

A corrected statement and proof of Lemma 3.19 from the published paper is provided below. Notice that the result stated here is slightly stronger than that stated in the paper due to the presence of the constant $\frac{1}{2}H_{Lip}$ in place of H_{Lip} . This means that the inequality in the published paper is correct, but not as tight as it could have been. Inclusion of the $\frac{1}{2}$ factor would have tightened this bound in the paper and in the subsequent results that use the bound from this lemma.

Lemma 3.19 *For all $k \in \mathcal{A}_\sigma$, the accepted step s_k satisfies*

$$\|s_k\|_2 \geq (\frac{1}{2}H_{Lip} + \sigma_{\max})^{-1/2} \|g_{k+1}\|_2^{1/2}.$$

Proof Let $k \in \mathcal{A}$. It follows with (2.1a) that

$$\begin{aligned} \|g_{k+1}\|_2 &= \|g(x_k + s_k) - (g_k + (H_k + \lambda_k I)s_k)\|_2 \\ &\leq \|g(x_k + s_k) - (g_k + H_k s_k)\|_2 + \lambda_k \|s_k\|_2. \end{aligned} \quad (\star)$$

By Taylor's theorem, the first term on the right-hand side of this inequality satisfies

$$\begin{aligned} \|g(x_k + s_k) - (g_k + H_k s_k)\|_2 &\leq \left\| \int_0^1 (H(x_k + \tau s_k) - H_k) s_k d\tau \right\|_2 \\ &\leq \int_0^1 \|H(x_k + \tau s_k) - H_k\|_2 d\tau \cdot \|s_k\|_2 \\ &\leq \int_0^1 \tau d\tau \cdot H_{Lip} \|s_k\|_2^2 = \frac{1}{2} H_{Lip} \|s_k\|_2^2, \end{aligned}$$

which, with (\star) and the fact that $\lambda_k \leq \sigma_k \|s_k\|_2$ for all $k \in \mathcal{A}_\sigma$, implies that

$$\begin{aligned} \|g_{k+1}\|_2 &\leq \frac{1}{2} H_{Lip} \|s_k\|_2^2 + \left(\frac{\lambda_k}{\|s_k\|_2} \right) \|s_k\|_2^2 \\ &\leq (\frac{1}{2} H_{Lip} + \sigma_k) \|s_k\|_2^2. \end{aligned}$$

This, along with Lemma 3.18, implies the result. \square