum_{i\in\mathcal{I}_2}\frac{1}{|\mathcal{I}_2|}p_i(t_j) & j = 1,\dots,m \\ & p_i(t_j) \ge 0 & \forall i \in \mathcal{I}, j = 1,\dots,m \end{array} \tag{3}$$

where for any $i \in \mathcal{I}$

$$W_i = \left(\sum_{j=1}^{m}(t_j - t_{j-1})C_i(t_j, p^a)\right). \tag{4}$$

The variables in problem (3) are $p_i(t_j)$ and $C_i(t_j, p_i(t_j))$ for every customer $i \in \mathcal{I}$, and every time $t_j \in \mathcal{T}$.

The objective for this part of the project is to produce an optimization problem that could be solved or approximately solved using the AIMMS tools, but also, that can be appropriately constructed based on the available data (cf., Section 2.2). Specifically

1. Suggest an strategy for the Major to solve this problem, given the information provided. For example, what kind of simplifications would you suggest in order to obtain a formulation of the problem that can be specified using the provided data, and at the same time whose solution can be addressed using the optimization solvers available in AIMMS? Discuss how do these simplifications affect the optimality of the solutions you find. Are the solutions you find optimal or near-optimal?

2. Analyze the pros and potential cons of the solution approach proposed above. In particular, can you say whether the policy obtained with your approach is better than just fixing the energy price to p^a? Clearly explain your statements and support them with appropriate numerical results.

In developing the answer to the first question above, it is likely that your team will make a number of assumptions/decisions to deal with the complexity of the model. Make sure that each of those assumptions/decisions is clearly laid out and justified in your report.

1.2 A more complex approach

Although problem (3) takes into account constraints that relate the consumption of energy by customers and the actual price, it does it in a simplistic way, by assuming that information about how the consumption of energy changes with the energy price at any time is known *a priori*. That is, formulation (3) requires the use of knowledge about $C_i(t, p_i(t))$. Given that in the past the Borough has used a constant energy price policy, assuming that such knowledge can be obtained is in fact unrealistic. For that reason, here the aim is to produce an optimization model that can recommend the price policy based **only** on information about the consumption of energy at the current energy price rate; that is about $C_i(t, p^a)$ (i.e., here you are asked to ignore the data that you might have used in the previous section about the consumption of energy for prices different than p^a). To do this, we will assume that the consumption of a customer for a given price is defined by the one that minimizes the consumer cost of energy minus the utility derived from consuming energy, while maintaining its total consumption of energy below an upper bound related to the current consumption level at the actual energy price rate p^a. Formally, we would like to consider the following variation of problem (3) to help the Major decide the pricing policy.

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{N}\sum_{j=1}^{m}(t_j - t_{j-1})C_i(t, p_i(t_j))p_i(t_j) \\
\text{st.} \quad & \{C_i(t_j, p_i(t_j)) : j = 1, \dots, m\} = \operatorname{argmin} \left\{ \begin{array}{l} \sum_{j=1}^{m}(t_j - t_{j-1})\big(C_i(t_j)p_i(t_j) - \lambda_i U_{ij}(C_i(t_j))\big) : \\ \sum_{j=1}^{m}(t_j - t_{j-1})C_i(t_j) \le \mu^+ W_i \end{array} \right\} \text{ for all } i \in \mathcal{I}, \\
& \sum_{i \in \mathcal{I}_1} \frac{1}{|\mathcal{I}_1|} p_i(t_j) \le \gamma \sum_{i \in \mathcal{I}_2} \frac{1}{|\mathcal{I}_2|} p_i(t_j), \; j = 1, \dots, m \\
& p_i(t_j) \ge 0, \; \forall i \in \mathcal{I}, j = 1, \dots, m,
\end{aligned} \tag{5}
$$

where

$U_{ij}(C_i(t_j)) =$	Utility obtained by the ith consumer from consuming the energy $C_i(t_j)$ (see Section 2.2 for details on how to define the Utility function).
$\lambda_i =$	Trade-off between utility and total price of energy for consumer $i \in \mathcal{I}$

Although this makes the problem more realistic (assuming rational behaviour of consumers), it makes it a *bilevel* optimization problem, which in general is difficult to solve. As stated the variables in the outer optimization problem (5) are $p_i(t_j)$ and $C_i(t_j, p_i(t_j))$ for every customer $i \in \mathcal{I}$, and every time $t_j \in \mathcal{T}$. The variables $p_i(t_j)$ also appear in the inner optimization problem together with the variables $C_i(t_j)$ for every customer $i \in \mathcal{I}$, and every time $t_j \in \mathcal{T}$.

As in Section 1.1, the objective for this part of the project is to produce an optimization problem that could be solved or approximately solved using the AIMMS tools, but also, that can be appropriately constructed based on the available data (cf., Section 2.2, and recall that in principle here the aim is to only use data about consumption at the current price rate p^a). Specifically:

1. Suggest an strategy for the Major to solve this problem, given the information provided. For example, what kind of simplifications would you suggest in order to obtain a formulation of the problem that can be specified using the provided data, and at the same time whose solution can be addressed using the optimization solvers available in AIMMS? Discuss how do these simplifications affect the optimality of the solutions you find. Are the solutions you find optimal or near-optimal?

2. Analyze the pros and potential cons of the solution approach proposed above. In particular, can you say whether the policy obtained with your approach is better than just fixing the energy price to p^a? Clearly explain your statements and support them with appropriate numerical results.

2 The Tools

The following tools will be available to your team to address the case problem.

2.1 Software

A full version of the AIMMS modeling platform along with solvers `CPLEX`, `GUROBI`, `MOSEK`, `XA`, `CONOPT`, `MINOS`, `SNOPT`, `LGO`, `AOA`, `PATH`, `CP Optimizer` and, through `COIN-OR`, `CBC`, and `IPOPT` are provided to the teams free of charge. You may use any combination of these to solve your models. In particular, `CP Optimizer` is a new addition to the available solvers in AIMMS that allows the user to use Constraint Programming techniques to solve optimization problems.

We encourage all teams to take advantage of the procedural aspect of the AIMMS modeling system and solve the problems in multiple stages. Please study the documentation about the features and capabilities of these solvers.

The quality of the AIMMS software your team produces will be taken into account in judging your teams submission. Have in mind that the user of this software is expected not to be an expert in the general area of Operations Research.

If you have any questions about the software please contact support@aimms.com.

2.2 Data values/sets

The Borough is interested in setting the energy price rates hourly for the next 24 hours based on available information. Specifically:

$$\begin{aligned} T &= 24\text{hr} \\ \mathcal{T} &= \{1:00, 2:00, 3:00, \ldots, 11:00, 12:00, 13:00, \ldots, 23:00, 24:00\}. \end{aligned}$$

Currently the Borough is charging its customers a constant energy price:

$$p^a = \$0.20/\text{kWh},$$

where kWh stands for kilowatt per hour. This rate is constant for all the customers at any time of the day.

The Borough has 16,000 customers which can be roughly classified into the following five (5) classes outlined in Table 1:

Customer Class	Percentage of Customers
Small residential	45.19%
Large residential	54.54%
Office building & commercial	0.040%
Shift industrial	0.200%
No-shift industrial	0.003%

Table 1: Distribution of energy consumption in the Borough

Table 1 also provides the percentage of customers in the Borough that belong to each of the classes.

In Table 1 "shift" and "no-shift" refers respectively to industries that use shifts of work (i.e., work is done during 24hr of the day), and that do not use shifts of work (i.e., most work is done during "office" hours).

Although the Borough is considering the future installation of smart meters that will allow them to track the consumption of each customer, such information is not available now. Instead, the Borough has data on the hourly energy consumption of **one** representative customer in each class at the current price rate p^a. This information is available in the `EXCEL` file:

`Consumption_profiles.xls`

which is available at the competition's website `http://coral.ie.lehigh.edu/~mopta/competition/`.

In solving the problem in Section 1.1 it is further assumed that there is information available about the energy consumption at a given time for different price rates. For that problem, similarly assume that the Borough has data on the hourly consumption for different price rates of **one** representative customer in each class. The following `EXCEL` files provide this information for each of the customer classes:

```
Small_residential_profiles.xls
Large_residential_profiles.xls
Office_building_and_commercial_profiles.xls
Shift_industrial_profiles.xls
No_shift_industrial_profiles.xls
```

which can be downloaded from the competition's website `http://coral.ie.lehigh.edu/~mopta/competition/` (you are welcome to merge all these files into one for use in your model).

The set $\mathcal{I}_1$ used in the constraints in (3) and (5) will be the union of the Borough customers in the classes: Small residential, Large residential, and Office building and commercial. Similarly, the set $\mathcal{I}_2$ used in the constraints in (3) and (5) will be the union of the Borough customers in the classes: shift industrial and no-shift industrial.

In the second part of the case (Section 1.2) it is necessary to specify the monetary utility the costumers derive from their energy consumption. This can be done by assuming that for any customer i at time t_j, the monetary utility function is of the form $U_{ij}(x) = 1 - e^{-a_{ij}x}$ (with x representing the energy consumption); that is, a Bernoulli type utility function, and setting up the a_{ij} such that $U_{ij}(C_i(t_j, p^a)) = u_i$, with $u_i = 90\%$ for small residential customers, $u_i = 85\%$ for large residential customers, $u_i = 95\%$ for office building & commercial customers, for $u_i = 75\%$, shift industrial customers, and $u_i = 80\%$ for no-shift industrial customers. Notice that for a customer i, u_i represents the percentage of the maximum utility that is obtained at the current consumption level. Your team is free to make the characterization of the utility in a different (or approximate) form as long as it results in a reasonable choice of functions and related parameters as a mean to improve the solvability and the quality of your algorithm for the problem. Make sure that the choice of λ_i properly weights the cost vs. utility trade-off.

With the data above, the only parameters that remain to be defined are μ^+, μ^-, γ, and λ_i for all $i \in \mathcal{I}$, used to set the constraints in (3) and (5). Instead of giving a single value for these parameters you are asked to investigate different reasonable values for these parameters and make conclusions about how changes in those values affect both the policy that your models are recommending as well as the difficulty to solve the corresponding optimization problem (in case the difficulty of your solution approach changes for different values of your parameters). Make sure you clearly illustrate with tables or plots the different policies that you might be recommending for different values of these parameters.

2.3 Relevant literature

You are free to browse and use the relevant literature for inspiration. Please cite all sources and carefully distinguish your ideas from those obtained in the literature.

3 Deliverables

Your team needs to deliver a solution to the problem described in this case study. In particular, your submission should include:

- implementation of your models in AIMMS, including a user interface, providing the user graphical and textual output. In particular, your model should produce appropriate graphs and/or tables that help to analyze the decisions your team is recommending.
- solutions of the problem instances defined by the data files (the more the better), as well as summary tables or graphs addressing the performance of your developed methodologies.
- a no more than 15 page report that discusses your models, the mathematical background of your techniques, the solutions that you obtained, and further recommendations. **In order to judge your numerical results, it is key that all mathematical programming, and algorithms you used are clearly presented in the report**.

Teams that only develop partial solutions to the case are still encouraged to submit their solution. Teams are also encouraged to address additional or alternative "realistic" factors, variants, or modifications of the underlying problem considered in the case. If data beyond the one provided with the case is necessary to consider these factors, variants, or modifications, you're asked to generate additional corresponding data either from relevant literature or sensible assumptions. Be sure to reference the literature or assumptions you used to generate the additional data.

The **deadline** for submission is **June 13, 2014 23:59 EDT**. If you have questions about the problem or the competition in general, please contact Luis F. Zuluaga and Alberto J. Lamadrid at luis.zuluaga@lehigh.edu and ajlamadrid@lehigh.edu.