The Benefit of Time-Dependent Ordering Policies Under Cyclic Supply Disruptions

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Outline

1. Motivation
2. Problem Formulation
3. Ordering Policies
4. Benefits of Time-Dependent Policies
5. Conclusions and Extensions
The Impact of Supplier Disruptions

key questions when we face stationary disruptions:

- What’s the optimal stationary policy?
- When to order?
- How much to order?
Seasonal Frequency of Hurricanes

Fig. 4. Number of tropical storms and hurricanes (open curve) and hurricanes (solid curve) observed on each day, 1 May–31 December, 1886–1980 (from Neumann et al., 1981).

(Source: Gray, Monthly Weather Review, 1984.)
Time-Dependent Parameters

- Failure rate: \( f(t) = \bar{f} \cdot (1 - RA_f \cdot \cos(2\pi(t + \phi_f))) \)
Key Questions for Time Dependent Systems

- Should we use a stationary or a time-dependent inventory ordering policy?
- What kinds of time-dependent policies should be used?
- What if we have bad estimates of the parameters?
Literature Review

- Zero Inventory Ordering (EOQ) Policy:

- (Q,r) Type Policy:

- Non-Stationary Random Disruption Probability:
PSA vs. SSA

- **PSA**: Pointwise Stationary Approximation
  - Estimate the performance measures at each time $t$ by taking the arrival and service rates at time $t$, then using those to find the steady-state performance measures
  - Best if rates change slowly

- **SSA**: Simple Stationary Approximation
  - Only consider the average arrival and service rate, then treat the system as always in steady-state with those average rates
  - Best if rates change quickly

We explore the middle ground between PSA and SSA.
Assumptions

- The supplier status is either Up or Down
- No leadtime
- Poisson demand with arrival rate $\lambda$
- Placing an order occurs a fixed cost $K$
- Holding cost $h$
- Lost sales with stockout cost $p$
- ZIO policies are not optimal under disruptions
  - In our paper, we use an $(r, Q)$-type policy ($r \geq 0$)
  - In this presentation, we demonstrate only ZIO policies
Motivation

Problem Formulation

Ordering Policies

Benefits of Time-Dependent Policies

Conclusions and Extensions

The Assumptions and IL Distribution

Cost Equation

Inventory Level Distribution

- Time-dependent joint distribution of IL, conditioning on supplier status:

\[
Pr(x, s; t) \equiv Pr(IL(t) = x \text{ and } S(t) = s \text{ at time } t)
\]

- Marginal distribution of IL:

\[
Pr(x; t) = Pr(x, U; t) + Pr(x, D; t)
\]

- We apply Kolmogorov Ordinary Differential Equations (ODEs) to obtain \(Pr(x, s; t)\) numerically.
IL Distribution: Movie

- Cyclic order size with 10% relative amplitude (RA)
- Constant demand rate
- No supply disruptions
Cost-Related Output Functions

- Expected number of items in inventory at time $t$:
  \[ L(t) \equiv E[IL(t)] \]

- Probability that the system is empty at time $t$:
  \[ \alpha(t) \equiv Pr(0, D; t) \]

- Instantaneous order rate at time $t$:
  \[ \beta(t) \equiv Pr(1, U; t)\lambda(t) + Pr(0, D; t)r \]
Using output functions from previous slide, expected cost between $t_1$ and $t_2$:

$$C(t_1, t_2) = \int_{t_1}^{t_2} [L(t)h + \alpha(t)\lambda(t)p + \beta(t)K] dt$$
## Summary of Policy Types

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<td>EOQ-SSA</td>
<td>EOQD-PSA</td>
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<td>EOQD-SSA</td>
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<td><strong>Tunable</strong></td>
<td>Q-nt (1 param)</td>
<td>EOQD-PSA-ph (1 param)</td>
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<tr>
<td></td>
<td>Q-t (3 params)</td>
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</tbody>
</table>
Use average value of time-variant parameters, e.g., supplier failure rate and/or demand rate (SSA):

- **EOQ-SSA**: Constant order quantity using EOQ formula
- **EOQD-SSA**: Constant order quantity using EOQ with disruption (EOQD) approximation developed by Snyder (2006) based on Parlar and Berkin (1991) and Berk and Arreola-Risa (1994)
Time-Independent Policy with Tunable Parameters

Use complete information about the time-variant parameters:

- **Q-nt**: The best constant order quantity under time-variant system, accounting for evolution of parameters over time (found numerically)
Calculate order quantity at every point in time using instantaneous rate of time-variant parameters (PSA):

- **EOQD-PSA**: Plug instantaneous failure and/or demand rates into EOQD formula to get instantaneous order quantity
- **EOQD-PSA-f**: Use sinusoid to fit EOQD-PSA order function
Time-Dependent Policies with Tunable Parameters

Use complete information about time-variant parameters:

- **EOQD-PSA-ph**: Adjust and optimize the phase of EOQD-PSA to compensate for lag
- **Q-t**: Let order quantity be sinusoid and optimize average order quantity, relative amplitude (RA), and phase
Optimal Order Quantity and Failure Rate (Q-t policy)

- RA of order quantity is much less than RA of failure rate
- Order quantity and failure rate don’t have the same phase

- Lag delay of disruption probability
- Peak of order rate ($\beta(t)$) is valley of disruption probability
Impact of Failure Rate RA on Total Cost

- Time-independent policy is almost entirely insensitive to fluctuations in failure rate as long as demand rate is constant.
- Time-dependent policy can utilize fluctuations in failure rate to decrease total cost.
Impact of Demand Rate RA on Total Cost

- Cost of Q-t policy decreases with demand RA
- Cost of all other policies increases eventually
- Cost of time-independent policies is not constant
We investigate robustness of Q-t and Q-nt policies in two cases:

- Failure rate RA and/or phase are estimated badly
- Distribution of repair time is assumed exponential but is actually phase-type
  - Less variable: Erlang-2
  - More variable: Hyperexponential-2

In both cases, Q-t (time-variant) policy outperforms Q-nt (time-invariant) policy in most trials.
One Example of Robustness: Failure RA and Phase

![Graph showing cost comparison]

- Highest cost under optimal Q−t
- Average cost under optimal Q−nt
- Average cost under optimal Q−t
- Lowest cost under optimal Q−t

**Graph Details**
- **X-axis**: Stockout Cost
- **Y-axis**: Total Cost
- Key points:
  - Highest cost
  - Average cost
  - Lowest cost

**Graph Title**
One Example of Robustness: Failure RA and Phase

**Graph Source**
Rong, Ross, Snyder

**Graph Content**
Supply Disruption with Time-Dependent Parameters
Conclusions

- In most cases, tunable, time-variant policies dominate:
  - Q-t policy performs best but is more numerically cumbersome
  - EOQD-PSA-ph balances cost and complexity
- EOQD-PSA-ph and Q-t policies utilize cyclic nature of failure rate to decrease cost
  - Q-t policy can achieve lower cost even as RA of demand rate increases
- Stationary policies are insensitive to fluctuations in failure rate
- EOQD is a good approximation to Q-nt policy.
  - Don’t need Q-nt-PSA-ph policy since we get rid of one parameter to optimize
Extensions and Future Research

- Does linear additivity property hold?
  - (Suppose there are several disruption sources with different sinusoids. Can we just optimize based on every individual cycle and take a linear combination? Or do we need to optimize simultaneously?)
- Non-zero leadtimes
- Phase-type demand arrivals
- Yield uncertainty
- Backorders
Further details can be found in:


Preprint available at

- [www.lehigh.edu/~lvs2/research.html](http://www.lehigh.edu/~lvs2/research.html)
## Computational Study: Base Settings

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<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Basic Setting</th>
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<tr>
<td>$K$</td>
<td>Fixed cost</td>
<td>$31$/order</td>
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<tr>
<td>$h$</td>
<td>Holding cost</td>
<td>$1$/item/year</td>
</tr>
<tr>
<td>$p$</td>
<td>Stockout cost</td>
<td>$11$/item</td>
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<tr>
<td>$r$</td>
<td>Repair rate</td>
<td>12/year</td>
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<tr>
<td>$\bar{f}$</td>
<td>Failure rate, mean</td>
<td>1/year</td>
</tr>
<tr>
<td>$RA_f$</td>
<td>Failure rate, relative amplitude</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi_f$</td>
<td>Failure rate, phase</td>
<td>0 years</td>
</tr>
<tr>
<td>$\bar{\lambda}$</td>
<td>Demand rate, mean</td>
<td>100/year</td>
</tr>
<tr>
<td>$RA_d$</td>
<td>Demand rate, relative amplitude</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>Demand rate, phase</td>
<td>0 years</td>
</tr>
</tbody>
</table>
Time-Independent Ordering Policies

- **EOQ:**
  \[
  Q(t) = EOQ(t) = \sqrt{2K\bar{\lambda}/h}
  \]

- **EOQD:**
  \[
  Q^* = \frac{\sqrt{(\rho\lambda h)^2 + 2h\mu(K\lambda\mu + \lambda^2 p\rho)} - \rho\lambda h}{h\mu}
  \]

- **Q-nt:** Numerically find the optimal constant Q value, evaluating the overall cost using the time-varying demand and failure rates.
Time Dependent Policy without Tunable Parameters

EOQD-PSA: Let $\rho(t) \equiv f(t)/(f(t) + r)$.

$$EOQD_{PSA}(t) = \sqrt{(\rho(t)\lambda(t)h)^2 + 2hr \cdot (K\lambda(t)r + \lambda(t)^2 p\rho(t))} / hr$$

$$- \frac{\rho(t)\lambda(t)h}{hr}$$

EOQD-PSA-f:

$$EOQD_{PSA-t}(t) = \frac{EOQD_{PSA} + EOQD_{PSA}}{2}$$

$$\left(1 - \frac{EOQD_{PSA} - EOQD_{PSA}}{EOQD_{PSA} + EOQD_{PSA}} \cos(2\pi t)\right)$$

Rong, Ross, Snyder  Supply Disruption with Time-Dependent Parameters
Time Dependent Policy with Tunable Parameters

- **Q-t:**
  \[ Q(t) = \bar{Q} \cdot (1 - RA_q \cos(2\pi(t + \phi_q))) \]

- **EOQD-PSA-ph:**
  \[ Q(t) = EOQD_{PSA}(t + \phi_q) \]
Here we need to point that time dependent policy can turn the time dependent system into our favor even if there is more disruption can happen.
Impact of Stockout Cost on Total Cost

The optimality of different policies

- EOQD−PSA−f
- EOQD−PSA
- EOQD−SSA
- EOQ−SSA
- Q−nt
- EOQD−PSA−ph
- Q−t

Stockout Cost vs. Total Cost graph showing the performance of different policies.