The Benefit of Time-Dependent Ordering Policies Under Cyclic Supply Disruptions

Ying Rong[1] Andrew Ross[2] Lawrence Snyder[1]

[1]Department of Industrial & Systems Engineering
Center for Value Chain Research
Lehigh University

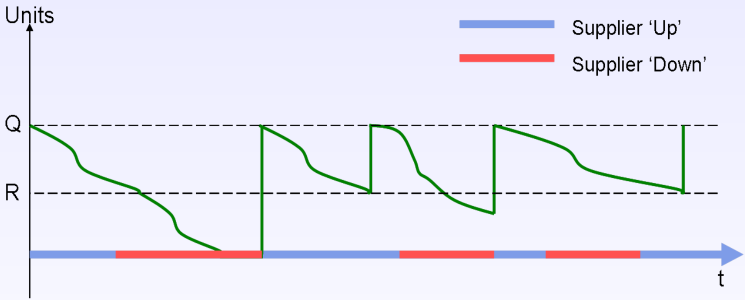
[2]Department of Mathematics
Eastern Michigan University

INFORMS, 2006

Outline

1. Motivation
2. Problem Formulation
3. Ordering Policies
4. Benefits of Time-Dependent Policies
5. Conclusions and Extensions

The Impact of Supplier Disruptions



Some key questions when we face stationary disruptions:

- What's the optimal stationary policy?
- When to order?
- How much to order?

Seasonal Frequency of Hurricanes

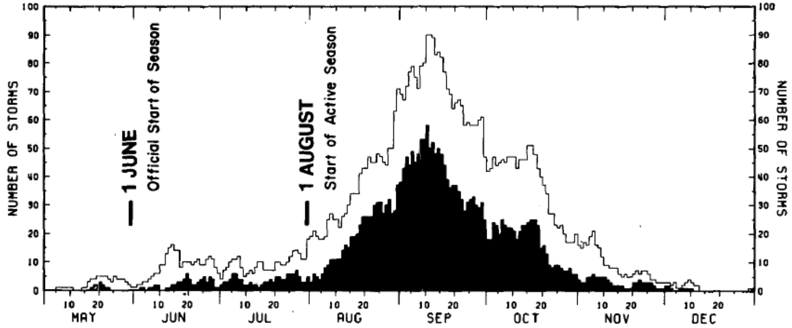
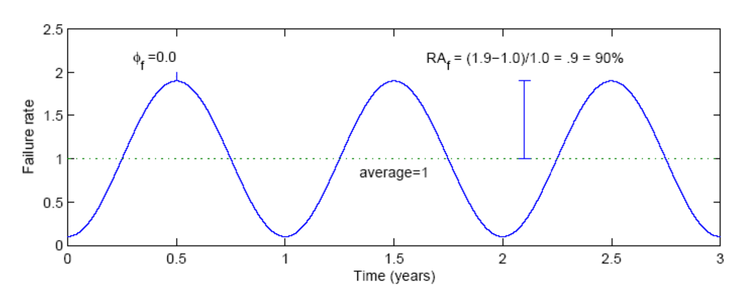


FIG. 4. Number of tropical storms and hurricanes (open curve) and hurricanes (solid curve) observed on each day, 1 May–31 December, 1886–1980 (from Neumann *et al.*, 1981).

(Source: Gray, *Monthly Weather Review*, 1984.)

Time-Dependent Parameters



- Failure rate: $f(t) = \bar{f} \cdot (1 - RA_f \cdot \cos(2\pi(t + \phi_f)))$

Key Questions for Time Dependent Systems

- Should we use a stationary or a time-dependent inventory ordering policy?
- What kinds of time-dependent policies should be used?
- What if we have bad estimates of the parameters?

Literature Review

- Zero Inventory Ordering (EOQ) Policy:
  - Parlar and Berkin (1991), Berk and Arreola-Risa (1994), Snyder (2006)
- (Q,r) Type Policy:
  - Parlar and Perry (1995), Gupta (1996), Parlar (1997), Mohebbi (2003), Heimann and Waage (2006)

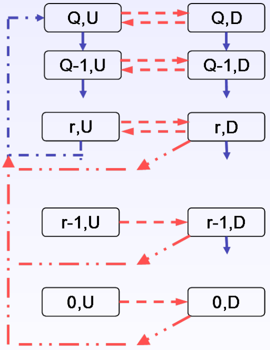
- Non-Stationary Random Disruption Probability:
  - Tomlin and Snyder (2006), Li, Xu and Hayya (2004)

PSA vs. SSA

- PSA: Pointwise Stationary Approximation
    - Estimate the performance measures at each time t by taking the arrival and service rates at time t, then using those to find the steady-state performance measures
    - Best if rates change slowly
- SSA: Simple Stationary Approximation
    - Only consider the average arrival and service rate, then treat the system as always in steady-state with those average rates
    - Best if rates change quickly

We explore the middle ground between PSA and SSA.

Assumptions



- The supplier status is either Up or Down
- No leadtime
- Poisson demand with arrival rate λ
- Placing an order occurs a fixed cost K
- Holding cost h
- Lost sales with stockout cost p
- ZIO policies are not optimal under disruptions
    - In our paper, we use an (r, Q)-type policy $(r \geq 0)$
    - In this presentation, we demonstrate only ZIO policies

Inventory Level Distribution

- Time-dependent joint distribution of IL, conditioning on supplier status:

$$\Pr(x, s; t) \equiv \Pr(IL(t) = x \text{ and } S(t) = s \text{ at time } t)$$

- Marginal distribution of IL:

$$\Pr(x; t) = \Pr(x, U; t) + \Pr(x, D; t)$$

- We apply Kolmogorov Ordinary Differential Equations (ODEs) to obtain $\Pr(x, s; t)$ numerically.

IL Distribution: Movie

- Cyclic order size with 10% relative amplitude (RA)
- Constant demand rate
- No supply disruptions

Cost-Related Output Functions

- Expected number of items in inventory at time t:

$$L(t) \equiv E[IL(t)]$$

- Probability that the system is empty at time t:

$$\alpha(t) \equiv \Pr(0, D; t)$$

- Instantaneous order rate at time t:

$$\beta(t) \equiv \Pr(1, U; t)\lambda(t) + \Pr(0, D; t)r$$

Cost Equation

- Using output functions from previous slide, expected cost between t_1 and t_2:

$$C(t_1, t_2) = \int_{t_1}^{t_2} [L(t)h + \alpha(t)\lambda(t)p + \beta(t)K] \, dt$$

Summary of Policy Types

	Time-Independent	Time-Dependent
Non-Tunable	EOQ-SSA EOQD-SSA	EOQD-PSA EOQD-PSA-t
Tunable	Q-nt (1 param)	EOQD-PSA-ph (1 param) Q-t (3 params)

Time-Independent Policies without Tunable Parameters

Use average value of time-variant parameters, e.g., supplier failure rate and/or demand rate (SSA):

- EOQ-SSA: Constant order quantity using EOQ formula
- EOQD-SSA: Constant order quantity using EOQ with disruption (EOQD) approximation developed by Snyder (2006) based on Parlar and Berkin (1991) and Berk and Arreola-Risa (1994)

Time-Independent Policy with Tunable Parameters

Use complete information about the time-variant parameters:

- Q-nt: The best constant order quantity under time-variant system, accounting for evolution of parameters over time (found numerically)

Time-Dependent Policies without Tunable Parameters

Calculate order quantity at every point in time using instantaneous rate of time-variant parameters (PSA):

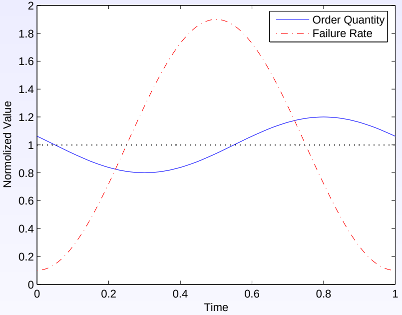
- EOQD-PSA: Plug instantaneous failure and/or demand rates into EOQD formula to get instantaneous order quantity
- EOQD-PSA-f: Use sinusoid to fit EOQD-PSA order function

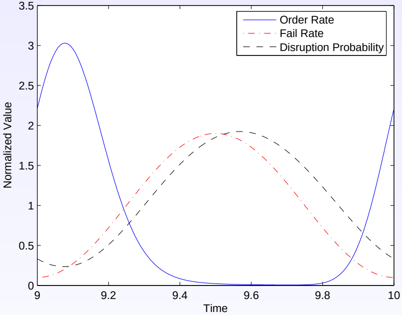
Time-Dependent Policies with Tunable Parameters

Use complete information about time-variant parameters:

- EOQD-PSA-ph: Adjust and optimize the phase of EOQD-PSA to compensate for lag
- Q-t: Let order quantity be sinusoid and optimize average order quantity, relative amplitude (RA), and phase

Optimal Order Quantity and Failure Rate (Q-t policy)

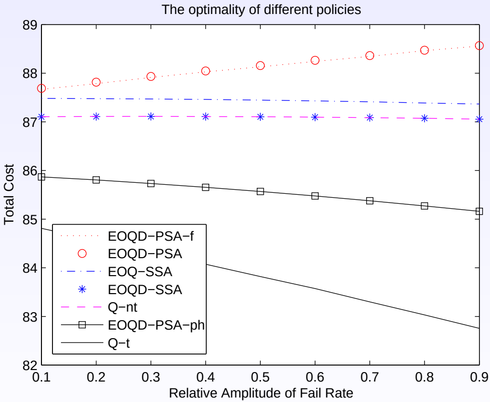




- RA of order quantity is much less than RA of failure rate
- Order quantity and failure rate don't have the same phase

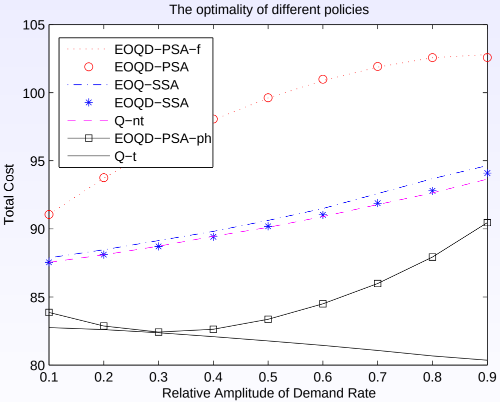
- Lag delay of disruption probability
- Peak of order rate ($\beta(t)$) is valley of disruption probability

Impact of Failure Rate RA on Total Cost



- Time-independent policy is almost entirely insensitive to fluctuations in failure rate as long as demand rate is constant
- Time-dependent policy can utilize fluctuations in failure rate to decrease total cost

Impact of Demand Rate RA on Total Cost



- Cost of Q-t policy decreases with demand RA
- Cost of all other policies increases eventually
- Cost of time-independent policies is not constant

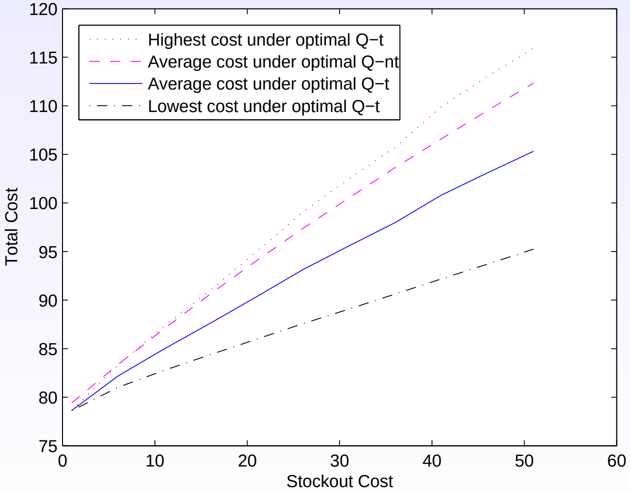
Robustness of Q-t Policy

We investigate robustness of Q-t and Q-nt policies in two cases:

- Failure rate RA and/or phase are estimated badly
- Distribution of repair time is assumed exponential but is actually phase-type
  - Less variable: Erlang-2
  - More variable: Hyperexponential-2

In both cases, Q-t (time-variant) policy outperforms Q-nt (time-invariant) policy in most trials.

One Example of Robustness: Failure RA and Phase



Conclusions

- In most cases, tunable, time-variant policies dominate:
  - Q-t policy performs best but is more numerically cumbersome
  - EOQD-PSA-ph balances cost and complexity
- EOQD-PSA-ph and Q-t policies utilize cyclic nature of failure rate to decrease cost
  - Q-t policy can achieve lower cost even as RA of demand rate increases
- Stationary policies are insensitive to fluctuations in failure rate
- EOQD is a good approximation to Q-nt policy.
  - Don't need Q-nt-PSA-ph policy since we get rid of one parameter to optimize

Extensions and Future Research

- Does linear additivity property hold?
  - (Suppose there are several disruption sources with different sinusoids. Can we just optimize based on every individual cycle and take a linear combination? Or do we need to optimize simultaneously?)
- Non-zero leadtimes
- Phase-type demand arrivals
- Yield uncertainty
- Backorders

Our Paper

Further details can be found in:

- Ross, A. M., Y. Rong, and L. V. Snyder. Supply disruptions with time-dependent parameters. Forthcoming in *Computers and Operations Research*

Preprint available at

- `www.lehigh.edu/∼lvs2/research.html`

Computational Study: Base Settings

Parameter	Definition	Basic Setting
K	Fixed cost	\$31/order
h	Holding cost	\$1/item/year
p	Stockout cost	\$11/item
r	Repair rate	12/year
$\bar{f}$	Failure rate, mean	1/year
RA_f	Failure rate, relative amplitude	0.9
ϕ_f	Failure rate, phase	0 years
$\bar{\lambda}$	Demand rate, mean	100/year
RA_d	Demand rate, relative amplitude	0
ϕ_d	Demand rate, phase	0 years

Time-Independent Ordering Policies

- EOQ:

$$Q(t) = EOQ(t) = \sqrt{2K\bar{\lambda}/h}$$

- EOQD:

$$Q^* = \frac{\sqrt{(\rho\lambda h)^2 + 2h\mu(K\lambda\mu + \lambda^2 p\rho)} - \rho\lambda h}{h\mu}$$

- Q-nt: Numerically find the optimal constant Q value, evaluating the overall cost using the time-varying demand and failure rates.

Time Dependent Policy without Tunable Parameters

- EOQD-PSA: Let $\rho(t) \equiv f(t)/(f(t)+r)$.

$$EOQD_{PSA}(t) = \frac{\sqrt{(\rho(t)\lambda(t)h)^2 + 2hr \cdot (K\lambda(t)r + \lambda(t)^2 p\rho(t))}}{hr} - \frac{\rho(t)\lambda(t)h}{hr}$$

- EOQD-PSA-f:

$$EOQD_{PSA-t}(t) = \frac{\overline{EOQD}_{PSA} + \underline{EOQD}_{PSA}}{2} \left(1 - \frac{\overline{EOQD}_{PSA} - \underline{EOQD}_{PSA}}{\overline{EOQD}_{PSA} + \underline{EOQD}_{PSA}} \cos(2\pi t)\right)$$

Time Dependent Policy with Tunable Parameters

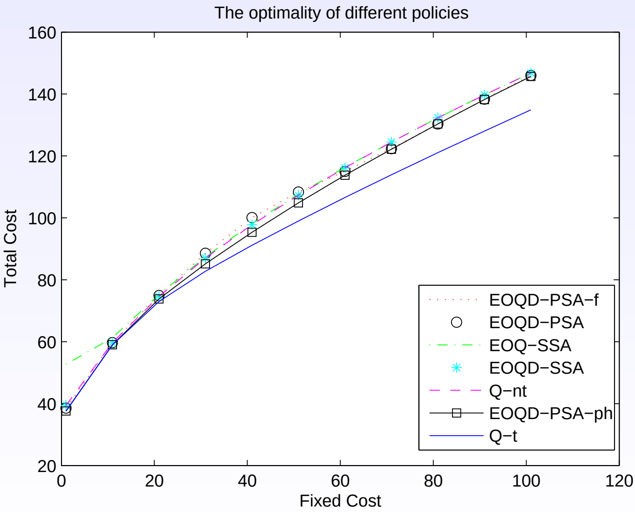
- Q-t:

$$Q(t) = \bar{Q} \cdot (1 - RA_q \cos(2\pi(t + \phi_q)))$$

- EOQD-PSA-ph:

$$Q(t) = EOQD_{PSA}(t + \phi_q)$$

Impact of Fixed Cost on Total Cost



Impact of Stockout Cost on Total Cost