Cable Trench Problem

Matthew V Galati

Ted K Ralphs

Joseph C Hartman

magh@lehigh.edu

Department of Industrial and Systems Engineering

Lehigh University, Bethlehem, PA

Cable Trench Problem

The Cable Trench Problem (CTP) is that of minimizing the cost of digging trenches and laying cable for a communications network given a central hub.

- Let $G = (N, A)$ be a connected digraph with specified depot $0 \in N$.
- Define c_{ij} as the cost/weight (typically distance) on arc (i, j).
- Define fixed charge variables (trench) x_{ij} as to whether or not to dig a trench between nodes i and j.
- Define flow variables (cable) y_{ij} as to the amount of cable to lay between nodes i and j.

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Node Routing

- A *node routing* is a directed subgraph G' of G satisfying the following properties:
  - G' is (weakly) connected.
  - The in-degree of each non-depot node is 1.
- It is a spanning arborescence.
- There is a unique path from the depot to each demand point (vertex).
- Cost Measures (least cost routing)
  - Sum the lengths of all arcs in G'.
  - Sum the length of all paths from the depot.
  - Some linear combination of these two.

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IP Formulation

$$Min \sum_{(i,j)\in A} \tau\, c_{ij}(x_{ij} + x_{ji}) + \gamma\, c_{ij}(y_{ij} + y_{ji})$$

$$\begin{aligned}
s.t. \quad & x(\delta(N \setminus \{i\})) = 1 & \forall i \in N \setminus \{0\} \quad (1)\\
& y(\delta(N \setminus \{i\})) - y(\delta(\{i\})) = d_i & \forall i \in N \setminus \{0\} \quad (2)\\
& y_{ij} \le M x_{ij} & \forall (i,j) \in A \quad (3)\\
& y_{ij} \ge 0 & \forall (i,j) \in A \quad (4)\\
& x_{ij} \in \{0,1\} & \forall (i,j) \in A \quad (5)
\end{aligned}$$

where:

- (1) degree constraint
- (2) flow balance / demand constraint
- (3) capacity constraint

Complexity

- This node routing problem is NP-complete in general.
- Cable Trench Problem ($\tau, \gamma > 0$)
  - $\gamma = 0 \Rightarrow$ Minimum Spanning Tree Problem.
  - $\tau = 0 \Rightarrow$ Shortest Paths Tree Problem.
- Other special cases.
  - $\gamma = 0$ and $x(\delta(\{i\}) = 1 \Rightarrow$ Traveling Salesman Problem (TSP).
  - $\gamma > 0$ and $x(\delta(\{i\}) = 1 \Rightarrow$ Variable Cost TSP (VCTSP).
  - $x(\delta(V \setminus \{0\}) = x(\delta(\{0\}) = k \Rightarrow$ k-TSP.

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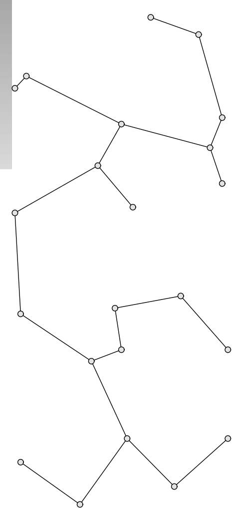
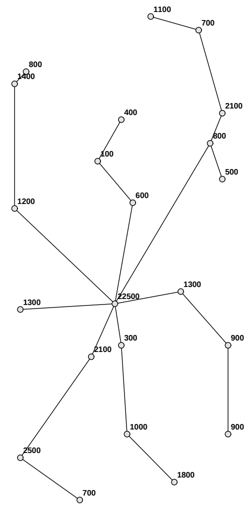
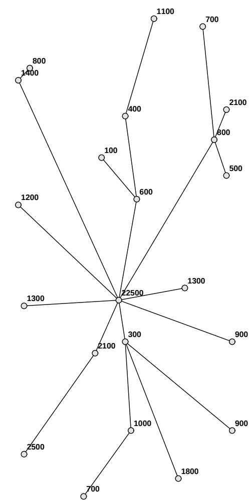
Sample Spanning Trees



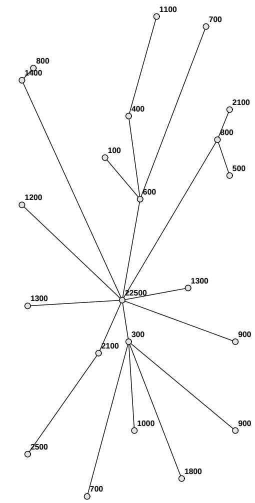
$\gamma = 0$



$\tau/\gamma = 10$



$\tau/\gamma = 0.1$



$\tau/\gamma = 0.001$

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Complexity

Theorem 1 *Among all minimum spanning trees finding the one that minimizes the path length between a particular set of vertices s and t is NP-Complete.*

Corollary 1 *Among all minimum spanning trees finding the one that minimizes the total path length between a particular vertex s and all other vertices in V is NP-Complete.*

Corollary 2 *The Cable Trench Problem is NP-Complete.*

Theorem 2 *Among all shortest path trees rooted at s finding the one that minimizes the total edge length is in P.*

Previous Work

- Vasko et. al - Kutztown University (to be published CAOR Nov 2001)
  - Heuristic upper bound for all values of τ/γ.
  - The solution to CTP is a sequence of spanning trees such that as τ/γ increases, the total edge length strictly decreases each time another spanning tree becomes optimum and the total path length strictly increases.
  - Total cost versus τ/γ is piecewise linear curve with strictly decreasing slopes.
- Related Areas
  - Fixed-Charge Network Flow
  - Capacitated Network Design

Valid Inequalities

- Rounded Capacity Constraints

$$\sum_{i \notin S,\, j \in S} x_{ij} \geq \lceil d(S)/C \rceil$$

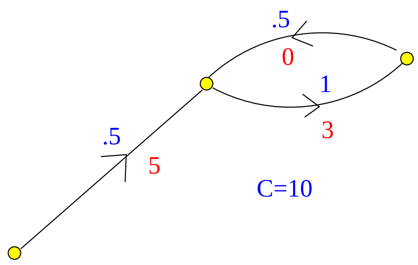
- Flow Linking Constraints

$$y_{ij} \leq (C - d_i)x_{ij} \Leftrightarrow x_{ij} \geq \frac{y_{ij}}{C - d_i}$$

$$y_{ij} - y(\delta(\{j\})) \leq x_{ij}d_j$$

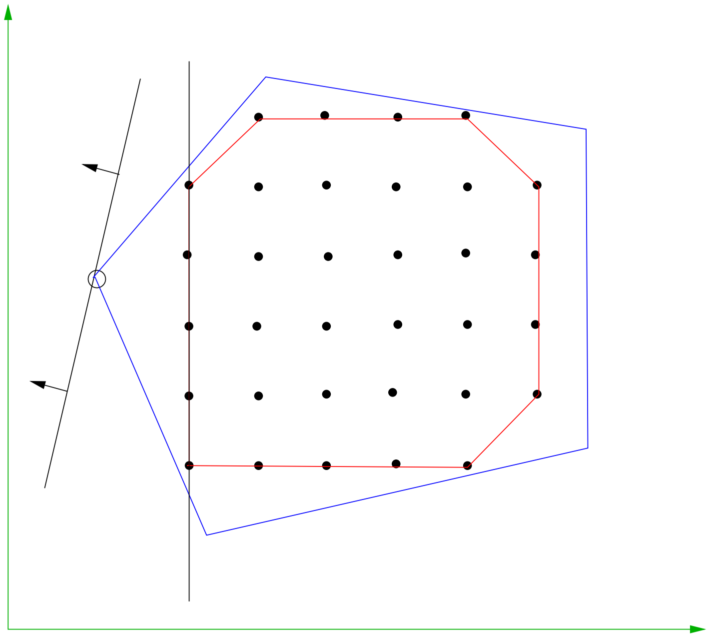
- Edge Cuts

$$x_{ij} + x_{ji} \leq 1$$



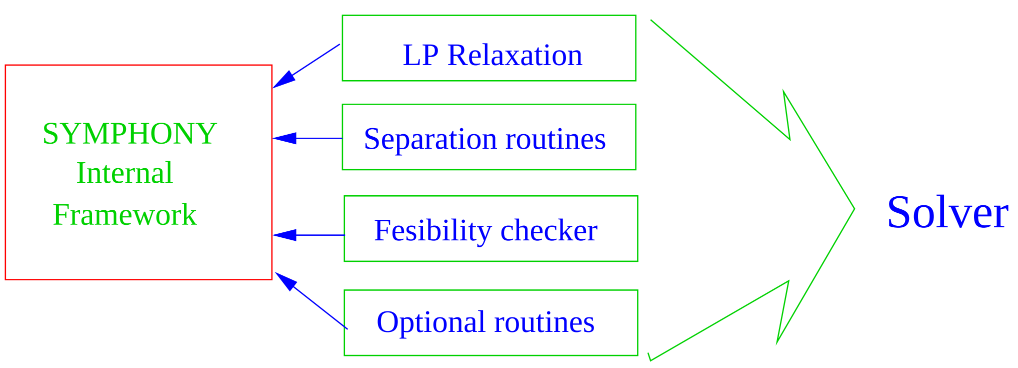
Separation

- Flow linking constraints and edge cuts can be included explicitly or separated in polynomial time.
- Separating rounded capacity constraints is NP-complete, but can be done effectively.



Implementation

- The implementation uses SYMPHONY, a parallel framework for branch, cut, and price (relative of COIN/BCP).
- In SYMPHONY, the user supplies:
  - the initial LP relaxation,separation subroutines,
  - feasibility checker, and other optional subroutines.
- SYMPHONY handles everything else.
- The source code and documentation are available from `www.BranchAndCut.org`
- Workshop on COIN/BCP (TB42) - Laszlo Ladanyi, Ted Ralphs



Conclusions and Future Research

- The flow linking constraints help to force integrality.
- The edge cuts also help impose structure and integrality.
- Future Research
  - Generalizations of the model (different types of "trenches", different grades of "cable").
  - Better (more specific) cuts for the case where τ/γ is not extreme.
  - Take advantage of connections to other models.
- Upper Bounds

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