Integrated Location Routing and Scheduling Problems

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Outline

LRS Problem

LRS problem integrates the decisions of determining

- the optimal number and locations of facilities,
- an optimal set of vehicle routes from facilities to customers
- an optimal assignment of routes to vehicles subject to scheduling constraints.

The objective is to minimize the total fixed costs and operating costs of facilities and vehicles.

Motivation

- For multiple customer routes, location and routing are interdependent.
- Assuming one-to-one relationship between vehicles and routes overestimates the required number of vehicles and costs.
  -Fixed costs of vehicles and drivers are high.
  -Companies might have fleets with constant size.
  -Delivery of items might be time sensitive and drivers might have working hour limits.

LRS Problem in Literature

- Integrates other NP-Hard problems such as CFLP, CVRP and MDVSP.
- Generalizes problems such as LRP, MDVRP, VRPMT.
- Lin et al. [2002] introduce the LRS problem.
- They divide the problem into 3 phases: facility location, vehicle routing and loading, and construct a heuristic algorithm which includes some metaheuristics.
- Lin and Kwok [2005] extend the study to a multi-objective LRS problem. They use a similar heuristic algorithm.

Problem Definition

Objective

to select a subset of the facilities, construct a set of delivery routes and to assign routes to vehicles with minimum total cost.

Constraints

- Capacitated facilities.
- Capacitated vehicles.
- Time limit for the vehicles.
- Each customer must be visited exactly once.
- Each route and vehicle must start at a facility and return to the same facility.

2 Formulations: Edge-based linear mixed integer model, set-partitioning based linear integer model

Pairing Concept

Pairing:

A set of routes that can be served sequentially by one vehicle within the vehicle's working hour limit.

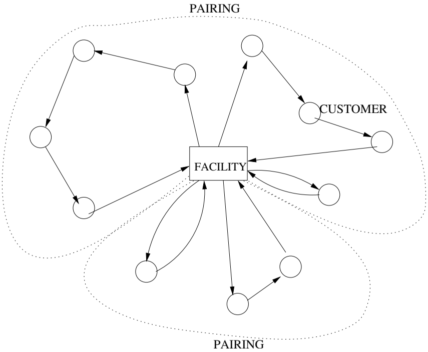

Figure: Example of pairings

A pairing is feasible if

- total demand of each route $\leq$ vehicle capacity,
- total travel time of the pairing $\leq$ vehicle working hour limit,
- each customer included in the pairing is visited once.

Set Partitioning-based model: Notation

Sets

$$N = \text{set of demand nodes}$$
$$M = \text{set of candidate facility locations}$$
$$P_j = \text{set of all feasible pairings for facility } j, \forall j \in M$$

Parameters

$$a_{ipj} = \begin{cases} 1 & \text{if demand node } i \text{ is in pairing } p \text{ of facility } j, \forall i \in N, j \in M, p \in P_j \\ 0 & \text{otherwise} \end{cases}$$
$$C_{jp} = \text{cost of pairing } p \text{ associated with facility } j, \forall j \in M, p \in P_j$$
$$FC_j = \text{fixed cost of opening facility } j, \forall j \in M$$
$$CapF_j = \text{capacity of facility } j, \forall j \in M$$

Decision Variables

$$Z_{jp} = \begin{cases} 1 & \text{if pairing p is selected for facility j, } \forall p \in P_j \text{ and } j \in M \\ 0 & \text{otherwise} \end{cases}$$
$$T_j = \begin{cases} 1 & \text{if facility j is selected, } \forall j \in M \\ 0 & \text{otherwise} \end{cases}$$

Set Partitioning Formulation

(SPP-LRS)

$$\text{Minimize} \quad \sum_{j \in M} FC_j T_j + \sum_{j \in M} \sum_{p \in P_j} C_{jp} Z_{jp} \qquad (1)$$

$$\text{subject to} \quad \sum_{j \in M} \sum_{p \in P_j} a_{ipj} Z_{jp} = 1 \qquad \forall i \in N \qquad (\pi_{\mathbf{i}}) \qquad (2)$$

$$\sum_{p \in P_j} \sum_{i \in N} a_{ipj} d_i Z_{jp} \leq CapF_j T_j \qquad \forall j \in M \qquad (\mu_{\mathbf{j}}) \qquad (3)$$

$$Z_{jp} \in \{0, 1\} \qquad \forall j \in M, \forall p \in P_j \qquad (4)$$

$$T_j \in \{0, 1\} \qquad \forall j \in M \qquad (5)$$

Simple Valid Inequalities

$$\sum_{j \in M} T_j \geq nREQ \tag{6}$$

$$\sum_{p \in P_j} a_{ipj} Z_{jp} \leq T_j \quad \forall j \in M, i \in N \quad (\sigma_{\mathbf{ji}}) \tag{7}$$

nREQ= minimum number of required (selected) facilities

Branch and Price Algorithm

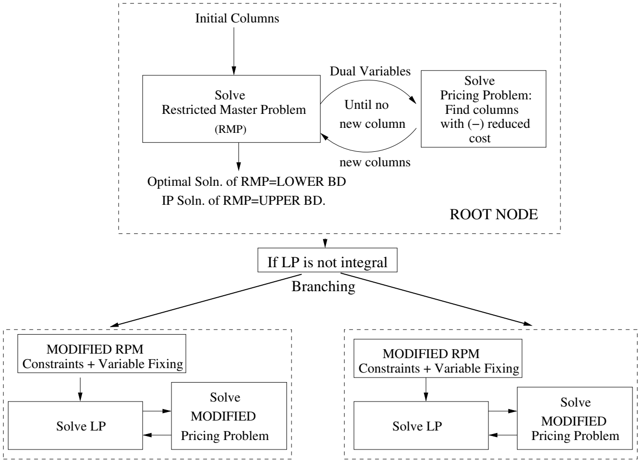

Figure: Branch-and-Price Tree

Pricing Problem as a Network Problem

Objective: Find a column with minimum reduced cost.
Constraints:

- All of the routes in the pairing start and end at the same facility.
- Total demand of each route $\leq$ vehicle capacity.
- Total travel time of the pairing $\leq$ time limit.
- Each customer node can be visited at most once.

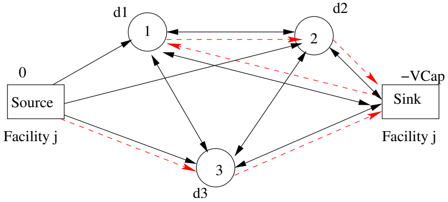

Figure: Constructed Network for facility j

- Can define the problem as an ESPPRC (not elementary wrt sink).
- Exact solution: labeling algorithm by Feillet et al. [2004] for ESPPRC.
- Heuristic solution: labeling algorithm with a LIMIT on the number of labels for each node.

Branching Rules

1. Branching on the facility location variables, OPEN \ CLOSED.
   Simple, no need to update the pricing problems.
2. Integrality of total # of vehicles at each facility.
   Just a fixed cost change in the total reduced cost of a column.
3. A customer can only be assigned to 1 facility, FORCE\ FORBID a customer for a facility.
   Add constraints dual variables of which can be easily incorporated into the pricing problem.
4. Modified Ryan and Foster branching rule suggested by Desrochers and Soumis [1989].
   Update the arc costs in pricing problem.

Branch and Price Algorithm

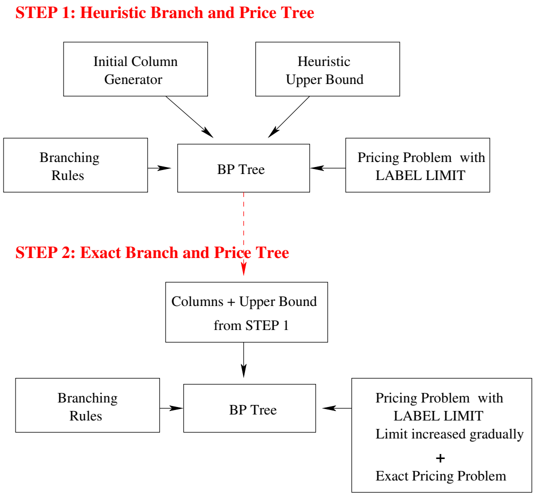

Figure: Branch and Price Algorithm

Implementation and Test Problems

- MINTO 3.1 and CPLEX 9.1.
- Initial columns and upper bound for heuristic BP are generated using a facility location heuristic, some VRP heuristics and a bin packing heuristic.
- Step 1 of the algorithm (Heuristic BP) is run for 3 CPU hours.
- Step 2 of the algorithm (Exact BP) is run for 6 CPU hours.
- 6 set of customer and candidate facility locations, customer demands generated from MDVRP benchmark problems developed by Cordeau et al. [1995].
- For each set of locations, 2 possible vehicle capacity, 2 possible time limit values are used. Therefore, 24 instances with 30 customers.
- Facility and vehicle fixed costs, vehicle operating cost are estimated.

Computational Results: 2-step Branch and Price Algorithm

Data	LP	IP	% Int. Gap	CPU Time		Solution	
				Step 1	Step 2	# of Fac.	# of veh.
p01-f-11	4821.07	4877.33	0.00%	1.36 hr	5.49 hr	2	4
p01-f-12	4715.3	4987.17	2.83%	3 hr	6 hr	2	4
p01-f-21	4582.27	4684.67	0.01%	3 hr	6 hr	2	4
p01-f-22	4482.2	4885	8.25%	3 hr	6 hr	2	4
p03-f-11	6467.35	6504.83	0.00%	1.76 hr	0.89 hr	3	4
p03-f-12	6370.12	6497.33	0.94%	3 hr	6 hr	3	4
p03-f-21	6263.17	6420.67	0.55%	2.5 hr	6 hr	3	4
p03-f-22	6193.52	6196.5	0.05%	0.82 hr	6 hr	3	3
p03-l-11	4896.04	5014.67	0.00%	1.58 hr	0.54 hr	2	5
p03-l-12	4756.95	5026.83	4.12%	3 hr	6 hr	2	5
p03-l-21	4679.23	4730.17	0.00%	1.43 hr	0.67 hr	2	4
p03-l-22	4562.29	4706.33	1.28%	2.4 hr	6 hr	2	4
p07-f-11	4765.63	4808.33	0.00%	1.82 hr	1.06 hr	2	4
p07-f-12	4654.19	4760.17	0.01%	2.65 hr	6 hr	2	4
p07-f-21	4558.21	4683.5	0.00%	1.74 hr	2.56 hr	2	4
p07-f-22	4453.2	4745	5.77%	3 hr	6 hr	2	4
p11-f-11	7915.87	8054.67	0.00%	1 min	1.19 min	3	7
p11-f-12	7791.1	8036.33	0.00%	2.32 min	4.17 min	3	7
p11-f-21	7531.81	7634.67	0.00%	0.4 min	0.21 min	3	6
p11-f-22	7469.84	7634.67	0.00%	0.85 min	0.47 min	4	7
p11-l-11	9929.75	10043	0.00%	0.25 min	0.18 min	4	7
p11-l-12	9837.09	9937.33	0.00%	0.42 min	0.31 min	4	7
p11-l-21	9645.66	9758.17	0.00%	0.8 min	0.74 min	4	7
p11-l-22	9416.78	9500.67	0.00%	1.08 min	0.75 min	4	6

Future Work

- Improve the algorithm to be able to solve larger instances.
- Investigate how to incorporate cuts into the algorithm, develop a branch, cut and price algorithm.
- Update the algorithm to solve the special cases of the LRS problem such as LRP, MDVRP, VRPMT.

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