IGERT Experience In Switzerland
Logistics at Galenica Pharmaceutical

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Outline

- École Polytechnique Fédérale de Lausanne, Suisse (EPFL) - Travel
- Galenica Pharmaceutical - Logistics
- Vehicle Routing Problem with Time Windows (VRPTW)
- Branch Cut and Price (BCP)
- BCP for VRPTW
  - Formulation
  - Pricing Problem
  - Cutting Planes
  - Branching
- Implementation of BCP for VRPTW
- (Preliminary) Computational Results
- Current Research

EPFL

- School of Basic Sciences - Institute of Mathematics
  - Recherche Opérationnelle Sud Ouest (ROSO) - Thomas Liebling
  - Recherche Opérationnelle Sud Est (ROSE) - Dominique de Werra
- Student Proposals - COIN-or.org
  - Preprocessing Techniques for Mixed Integer Programming
  - A Cut Generator Library for Integer Programming
- Seminar Series: Martine Labbé - Universite Libre De Bruxelles
  - Branch-and-Cut for Network Design in Telecommunications
  - Two-Connected Network with Bounded Rings
- 3ème Cycle Romand de Recherche Opérationnelle - Zinal, Switzerland
  - Alexander Schrijver of CWI (National Research Institute for Mathematics and Computer Science in the Netherlands), Amsterdam
  - Kurt Mehlhorn of Max-Planck-Institut für Informatik, Saarbrücken

Common Optimization INterface for Operations Research

- An initiative to spur the development of open source software for the operations research community.
- Current projects in the COIN repository www.coin-or.org:
  - BCP: a parallel branch-cut-price framework
  - CGL: a cut generation library
  - DFO: a package for solving general nonlinear optimization problems when derivatives are unavailable
  - VOL: the volume algorithm
  - OSI: an open solver interface layer
  - OTS: an open framework for tabu search
  - IPOPT: an interior point algorithm for general large-scale nonlinear optimization
  - CLP: a native simplex solver

- ROSO Seminar Series (May 2002):
  Decomposition-based Methods for Large-scale Discrete Optimization
- Galenica Pharmaceutical - Logistics Optimization
  - Current system uses a greedy insertion heuristic
- Contribution
  - Improve the quality of the solutions provided by the heuristic methods
  - Provide some measurement or proof of optimality

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Galenica Logistics

- Major pharmaceutical corporation headquartered in Bern,Switzerland
- Several distribution centers (DCs) across the country
- Apriori set of customers (and demand) to be serviced each day of the week (from a given DC)
- Each customer has a (tight) fixed time window when delivery may occur (considered *hard* constraints).
- Routing between customers calculated by a street-level route generator.
- Cost is a linear combination of distance and time.

Galenica Logistics - Challenges

- Design
  - Delivery length is typically small so the same truck can make several cycles per day (load, depart from DC, delivery, return to DC).
  - It is possible to use trailers with some trucks.
  - What is the effect of *soft* time windows?
- Depot
  - Trucks sometimes require 60m maintenance on return to depot.
  - During loading there are only n slots available at one time.
- Trucks
  - Some trucks are only available within a specificed time window.
  - Trucks are capacitated by both weight and volume.
- Route
  - If the route exceeds 4 hours, the driver must take a 15m break.
  - Idle time at the customer must be less than 15m.

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Vehicle Routing with Time Windows

- Set of customers C set of homogenous vehicles K with capacity q.
- Let N define the set of nodes $0, 1, 2, ..., n+1$.
- The set of arcs A is defined as $\{(i, j) : i \neq j, i \neq n+1, j \neq 0\}$.
- For each arc $(i, j) \in A$, we have cost c_{ij} and time t_{ij} which includes the service time at customer i.
- Each customer $i \in C$ has a demand d_i and a time window $[a_i, b_i]$.
- Define the variable $x_{ijk} = 1$ if vehicle $k \in K$ drives along arc $(i, j) \in A$. Let s_{ik} define the time vehicle k starts to service customer i.

Vehicle Routing with Time Windows

- Design a set of minimal cost routes, one for each vehicle,

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ijk}$$

- such that, each customer is serviced exactly once,

$$\sum_{k \in K} \sum_{j \in N} x_{ijk} = 1 \ \forall i \in C \tag{1}$$

- every route originates at vertex 0 and ends at vertex $n+1$,

$$\sum_{j \in N} x_{ojk} = 1 \ \forall k \in K \tag{2}$$

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0 \ \forall h \in C, k \in K \tag{3}$$

$$\sum_{i \in N} x_{i,n+1,k} = 1 \ \forall k \in K \tag{4}$$

Vehicle Routing with Time Windows

- the time windows and capacity constraints are observed and

$$\sum_{i \in C} d_i \sum_{j \in N} x_{ijk} \leq q \quad \forall k \in K \tag{5}$$

$$s_{ik} + t_{ij} - M(1 - x_{ijk}) \leq s_{jk} \quad \forall i, j \in N, k \in K \tag{6}$$

$$a_i \leq s_{ik} \leq b_i \quad \forall i \in N, k \in K \tag{7}$$

- assignments are integral.

$$x_{ijk} \in \{0, 1\} \ \forall i, j \in N, k \in K \tag{8}$$

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LP-based Branch and Bound

- Consider problem P:

$$
\begin{aligned}
min\ c^T x & \\
s.t. \quad Ax & \leq b \\
x_i & \in \mathbb{Z}\ \forall\, i \in I
\end{aligned}
$$

  where $(A, b) \in \mathbb{R}^{m \times n+1}, c \in \mathbb{R}^n$.

- Let $\mathcal{P} = conv\{x \in \mathbb{R}^n : Ax \leq b, x_i \in \mathbb{Z}\ \forall\ i \in I\}$.
- Basic Algorithmic Approach
  - Use LP relaxations to produce lower bounds.
  - Branch using hyperplanes.
- Basic Algorithmic Elements
  - A method for producing and tightening the LP relaxations.
  - A method for branching.

Branch Cut and Price

- Weyl-Minkowski
  - $\exists(\bar{A}, \bar{b}) \in \mathbb{R}^{\bar{m} \times \bar{n}+1}$ s.t. $\mathcal{P} = \{x \in \mathbb{R}^n : \bar{A}x \leq \bar{b}\}$
  - We want the solution to $min\{c^T x : \bar{A}x \leq \bar{b}\}$.
  - Solving this LP isn't practical (or necessary).
- BCP Approach
  - Form LP relaxations using submatrices of $\bar{A}$.
  - The submatrices are defined by sets $\mathcal{V} \subseteq [1..\bar{n}]$ and $\mathcal{C} \subseteq [1..\bar{m}]$.
- BCP Elements
  - Pricing Algorithm - Variable Generation
  - Cutting Plane Algorithm - Constraints Generation

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Set Partitioning Formulation

- Set of elementary feasible routes $\hat{R}$
- For each route $r \in \hat{R}$, let β_{ir} be the number of times route r visits customer i with cost c_r.

$$\min \sum_{r \in \hat{R}} c_r y_r$$

$$\sum_{r \in \hat{R}} \beta_{ir} y_r = 1 \qquad \forall i \in C \tag{9}$$

$$\left\lceil \sum_{i \in C} d_i / q \right\rceil \leq \sum_{r \in \hat{R}} y_r \leq |K| \tag{10}$$

$$y_r \in \{0, 1\} \qquad \forall r \in \hat{R} \tag{11}$$

Pricing Problem

- Reduced cost of variable y_r is $c_r - \sum_{i \in C \cup \{0\}} \pi_i \beta_{ir}$
- Corresponds to the modified arc costs $\hat{c}_{ij} = c_{ij} - \pi_i$
- Does there exist a member of $\hat{R}$ with negative reduced cost?
- Instance of *Elementary Shortest Path Problem with Time Windows and Capacity Constraints* (ESPPTWCC).
- ESPPTWCC is NP-hard and there is no known efficient algorithm.
- Let R be the set of all feasible routes (nonelementary routes are allowed).
- Now an instance of SPPTWCC (also NP-hard) but there exists an efficient pseudo-polynomial dynamic programming [Desrochers et al].

SPP with Resource Constraints

- Ideas of label setting and dominance from *Dijkstras* for SPP.
- Assumption: time and capacity are discretized.
- Define a state as $c(i,t,d)$ for vertex i, current time t and accumulated demand d.

$$c(0,0,0) = 0$$

$$c(j,t,d) = min_i\{\hat{c} + c(i,t',d')|t' + t_{ij} = t \wedge d' + d_i = d\}$$

- The number of possible states is $\Gamma = \sum_{i \in N}(b_i - a_i)(q-1)$.
- Dominance: $(i,t_1,d_1) \prec (i,t_2,d_2)$ if and only if $c(i,t_1,d_1) \leq c(i,t_2,d_2), t_1 \leq t_2$ and $d_1 \leq d_2$.

SPP with Resource Constraints

```
<Initialization>
```

$NPS = \{(0,0,0)\}, c(0,0,0) = 0$

`repeat`

$(i,t,d) = BestLabel(NPS)$

`for` $j := 1$ `to` $n+1$ `do`

`if` $(i \neq j$ `and` $t + t_{ij} \leq b_j \wedge d + d_j \leq q)$ `then`

```
<Label Feasible>
```

`if` $c(j, max\{t + t_{ij}, a_j\}, d + d_j) > c(i,t,d) + \hat{c}_{ij}$ `then`

```
<New Label Better>
```

$InsertLabel(NPS, (j, max\{t + t_{ij}, a_j\}, d + d_j))$

$c(j, max\{t + t_{ij}, a_j\}, d + d_j) = c(i,t,d) + \hat{c}_{ij}$

`until`$(i = n+1)$

`return`

SPP with Resource Constraints

- Can find a lot of path cycling between “good” vertices.
- Houck et al: 2-cycle Elimination $i \rightarrow j \rightarrow i$ extend the label to $(i, t, d, pred)$ which increases the number of states by 2.
- Irnich: k-cylce Elimination can increase the number of states by $\binom{k}{2} + 1$.
  - Example: $k = 4 : i \rightarrow j \rightarrow k \rightarrow l \rightarrow i$
- Data Structures
  - N Linked Lists
  - Generalized Buckets

k-Path Cuts

- Let $k(S)$ denote the least number of vehicles needed to serve each customer in S: NP-hard.
- Define the flow on an arc $x_{ij} = \sum_{r \in R} \gamma_{ij}^r y_r$.
- Let k be some integer no larger than $k(S)$.
- Define the general *k-Path Cut* as

$$x(S) = \sum_{(i,j) \in \delta(S)} x_{ij} \geq k(S) \geq k$$

- $x(S) \geq 1$: Directed version of TSP subtour elimination constraints.
- Separation: Polynomial using max flow / min cut.
  - Appelgate, Bixby, Chvátal and Cook - CONCORDE

Separation of k-Path Cuts

- $k = 2:$ Is it valid? Determine a set S where $x(S) < 2$.
  - Larsen: Greedy search of neighborhood until $x(S) \geq 2$.
  - Rich/Cook: Random contraction algorithm of Karger - with high probability, finds all cuts with weight within a multiplicative factor α of the min cut in $O(n^{2\alpha} \log^3 n)$.

- $k = 2:$ Is it violated? Check if $k(S) \geq 2$.
  - Step 1: Is there sufficient capacity on a single vehicle to service S?
  - Step 2: Is there a feasible route that leaves the depot, visits each customer in S once and returns to the depot?
  - Step 2 is an instance of a TSPTW-feasibility problem: NP-complete.
  - TSPTW can be solved efficiently using a DP similiar to SPPTWCC.

- $k > 2:$ Rich/Cook
  - Is it valid? Greedy or Karger
  - Is it violated? Instance of VRPTW-feasibility!!

Branching

- If number of vehicles $\sum_{r \in R} y_r$ is fractional branch on constraint (10).
- Else, choose the arc (i, j) which maximizes the $c_{ij} \min(x_{ij}, 1 - x_{ij})$.
  - $x_{ij} = 0$, Fix routes containing arc (i, j) to 0.
  - $x_{ij} = 1$, Fix any route that visits i or j without using arc (i, j) to 0.
- Alternatives: Branching on resource constraints (TWs, capacity).

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Implementation - COIN/BCP

- Summer Sessions: http://sagan.ie.lehigh.edu/coin/coin.html
- Modules: TM, LP, VG, CG
- Tree Manager
  - `create_root` - initial set of variables (routes)
- Linear Program
  - `cuts_to_rows` - expand constraints given the current LP
  - `vars_to_cols` - expand variables given the current LP
  - `select_branching_candidates` - branching
- Variable Generator
  - `generate_vars` - generate variables (SPPTWCC)
- Cut Generator
  - `generate_cuts` - generate cuts (k-Path Cuts)

Galenica Logistics - Challenges

- Design
  - Delivery length is typically small so the same truck can make several cycles per day (load, depart from DC, delivery, return to DC).
    - 3 deliveries per day: morning, noon, night
  - It is possible to use trailers with some trucks. ??
  - What is the effect of *soft* time windows?
    - Increase the TW and set a cost $p(s_i)$. Dominance criterion remains if cost is non-decreasing, i.e., penalty on late, not idle.
- Depot
  - Trucks sometimes require 60m maintenance on return to depot.
  - During loading there are only n slots available at one time.
    - Allow lead time between deliveries for needed maintenance.

Galenica Logistics - Challenges

- Trucks
  - Some trucks are only available within a specificed time window.
  - Trucks are capacitated by both weight and volume.
    - Heterogenous fleet means solving K instances of SPPRC at each iteration.
    - 2 dimensional resource constraints (weight, volume): $\Gamma = \sum_{i \in N} (b_i - a_i)(q_1 - 1)(q_2 - 1)$
- Route
  - If the route exceeds 4 hours, the driver must take a 15m break. ??
  - Idle time at the customer must be less than 15m.
    - As a hard constraint can be restricted in column generator.

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Computational Results

- Benchmark Solomon Instances
  - $c1$ instances: Clustered geography, short horizon
    (5-10 customers per route)
  - $r1$ instances: Random geography, short horizon
  - $rc1$ instances: Random and Clustered geography, short horizon
  - $c2$ instances: Clustered geography, long horizon
    (more than 30 customers per route)
  - $r2$ instances: Random geography, long horizon
  - $rc2$ instances: Random and Clustered geography, long horizon
- Rich/Cook: 300 MHz Pentium II with 256MB - Cplex 5.0
- Galati : - OSL V3.0

Computational Results

- Solomon Instances - $c1.25$ instances

	Rich/Cook				Galati			
problem	nodes	cols	cuts	time	nodes	cols	cuts	time
c101.25	1	499	0	0.57	1	1146	0	0.57
c102.25	1	726	0	1.83	1	3713	0	2.63
c103.25	1	616	0	3.34	1	3430	0	4.41
c104.25	1	570	0	5.41	1	2898	0	15.62
c105.25	1	445	0	0.58	1	1487	0	0.66
c106.25	1	363	0	0.27	1	993	0	0.42
c107.25	1	472	0	0.50	1	1227	0	0.45
c108.25	1	380	0	0.69	1	2029	0	1.02
c109.25	1	383	0	1.20	3	1357	9	3.05
Total	9	4454	0	14.39	11	18280	9	28.83

Computational Results

- Solomon Instances - $c1.50$ instances

	Rich/Cook				Galati			
problem	nodes	cols	cuts	time	nodes	cols	cuts	time
c101.50	1	956	0	1.52	1	2281	0	1.81
c102.50	1	1150	0	6.88	1	4684	0	5.90
c103.50	1	1192	0	27.00	1	6525	0	61.97
c104.50	4	2043	1	169.22	401	22387	1	1210.32
c105.50	1	1222	0	2.53	1	3341	0	2.80
c106.50	1	744	0	1.30	1	2440	0	1.81
c107.50	1	1176	0	3.58	1	3466	0	2.81
c108.50	1	1142	0	5.43	1	4363	0	4.81
c109.50	1	999	0	9.53	3	3382	9	14.57
Total	12	10624	1	226.99	411	52869	10	1306.8
Total/104	7	7625	0	56.25	9	28201	9	94.67

Computational Results

- Solomon Instances - $r1.25$ instances

	Rich/Cook				Galati			
problem	nodes	cols	cuts	time	nodes	cols	cuts	time
r101.25	1	84	0	0.36	1	173	0	0.08
r102.25	1	152	2	0.43	1	567	3	0.26
r103.25	1	218	0	0.47	1	714	0	0.24
r104.25	1	277	0	1.02	1	1066	0	0.58
r105.25	1	124	0	0.17	1	320	0	0.08
r106.25	1	236	1	0.54	1	725	6	0.87
r107.25	1	278	2	0.89	2	929	0	0.81
r108.25	2	305	2	2.21	2	966	0	1.03
r109.25	1	192	0	0.25	1	515	0	0.14
r110.25	16	603	4	4.13	22	1506	1	5.14
Total	26	2469	11	10.47	33	7481	10	9.23

Computational Results

- Solomon Instances - $r1.50$ instances

	Rich/Cook				Galati			
problem	nodes	cols	cuts	time	nodes	cols	cuts	time
r101.50	1	292	1	1.67	3	680	1	1.46
r102.50	1	418	0	1.43	1	1847	0	1.19
r103.50	49	2051	2	47.37	182	10431	0	78.56
r105.50	8	609	2	5.51	68	1929	15	29.00
r106.50	1	499	5	4.19	1	1584	3	2.11
r107.50	42	2262	3	67.07	180	10997	0	89.79
r109.50	140	1955	8	91.88	1200	25340	0	271.00
r110.50	3	666	3	12.16	690	18098	0	213.00
Total	245	8752	24	231.28	2325	70906	19	686.11
r104.50	74	3382	12	489.87	–	–	–	3600.00
108.50	–	–	–	–	–	–	–	–

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Current Research

- DECOMP: A general separation algorithm for large-scale discrete optimization problems using decomposition methods.
- Optimization Seminar Series - http://sagan.ie.lehigh.edu/ipseminar