SQP Methods for Constrained Stochastic Optimization

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involving joint work with

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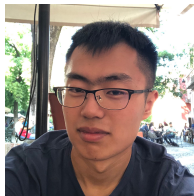**Baoyu Zhou**, Lehigh University

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Collaborators

"Sequential Quadratic Optimization for Nonlinear Equality Constrained Stochastic Optimization" https://arxiv.org/abs/2007.10525.

- ▶ to appear in SIAM Journal on Optimization

Outline

Motivation

SG and SQP

Adaptive (Deterministic) SQP

Stochastic SQP

Extensions

Conclusion

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Constrained optimization (deterministic)

Consider

$$\min_{x \in \mathbb{R}^n}\ f(x)$$
$$\text{s.t. } c_{\mathcal{E}}(x) = 0$$
$$c_{\mathcal{I}}(x) \leq 0$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $c_{\mathcal{E}} : \mathbb{R}^n \to \mathbb{R}^{m_{\mathcal{E}}}$, and $c_{\mathcal{I}} : \mathbb{R}^n \to \mathbb{R}^{m_{\mathcal{I}}}$ are smooth

- ▶ Physically-constrained, resource-constrained, etc.
- ▶ Long history of algorithms (penalty, SQP, interior-point, etc.)
- ▶ Comprehensive theory (even with lack of constraint qualifications)
- ▶ Effective software (Ipopt, Knitro, LOQO, etc.)

Constrained optimization (stochastic constraints)

Consider

$$\begin{aligned}
\min_{x \in \mathbb{R}^n} \ & f(x) \\
\text{s.t. } & c_{\mathcal{E}}(x) = 0 \\
& c_{\mathcal{I}}(x, \omega) \lesssim 0
\end{aligned}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $c_{\mathcal{E}} : \mathbb{R}^n \to \mathbb{R}^{m_{\mathcal{E}}}$, and $c_{\mathcal{I}} : \mathbb{R}^n \times \Omega \to \mathbb{R}^{m_{\mathcal{I}}}$

- ▶ Various modeling paradigms:
- ▶ . . . stochastic optimization
- ▶ . . . (distributionally) robust optimization
- ▶ . . . chance-constrained optimization

Constrained optimization (stochastic objective)

Consider

$$\min_{x \in \mathbb{R}^n} \ f(x) \equiv \mathbb{E}[F(x, \omega)]$$
$$\text{s.t. } c_{\mathcal{E}}(x) = 0$$
$$c_{\mathcal{I}}(x) \leq 0$$

where $f : \mathbb{R}^n \times \mathbb{R}$, $F : \mathbb{R}^n \times \Omega \to \mathbb{R}$, $c_{\mathcal{E}} : \mathbb{R}^n \to \mathbb{R}^{m_{\mathcal{E}}}$, and $c_{\mathcal{I}} : \mathbb{R}^n \to \mathbb{R}^{m_{\mathcal{I}}}$

- ▶ ω has probability space $(\Omega, \mathcal{F}, P)$
- ▶ $\mathbb{E}[\cdot]$ with respect to P
- ▶ Classical applications with objective uncertainty, constrained DNNs, etc.
- ▶ Very few algorithms so far (mostly penalty methods)

Questions

Need to establish what we want/expect from an algorithm.

Note: We are interested in the fully stochastic regime.[†]

We assume:

▶ Feasible methods are not tractable.

▶ ... no projection methods, Frank-Wolfe, etc.

▶ "Two-phase" methods are not effective.

▶ ... should not search for feasibility, then optimize.

▶ Only enforce convergence in expectation.

Finally, want to use techniques that can generalize to diverse settings.

[†]Alternatively, see Na, Anitescu, Kolar (2021)

This talk

Consider *equality constrained* stochastic optimization:

$$\min_{x \in \mathbb{R}^n} \ f(x) \equiv \mathbb{E}[F(x, \omega)]$$
$$\text{s.t. } c_{\mathcal{E}}(x) = 0$$

- ▶ *Adaptive* SQP method for deterministic setting
- ▶ *Stochastic* SQP method for stochastic setting
- ▶ Convergence in expection (comparable to SG for unconstrained setting)
- ▶ Numerical experiments are very promising
- ▶ Various open questions!

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Stochastic gradient method (SG)

Invented by Herbert Robbins and Sutton Monro (1951)

Sutton Monro, former Lehigh faculty member

Stochastic gradient (*not* descent)

$$\min_{x \in \mathbb{R}^n} \ f(x) \equiv \mathbb{E}[F(x, \omega)]$$

where $g := \nabla f : \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous with constant L

Algorithm SG : Stochastic Gradient

1: choose an initial point $x_0 \in \mathbb{R}^n$ and stepsizes $\{\alpha_k\} > 0$
2: **for** $k \in \{0, 1, 2, \dots\}$ **do**
3:     set $x_{k+1} \leftarrow x_k - \alpha_k \bar{g}_k$, where $\mathbb{E}_k[\bar{g}_k] = g_k$ and $\mathbb{E}_k[\|\bar{g}_k - g_k\|_2^2] \leq M$
4: **end for**

Not a descent method! ...but *eventual descent in expectation*:

$$f(x_{k+1}) - f(x_k) \leq g_k^T(x_{k+1} - x_k) + \tfrac{1}{2}L\|x_{k+1} - x_k\|_2^2$$
$$= -\alpha_k g_k^T \bar{g}_k + \tfrac{1}{2}\alpha_k^2 L\|\bar{g}_k\|_2^2$$
$$\implies \mathbb{E}_k[f(x_{k+1})] - f(x_k) \leq -\alpha_k\|g_k\|_2^2 + \tfrac{1}{2}\alpha_k^2 L\mathbb{E}_k[\|\bar{g}_k\|_2^2].$$

Markov process: x_{k+1} depends only on x_k and random choice at iteration k.

SG theory

Theorem SG

Since $\mathbb{E}_k[\bar{g}_k] = g_k$ and $\mathbb{E}_k[\|\bar{g}_k - g_k\|_2^2] \leq M$ for all $k \in \mathbb{N}$:

$$\alpha_k = \frac{1}{L} \qquad \implies \mathbb{E}\left[\frac{1}{k}\sum_{j=1}^{k}\|g_j\|_2^2\right] \leq \mathcal{O}(M)$$

$$\alpha_k = \Theta\left(\frac{1}{k}\right) \quad \implies \mathbb{E}\left[\frac{1}{\left(\sum_{j=1}^{k}\alpha_j\right)}\sum_{j=1}^{k}\alpha_j\|g_j\|_2^2\right] \to 0$$

$$\implies \liminf_{k\to\infty}\ \mathbb{E}[\|g_k\|_2^2] = 0$$

SG illustration

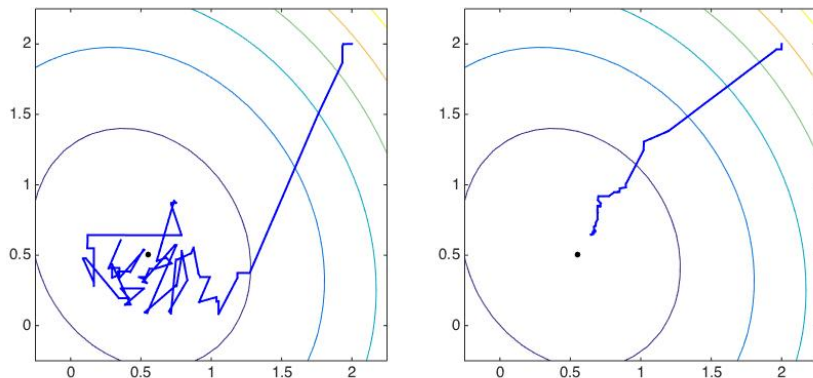Figure: SG with fixed stepsize (left) vs. diminishing stepsizes (right)

Sequential quadratic optimization (SQP)

Consider

$$\min_{x \in \mathbb{R}^n} f(x)$$
$$\text{s.t. } c(x) = 0$$

with $g \equiv \nabla f$, $J \equiv \nabla c$, and H (positive definite on $\text{Null}(J)$), two viewpoints:

$$\begin{bmatrix} g(x) + J(x)^T y \\ c(x) \end{bmatrix} = 0 \qquad \text{or} \qquad \min_{d \in \mathbb{R}^n} f(x) + g(x)^T d + \tfrac{1}{2} d^T H d$$
$$\text{s.t. } c(x) + J(x)d = 0$$

both leading to the same "Newton-SQP system":

$$\begin{bmatrix} H_k & J_k^T \\ J_k & 0 \end{bmatrix} \begin{bmatrix} d_k \\ y_k \end{bmatrix} = - \begin{bmatrix} g_k \\ c_k \end{bmatrix}$$
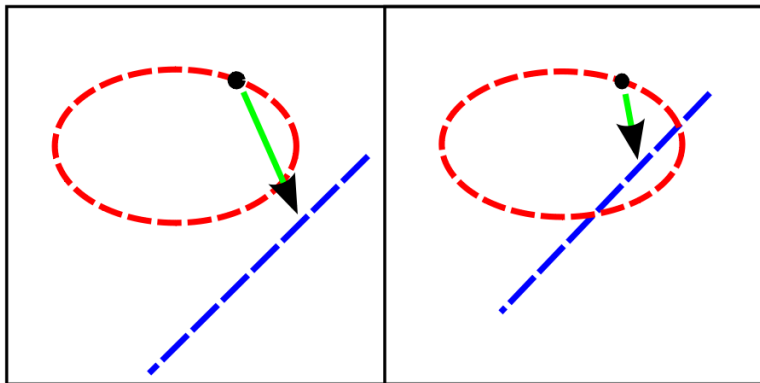
SQP illustration

Figure: Illustrations of SQP subproblem solutions

SQP

- Algorithm guided by merit function, with adaptive parameter τ, defined by

$$\phi(x, \tau) = \tau f(x) + \|c(x)\|_1$$

a model of which is defined as

$$q(x, \tau, d) = \tau(f(x) + g(x)^T d + \tfrac{1}{2} \max\{d^T H d, 0\}) + \|c(x) + J(x)d\|_1$$

- For a given $d \in \mathbb{R}^n$ satisfying $c(x) + J(x)d = 0$, the reduction in this model is

$$\Delta q(x, \tau, d) = -\tau(g(x)^T d + \tfrac{1}{2} \max\{d^T H d, 0\}) + \|c(x)\|_1,$$

and it is easily shown that

$$\phi'(x, \tau, d) \leq -\Delta q(x, \tau, d)$$

SQP with backtracking line search

Algorithm SQP-B

1: choose $x_0 \in \mathbb{R}^n$, $\tau_{-1} \in \mathbb{R}_{>0}$, $\sigma \in (0,1)$, $\eta \in (0,1)$
2: **for** $k \in \{0, 1, 2, \dots\}$ **do**
3:   solve

$$\begin{bmatrix} H_k & J_k^T \\ J_k & 0 \end{bmatrix} \begin{bmatrix} d_k \\ y_k \end{bmatrix} = - \begin{bmatrix} g_k \\ c_k \end{bmatrix}$$

4:   set τ_k to ensure $\Delta q(x_k, \tau_k, d_k) \gg 0$, offered by

$$\tau_k \leq \frac{(1-\sigma)\|c_k\|_1}{g_k^T d_k + \max\{d_k^T H_k d_k, 0\}} \quad \text{if} \;\; g_k^T d_k + \max\{d_k^T H_k d_k, 0\} > 0$$

5:   backtracking line search to ensure $x_{k+1} \leftarrow x_k + \alpha_k d_k$ yields

$$\phi(x_{k+1}, \tau_k) \leq \phi(x_k, \tau_k) - \eta \alpha_k \Delta q(x_k, \tau_k, d_k)$$

6: **end for**

Convergence theory

Assumption

- *f, c, g, and J bounded and Lipschitz*
- *singular values of J bounded below (i.e., the LICQ)*
- $u^T H_k u \geq \zeta \|u\|_2^2$ *for all* $u \in \text{Null}(J_k)$ *for all* $k \in \mathbb{N}$

Theorem SQP-B

- $\{\alpha_k\} \geq \alpha_{\min}$ *for some* $\alpha_{\min} > 0$
- $\{\tau_k\} \geq \tau_{\min}$ *for some* $\tau_{\min} > 0$
- $\Delta q(x_k, \tau_k, d_k) \to 0$ *implies*

$$\|d_k\|_2 \to 0, \quad \|c_k\|_2 \to 0, \quad \|g_k + J_k^T y_k\|_2 \to 0$$

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Toward stochastic SQP

- In a stochastic setting, line searches are (likely) intractable
- However, for ∇f and ∇c, may have Lipschitz constants (or estimates)
- Step #1: Design an adaptive SQP method with

  stepsizes determined by Lipschitz constant estimates

- Step #2: Design a stochastic SQP method on this approach

Primary challenge: Nonsmoothness

In SQP-B, stepsize is chosen based on reducing the merit function.

The merit function is nonsmooth! An upper bound is

$$\phi(x_k + \alpha_k d_k, \tau_k) - \phi(x_k, \tau_k)$$
$$\leq \alpha_k \tau_k g_k^T d_k + |1 - \alpha_k| \|c_k\|_1 - \|c_k\|_1 + \tfrac{1}{2}(\tau_k L_k + \Gamma_k)\alpha_k^2 \|d_k\|_2^2$$

where L_k and Γ_k are Lipschitz constant estimates for f and $\|c\|_1$ at x_k

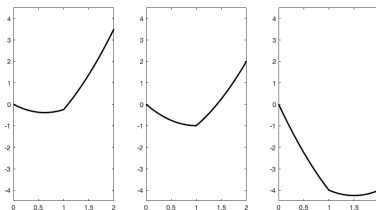

Figure: Three cases for upper bound of ϕ

Idea: Choose α_k to ensure sufficient decrease using this bound

SQP with adaptive stepsizes

Algorithm SQP-A

1: choose $x_0 \in \mathbb{R}^n$, $\tau_{-1} \in \mathbb{R}_{>0}$, $\sigma \in (0, 1)$, $\eta \in (0, 1)$

2: **for** $k \in \{0, 1, 2, \dots\}$ **do**

3:     solve
$$\begin{bmatrix} H_k & J_k^T \\ J_k & 0 \end{bmatrix} \begin{bmatrix} d_k \\ y_k \end{bmatrix} = - \begin{bmatrix} g_k \\ c_k \end{bmatrix}$$

4:     set τ_k to ensure $\Delta q(x_k, \tau_k, d_k) \gg 0$, offered by

$$\tau_k \leq \frac{(1 - \sigma)\|c_k\|_1}{g_k^T d_k + \max\{d_k^T H_k d_k, 0\}} \quad \text{if} \ \ g_k^T d_k + \max\{d_k^T H_k d_k, 0\} > 0$$

5:     set

$$\widehat{\alpha}_k \leftarrow \frac{2(1 - \eta)\Delta q(x_k, \tau_k, d_k)}{(\tau_k L_k + \Gamma_k)\|d_k\|_2^2} \quad \text{and}$$

$$\widetilde{\alpha}_k \leftarrow \widehat{\alpha}_k - \frac{4\|c_k\|_1}{(\tau_k L_k + \Gamma_k)\|d_k\|_2^2}$$

6:     set

$$\alpha_k \leftarrow \begin{cases} \widehat{\alpha}_k & \text{if } \widehat{\alpha}_k < 1 \\ 1 & \text{if } \widetilde{\alpha}_k \leq 1 \leq \widehat{\alpha}_k \\ \widetilde{\alpha}_k & \text{if } \widetilde{\alpha}_k > 1 \end{cases}$$

7:     set $x_{k+1} \leftarrow x_k + \alpha_k d_k$ and continue or update L_k and/or Γ_k and return to step 5

8: **end for**

Convergence theory

Exactly the same as for SQP-B, except different stepsize lower bound

- For SQP-A:

$$\alpha_k = \frac{2(1-\eta)\Delta q(x_k, \tau_k, d_k)}{(\tau_k L_k + \Gamma_k)\|d_k\|_2^2} \geq \frac{2(1-\eta)\kappa_q \tau_{\min}}{(\tau_{-1}\rho L + \rho\Gamma)\kappa_\Psi} > 0$$

- For SQP-B:

$$\alpha_k > \frac{2\nu(1-\eta)\Delta q(x_k, \tau_k, d_k)}{(\tau_k L + \Gamma)\|d_k\|_2^2} \geq \frac{2\nu(1-\eta)\kappa_q \tau_{\min}}{(\tau_{-1}L + \Gamma)\kappa_\Psi} > 0$$

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Stochastic setting

Consider the stochastic problem:

$$\min_{x \in \mathbb{R}^n} \ f(x) \equiv \mathbb{E}[F(x, \omega)]$$
$$\text{s.t. } c(x) = 0$$

Let us assume only the following:

Assumption

For all $k \in \mathbb{N}$, one can compute $\bar{g}_k$ with

$$\mathbb{E}_k[\bar{g}_k] = g_k \quad and \quad \mathbb{E}_k[\|\bar{g}_k - g_k\|_2^2] \leq M$$

Search directions computed by:

$$\begin{bmatrix} H_k & J_k^T \\ J_k & 0 \end{bmatrix} \begin{bmatrix} \bar{d}_k \\ \bar{y}_k \end{bmatrix} = - \begin{bmatrix} \bar{g}_k \\ c_k \end{bmatrix}$$

Important: Given x_k, the values (c_k, J_k, H_k) are deterministic

Stochastic SQP with adaptive stepsizes

(For simplicity, assume Lipschitz constants L and Γ are known.)

Algorithm : Stochastic SQP

1: choose $x_0 \in \mathbb{R}^n$, $\bar{\tau}_{-1} \in \mathbb{R}_{>0}$, $\sigma \in (0, 1)$, $\{\beta_k\} \in (0, 1]$
2: **for** $k \in \{0, 1, 2, \dots\}$ **do**
3:     solve

$$\begin{bmatrix} H_k & J_k^T \\ J_k & 0 \end{bmatrix} \begin{bmatrix} \bar{d}_k \\ \bar{y}_k \end{bmatrix} = - \begin{bmatrix} \bar{g}_k \\ c_k \end{bmatrix}$$

4:     set $\bar{\tau}_k$ to ensure $\Delta \bar{q}(x_k, \bar{\tau}_k, \bar{d}_k) \gg 0$, offered by

$$\bar{\tau}_k \leq \frac{(1 - \sigma)\|c_k\|_1}{\bar{g}_k^T \bar{d}_k + \max\{\bar{d}_k^T H_k \bar{d}_k, 0\}} \quad \text{if} \ \ \bar{g}_k^T \bar{d}_k + \max\{\bar{d}_k^T H_k \bar{d}_k, 0\} > 0$$

5:     set

$$\bar{\bar{\alpha}}_k \leftarrow \frac{\beta_k \Delta \bar{q}(x_k, \bar{\tau}_k, \bar{d}_k)}{(\bar{\tau}_k L + \Gamma)\|\bar{d}_k\|_2^2} \quad \text{and}$$

$$\bar{\bar{\alpha}}_k \leftarrow \bar{\bar{\alpha}}_k - \frac{4\|c_k\|_1}{(\bar{\tau}_k L + \Gamma)\|\bar{d}_k\|_2^2}$$

6:     set

$$\bar{\alpha}_k \leftarrow \begin{cases} \bar{\bar{\alpha}}_k & \text{if } \bar{\bar{\alpha}}_k < 1 \\ 1 & \text{if } \bar{\bar{\alpha}}_k \leq 1 \leq \bar{\bar{\alpha}}_k \\ \bar{\bar{\alpha}}_k & \text{if } \bar{\bar{\alpha}}_k > 1 \end{cases}$$

7:     set $x_{k+1} \leftarrow x_k + \bar{\alpha}_k \bar{d}_k$
8: **end for**

Stepsize control

The sequence $\{\beta_k\}$ allows us to consider, like for SG,

- a fixed stepsize
- diminishing stepsizes (e.g., $\Theta(1/k)$)

Unfortunately, additional control on the stepsize is needed

- too small: insufficient progress
- too large: ruins progress toward feasibility / optimality

We never know when the stepsize is too small or too large!

Idea: Project $\bar{\bar{\alpha}}_k$ and $\bar{\bar{\tilde{\alpha}}}_k$ onto

$$\left[\frac{\beta_k \bar{\tau}_k}{\bar{\tau}_k L + \Gamma}, \frac{\beta_k \bar{\tau}_k}{\bar{\tau}_k L + \Gamma} + \theta \beta_k^2 \right]$$

where $\theta \in \mathbb{R}_{>0}$ is a user-defined parameter

Fundamental lemmas

Lemma

For all $k \in \mathbb{N}$, for any realization of $\bar{g}_k$, one finds

$$\phi(x_k + \bar{\alpha}_k \bar{d}_k, \bar{\tau}_k) - \phi(x_k, \bar{\tau}_k)$$
$$\leq \underbrace{-\bar{\alpha}_k \Delta q(x_k, \bar{\tau}_k, d_k)}_{\mathcal{O}(\beta_k), \text{ "deterministic"}} + \underbrace{\tfrac{1}{2}\bar{\alpha}_k \beta_k \Delta \bar{q}(x_k, \bar{\tau}_k, \bar{d}_k)}_{\mathcal{O}(\beta_k^2), \text{ stochastic/noise}} + \underbrace{\bar{\alpha}_k \bar{\tau}_k g_k^T (\bar{d}_k - d_k)}_{\text{due to adaptive } \bar{\alpha}_k}$$

Lemma

For all $k \in \mathbb{N}$, for any realization of $\bar{g}_k$, one finds

$$\mathbb{E}_k[\bar{d}_k] = d_k, \quad \mathbb{E}_k[\bar{y}_k] = y_k, \quad and \quad \mathbb{E}_k[\|\bar{d}_k - d_k\|_2] = \mathcal{O}(\sqrt{M})$$

as well as

$$g_k^T d_k \geq \mathbb{E}_k[\bar{g}_k^T \bar{d}_k] \geq g_k^T d_k - \zeta^{-1} M \quad and \quad d_k^T H_k d_k \leq \mathbb{E}_k[\bar{d}_k^T H_k \bar{d}_k]$$

Good merit parameter behavior

Lemma

If $\{\bar{\tau}_k\}$ eventually remains fixed at sufficiently small $\tau_{\min} > 0$, then for large k

$$\mathbb{E}_k[\bar{\alpha}_k \bar{\tau}_k g_k^T (\bar{d}_k - d_k)] = \beta_k^2 \tau_{\min} \mathcal{O}(\sqrt{M})$$

Theorem

If $\{\bar{\tau}_k\}$ eventually remains fixed at sufficiently small $\tau_{\min} > 0$, then for large k

$$\beta_k = \Theta(1) \implies \alpha_k = \frac{\tau_{\min}}{\tau_{\min} L + \Gamma} \implies \mathbb{E}\left[\frac{1}{k} \sum_{j=1}^{k} \Delta q(x_j, \tau_{\min}, d_j)\right] \leq \mathcal{O}(M)$$

$$\beta_k = \Theta\left(\frac{1}{k}\right) \implies \mathbb{E}\left[\frac{1}{\left(\sum_{j=1}^{k} \beta_j\right)} \sum_{j=1}^{k} \beta_j \Delta q(x_j, \tau_{\min}, d_j)\right] \to 0$$

Good merit parameter behavior

Lemma

If $\{\bar{\tau}_k\}$ eventually remains fixed at sufficiently small $\tau_{\min} > 0$, then for large k

$$\mathbb{E}_k[\bar{\alpha}_k \bar{\tau}_k g_k^T (\bar{d}_k - d_k)] = \beta_k^2 \tau_{\min} \mathcal{O}(\sqrt{M})$$

Theorem

If $\{\bar{\tau}_k\}$ eventually remains fixed at sufficiently small $\tau_{\min} > 0$, then for large k

$$\beta_k = \Theta(1) \implies \alpha_k = \frac{\tau_{\min}}{\tau_{\min} L + \Gamma} \implies \mathbb{E}\left[\frac{1}{k} \sum_{j=1}^{k} (\|g_j + J_j^T y_j\|_2 + \|c_j\|_2)\right] \le \mathcal{O}(M)$$

$$\beta_k = \Theta\left(\frac{1}{k}\right) \implies \mathbb{E}\left[\frac{1}{\left(\sum_{j=1}^{k} \beta_j\right)} \sum_{j=1}^{k} \beta_j (\|g_j + J_j^T y_j\|_2 + \|c_j\|_2)\right] \to 0$$

Poor merit parameter behavior

$\{\bar{\tau}_k\} \searrow 0$:

- ▶ cannot occur if $\|\bar{g}_k - g_k\|_2$ is bounded uniformly
- ▶ occurs with small probability if distribution of $\bar{g}_k$ has *fast* decay(?)

$\{\bar{\tau}_k\}$ remains too large:

- ▶ can only occur if realization of $\{\bar{g}_k\}$ is *one-sided for all k*
- ▶ if there exists $p \in (0, 1]$ such that, for all k in infinite $\mathcal{K}$,

$$\mathbb{P}_k \left[\bar{g}_k^T \bar{d}_k + \max\{\bar{d}_k^T H_k \bar{d}_k, 0\} \geq g_k^T d_k + \max\{d_k^T H_k d_k, 0\} \right] \geq p$$

then occurs with probability zero

Numerical results

CUTE problems with noise added to gradients with different noise levels

- ▶ Stochastic SQP: 10^3 iterations
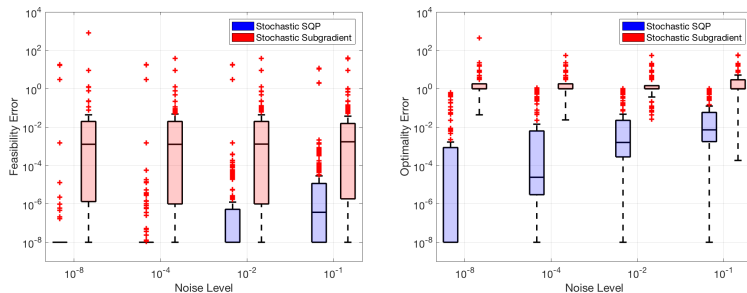- ▶ Stochastic Subgradient: 10^4 iterations and tuned over 11 values of τ

Figure: Box plots for feasibility errors (left) and optimality errors (right).

Outline

Current work: Complexity bounds

Iteration complexity

▶ adaptiveness of merit parameter is the main challenge!

Conjecture: Suppose the algorithm is run

▶ $k_{\max}$ iterations with
▶ $\beta_k = \gamma/\sqrt{k_{\max} + 1}$ and
▶ the merit parameter is reduced at most $s_{\max} \ll k_{\max}$ times.

Let k_* be sampled uniformly over $\{1, \ldots, k_{\max}\}$. With probability $1 - \delta$,

$$\mathbb{E}[\Delta q(x_{k_*})] \leq \frac{\tau_{-1}(f_0 - f_{\inf}) + \|c_0\|_1 + \gamma M^2 + \mathcal{O}(s_{\max} + \log(1/\delta))}{\sqrt{k_{\max} + 1}}$$

Current work: No LICQ

Remove constraint qualification

- ▶ infeasible and/or degenerate problems
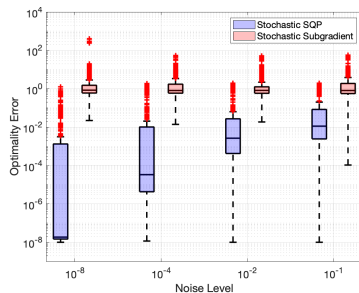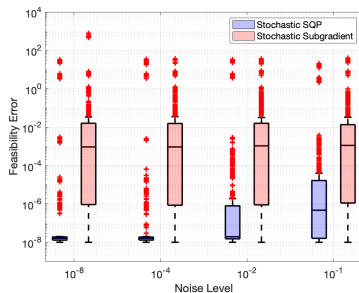- ▶ step decomposition method

Figure: Box plots for feasibility errors (left) and optimality errors (right).

Current work: Matrix-free, inequality constraints

Matrix-free methods

- ▶ inexact subproblem solves
- ▶ stochasticity and inexactness(!)
- ▶ applicable for large-scale, e.g., PDE-constrained

Inequality constraints

- ▶ SQP
- ▶ interior-point

Outline

Summary

Consider *equality constrained* stochastic optimization:

$$\min_{x \in \mathbb{R}^n} \ f(x) \equiv \mathbb{E}[F(x, \omega)]$$
$$\text{s.t. } c_{\mathcal{E}}(x) = 0$$

- *Adaptive* SQP method for deterministic setting
- *Stochastic* SQP method for stochastic setting
- Convergence in expection (comparable to SG for unconstrained setting)
- Numerical experiments are very promising
- Various extensions (coming soon)