

2nd AIMMS-MOPTA Optimization Modelling Competition

Tax-aware portfolio optimization

Case study

Classical models used in portfolio optimization focus on return and risk. More complicated models take into account the effect of trading costs. In this case study your team will have to develop a tool to optimize a portfolio in the presence of different tax rules.

1 Basics

1.1 General framework

We use the classical Markowitz model of portfolio optimization. We are trading only stocks S_1, \dots, S_n . At time t the expected (daily) return of stock i is r_i^t , the standard deviation of the return is σ_i^t . The correlation between the returns of stocks $i \neq j$ is ρ_{ij}^t . To abbreviate notation we introduce the return vector $r^t = (r_1^t, \dots, r_n^t)$ and the covariance matrix $Q^t = [\rho_{ij}^t \sigma_i^t \sigma_j^t]_{i,j=1,\dots,n}$. Assume that at time t we have a good estimate of r^t and Q^t , at least for the near future.

1.2 Transaction costs

Each transaction has a cost composed of two terms: a fixed cost and a variable portion. The fixed cost is flat fee of 10 dollars per transaction. The variable fee is due to the difference between the selling and bidding price of a stock, and is 0.5% of the traded volume. This means that if a stock is quoted at 30 dollars and the investor buys 10 shares, then they are going to pay $10 + 0.005 \times 30 \times 10 = 11.50$ dollars on top of the 300 dollars the 10 shares would cost otherwise. If they sell 20 shares at the same 30 dollar price then they get 600 dollars, minus the $10 + 0.005 \times 600 = 13$ dollar transaction cost.

1.3 Tax rules

To calculate the tax on an investment income the following rules apply for this case study. These are a simplified version of the American regulations.

1.3.1 Ordinary income

The income in a given calendar year is divided into two parts: ordinary income and capital gains. The ordinary income comes from the investor's day job and is assumed to be 400000 dollars per year (so it is in the highest tax bracket). The tax payable on this part is 35% of the income over 372950 dollars (plus of course some fixed sum for the portion below 372950 dollars, but you can ignore that part, as it is constant). In the second year of the study this tax rate changes to 40%. Note that losses can decrease the taxable income up to a 3000 dollar limit, see §1.3.3 for more details.

1.3.2 Capital gains

If the investor sells a stock with a profit, they realize a capital gain. It is important that in order to have a gain, the shares need to be sold, it's not enough that the prices are up. The advantage is the control over the timing of the transactions.

The time (number of calendar days) while the investor has a certain stock is the *holding period*. Capital gains can be short-term or long-term. We talk about long-term if the shares are held for at least a year and a day before they are sold, otherwise it is considered short-term.

To calculate the gain or loss you first establish the *basis*, which is the price at which the shares were bought, including all the transaction costs. The *amount realized* is the price at which the shares are sold, again taking into account all the transaction costs. The difference between the amount realized and the basis is the gain/loss.

1.3.3 Capital losses

The losses can also be short-term or long-term. In general losses can be used to offset gains, and thus reduce taxes. Short-term losses are offsetting short-term gains, while long-term losses can be used to offset long-term gains. The way this works is the following procedure:

1. First you calculate the short and long-term gains and losses.
2. You subtract the losses from the gains within each category. If there are more losses than gains, then there will be some losses left to use.
3. If there are losses left to use in either category, then you combine them and subtract that from the investor's regular income. You can only subtract up to 3000 dollars with this, the rest of the losses will carry forward to the next year.
4. The gains that remain after this are taxed with capital gains tax.

1.3.4 Taxing capital gains

Short-term gains (or what is left of them after subtracting the losses) are taxed at the same rate as the ordinary income (35% and 40% in the two years of the study). Long-term gains (again after subtracting losses) receive special tax rates, which in our case is 15% in the first year and 19% in the second year.

1.3.5 The wash sale rule

There is one final rule that complicates matters. Consider the following example: You've owned some shares for a long time, and they are losing their value. You want to realize the loss to offset some gains elsewhere in your portfolio, so you sell your shares. Soon you realize that the stock is bounding back, so you want to buy in again, so you buy shares from the same stock a few days later. This way you realized your loss, but you still have the shares, so if they rebound, you still win. The wash sale rule is there to prevent you from claiming the loss in this situation.

You cannot claim a loss from a sale if you bought any shares of the same stock *within 30 days before or after* the sale. This means that the critical period is 61 calendar (not trading!)

days long: 30 days before the sale, the day of the sale, and 30 days after the sale. Instead, the following will happen:

- The basis of the stock you bought is increased with the amount you would have claimed as a loss.
- The holding period of the shares you bought will include the holding period of the previously held stock.

As an example, imagine the following scenario:

January 1, 2009: You buy 10 shares of stock X at 15 dollars, thus your basis is $10 + 1.005 \times 150 = 160.75$ dollars.

December 15, 2009: You sell your 10 shares of X at 12 dollars, so your amount realized is $0.995 \times 120 - 10 = 109.40$ dollars. Your (short term) loss would be $160.75 - 109.40 = 51.35$ dollars.

December 20, 2009: You buy 10 shares of X at 13 dollars, costing $10 + 1.005 \times 130 = 140.65$ dollars. This makes your previous sale a wash sale and as a result the basis of your 10 shares of X will be $140.65 + 51.35 = 192$ dollars. You cannot claim any losses on the December 15 sale.

January 7, 2010: You decide to sell your 10 shares of X at 12.50 dollars. The amount realized is $0.995 \times 125 - 10 = 114.375$ dollars. As the basis on these shares was 192 dollars, your loss is 77.625 dollars. Moreover, as the holding period of the shares you sold on December 15 was added to the holding period of the shares you bought on December 20, this loss is now a long-term loss.

The wash sale can also happen *after* you've bought the replacement stock:

January 1, 2009: You buy 10 shares of stock X at 15 dollars, thus your basis is $10 + 1.005 \times 150 = 160.75$ dollars.

December 15, 2009: You buy 10 shares of X at 13 dollars, costing $10 + 1.005 \times 130 = 140.65$ dollars.

December 20, 2009: You sell your 10 shares of X from January 1 at 12 dollars, so your amount realized is $0.995 \times 120 - 10 = 109.40$ dollars. Your (short term) loss would be $160.75 - 109.40 = 51.35$ dollars, but this sale is a wash sale and you cannot claim any losses from it.

The wash sale rule is a lot more complicated in practice, with countless special cases and exceptions. For the sake of this competition we use this simplified form.

1.4 A few technicalities

1.4.1 Identifying shares in a sale

Since we take tax into account, we need to keep track of when we bought certain shares and at what price. If you have two batches of shares bought at different times, then when it comes to selling some shares you have to specify which batch you want to sell. It makes a difference in several levels: both the holding period and the basis are different.

1.4.2 Selling parts of a lot

When you sell only part of a lot, then the fixed portion of the transaction cost is divided up among the lots. For example, if you bought 10 shares of X for 20 dollars each, then your basis for the lot is $10 + 1.005 \times 200 = 211$ dollars. If you sell 6 shares of this lot, then the basis for those 6 shares will be $\frac{6}{10} \times 211 = 126.60$ dollars. As a consequence, the basis for the remaining 4 shares is $\frac{4}{10} \times 211 = 84.40$ dollars.

1.4.3 Selling multiple lots

If you have multiple lots of the same shares you can also sell them together in one transaction if you want. This way you only incur the fixed transaction cost once. You have to be careful in that you may have a loss from one lot and a gain from the other one. Take the following example:

January 1: Buy 10 shares of X at 9 dollars each. The basis for this lot is thus $10 + 1.005 \times 90 = 100.45$ dollars.

January 10: Buy 10 shares of X at 12 dollars each. The basis for this lot is thus $10 + 1.005 \times 120 = 130.60$ dollars.

January 20: Sell 20 shares of X, coming from the January 1 and January 10 lots, at 11 dollars. The amount realized is thus $220 \times 0.995 - 10 = 208.90$. This is divided among the shares in the lot equally, so on the 10 shares from January 1 you have an amount realized of 104.45 dollars, which makes for a capital gain of 4 dollars. On the other hand, the January 10 lot creates a loss of 26.15 dollars. These are both short-term gains and losses, so you can actually combine them. On the other hand, if you sold the same 20 shares on January 5 in the following year, then the gain of the January 1 lot would be long-term, while the loss of the January 10 lot would be short-term.

2 Goals

The goal of your optimization effort is to create a tool that allows the user to make regular decisions about rebalancing their portfolio keeping the tax aspect in mind. The user wants to consider the tax they pay, the total return, the risk, the return after taxes, the carryforward of your losses or anything else that you find suitable. They may want to minimize/maximize any of these components, while limiting one or more of the other components.

The basic building block is a daily decision: given a current portfolio, the market prices of the day, and the estimates of the mean and covariance of the daily returns, make a decision about what to buy and sell. You can also decide not to trade anything on a given day.

To test your daily decisions, you are given daily closing prices of 20 stocks over a two-year period. Obviously, you cannot use the actual future prices when making a decision, only the price of the day and the return estimates. These estimates were derived from the actual data and are quite accurate. Once you are done with trades for a day, you should move on to the following day, use the new prices (and new estimates, if you've entered a new period),

and make another decision. At the end of the calendar year you pay the taxes and enter the next year.

You are not allowed to use estimates other than the ones given to you in the data, or to update them using past pricing data.

2.1 Software

A full version of the AIMMS modelling environment along with the solvers Gurobi, Mosek, Conopt, IPOPT, CBC and AOA are provided to the teams free of charge. You may use any combination of these to solve the problem. Please study the documentation about the features and capabilities of these solvers.

If you have any questions about the software please contact mopta@aimms.com for assistance.

2.2 Data set

The stocks for the case study are from the US stock markets and are quoted in US dollars. Trading takes place on trading days, which are the days for which a price is given. The data is from 2005 and 2006.

The initial portfolio is as follows:

Stock	Bought at (USD)	Quantity	Held for (days)
MSFT	26.00	3000	400
MSFT	28.00	2000	10
F	15.00	1000	50
CRAY	18.00	2000	200
GOOG	200.00	3000	2
GOOG	100.00	500	120
AAPL	27.00	3000	250
NVDA	9.00	1000	350
DELL	41.00	2000	70

The value of the initial portfolio is about one million dollars. The initial bankroll to make new purchases is 300000 USD. The investor would like to take out 5000 dollars from their portfolio every month. The bankroll must be nonnegative at all times.

The initial carryforward of losses from previous years is 500 dollars.

An estimate of the daily returns of the stocks (for trading days) and the covariances of the returns are given to you. These estimates are quite accurate and are updated every second month. You can only use the current estimate for the calculations and are not allowed to change or update the estimates in any way.

Your model will also be tested on a different dataset, so please make it easy to change the data and avoid hardwiring any constants in your model. Do not over optimize for the given dataset.

2.3 Simplifications

We encourage you to build your model incrementally by leaving out certain features in the initial phases. Here is a list of what you could do to simplify the problem:

- ignore the fixed portion of the transaction cost;
- ignore the wash-sale rule, either by forbidding wash-sales, or by claiming the loss regardless of a wash-sale;
- forbid selling parts of a lot, or selling multiple lots at once;
- make decisions less frequently, every other day or once a week;
- only consider a subset of the stocks.

It may turn out that some of these complications can be safely ignored when making the decisions, i.e., their effect is negligible.

2.4 More information

If you want to read more about the tax rules and possible tax-optimization strategies then please visit <http://www.fairmark.com/capgain/>.

3 Deliverables

Your team needs to deliver a complete solution to the problem described in this case study, including

- an implementation of the model in AIMMS, including a graphical user interface, allowing the investor to make decisions easily;
- a solution of the model for the given data set;
- a 15 page report about the mathematical background of the model, the solution techniques, results and recommendations to the decision makers.

The deadline for submission is May 31, 2010 23:59 New York time. If you have questions about the problem or the competition in general, please contact Imre Pólik at imre@polik.net.