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OPTIMIZING WAVE FARM LAYOUTS UNDER UNCERTAINTY

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ABSTRACT

Ocean wave energy represents a large untapped source of energy in the U.S., with the advantage of being in close proximity to the coastal load centers in the U.S., which makes the transmission of the generated energy more efficient than, say, wind farms located in the remote geographical center of the country. Wave energy is more predictable and stable than wind and solar energy. Optimal harvest of this potentially vast source of renewable energy through arrays of multiple wave energy converters (WECs), known as wave farms, is an important operational and quantitative problem. The problem of efficiently locating wave energy converter (WEC) devices within a wave farm can have a significant impact on the total power of the farm due to the interactions among the incident ocean waves and the scattered and radiated waves produced by the WECs. However, ocean environments are stochastic, and these uncertainties can have a substantial degrading effect on the power output of the wave farm. Thus, we need optimization models that design layouts that perform well, even as the sea state changes.

The wave energy converter location problem (WECLP) for single deterministic sinusoidal waves has been studied in the literature [1–8]. However, there is little research devoted to the design of wave farms under uncertainty and in real ocean environments [5, 6]. In the present work, we study the problem of determining the optimal layout of WECs in an array under uncertainty, from an optimization and modeling point of view.

We propose modeling approaches for mitigating the effect of uncertainties assuming single component but stochastic sinusoidal waves, as well as irregular waves with a spectral representation, under a modification of the performance measure known as the q-factor. We formulate the problems and study the properties and theoretical characteristics of the proposed models for a simple 2-WEC case.

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1 Introduction

In order to have efficient and reliable power output, we need to study the effect of stochastic ocean environments on the design and performance of a wave farm. Depending on the interaction between waves and WECs in the ocean, a wave farm's configuration or layout can have a significant effect on the power output of the farm. Generally, when the incident wave (i.e., the incoming ocean wave) hits a WEC, it produces scattered and radiated waves that interact with each other and with the incident wave. Downstream WECs experience the combined wave and, in turn, produce their own scattered and radiated waves. Many works in the literature study the optimal configuration of a wave farm under deterministic ocean environments in order to maximize the power produced. These works provide a theoretical background for the modeling, analysis and optimization of a wave farm's layout [1–4, 9]. In these works, WECs absorb mechanical power from the waves and convert it to electrical power [2–4, 7, 8, 10, 11]. There are two common approaches in the literature to compute the absorbed mechanical power, or simply "power," from a wave farm, an exact method and the *point-absorber approximation*. The exact approach requires a boundary element code such as WAMIT [12] which is computationally expensive. Instead, by considering the assumption of linear wave theory, we can use the point-absorber approximation in our formulation, where the devices are assumed to be small enough with respect to the wavelength of the incident waves that the scattered waves can be neglected. It is common in the literature to optimize the *q-factor*, which is the ratio between the total power absorbed by N WECs in a wave farm to the power that would be absorbed by N WECs acting in isolation:

$$q = \frac{\sum_{n=1}^{N} P_n}{N P_0}, \qquad (1)$$

where P_n is the power absorbed by the nth device in the array and P_0 is the power absorbed by a single device acting in isolation (a constant). The total mean power absorbed by an array of N identical WECs oscillating in one mode of motion, such as heave, under the standard assumptions of linear wave theory is given by [2,3]:

$$P = \frac{1}{4}(\boldsymbol{U}^*\boldsymbol{X} + \boldsymbol{X}^*\boldsymbol{U}) - \frac{1}{2}\boldsymbol{U}^*\boldsymbol{B}\boldsymbol{U}, \quad (2)$$

where $\boldsymbol{U}$ is a column vector of complex velocity amplitudes (determined from the equations of motion); $\boldsymbol{X}$ is a column vector of complex exciting forces (i.e., forces acting on a floating system due to the waves) of both the incident and scattered waves; $\boldsymbol{B}$ is a matrix of real damping coefficients (i.e., parameters that quantify the reduction of oscillations in an oscillatory system); and an asterisk denotes complex conjugate transpose. For fixed WEC locations, equation (2) can be interpreted as a control problem with control variables in vector $\boldsymbol{U}$. Although, this problem can be solved in closed form, the optimal control vector $\boldsymbol{U}$ may not be attainable. However, our interest is in optimizing the WEC locations, in which case it is convenient to use an approximation for the optimal absorbed power.

Under the point-absorber approximation and assuming that the incident waves consist of a single sinusoid, characterized by the wave angle β and wavenumber $k = 2\pi/\lambda$, where λ is the wavelength, the optimal q-factor in (1) has an analytical expression [3]:

$$q = \frac{1}{N}\boldsymbol{L}^*\boldsymbol{J}^{-1}\boldsymbol{L}, \quad (3)$$

where $\boldsymbol{L}$ is an N-dimensional column vector with $L_m = e^{ikd_m\cos(\beta-\alpha_m)}$ and $\boldsymbol{J}$ is an $N \times N$ matrix with $J_{mn} = J_0(kd_{mn})$; $i = \sqrt{-1}$; J_0 is the Bessel function of the first kind with order 0; and d_{mn} is the distance between device m and device n.

The q-factor defined in (3) is a nonconvex function of the WEC locations. See Figure 1, which plots the q-factor vs. the location of device 1 in the 5-device layout S5A given by [8] (blue circles in Figure 3) assuming the incident wave angle $\beta = 0$; the location is plotted in Cartesian coordinates scaled by the wavenumber k.

The methods and analysis provided thus far are based on deterministic ocean states; however, ocean environments are stochastic, and these uncertainties can have a substantial degrading effect on the power of the wave farm. In fact, many authors (e.g., [1,2,5–8,10]) have lamented the fact that a wave farm optimized for a particular wave environment (wave heading angle or wavenumber) performs quite poorly when the environment changes just a little. For example, the best-known 5-device layout (Figure 3) in [8] performs quite well if the incident waves

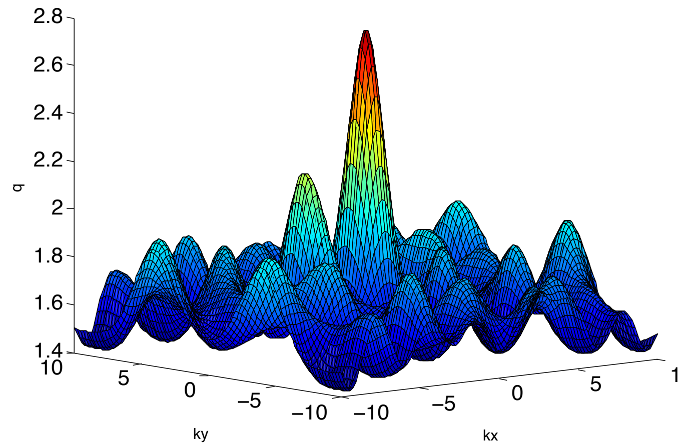


FIGURE 1. q-factor vs. location of device 1 in 5-WEC layout.

arrive at an angle of $\beta = 0$, but the performance degrades almost immediately as β changes; see the solid blue curve in Figure 2. Thus, we need optimization models that design layouts that perform well, even as the sea state changes. We propose stochastic and robust models for mitigating the effect of uncertainty on the total power by considering stochastic single component ocean waves, and also consider an average interaction factor for spectral wave climates.

In the first model, we maximize the expected value of the q-factor when the wave direction, β, is stochastic with known distribution:

$$\begin{aligned} \max_{(d,\alpha)} \quad & E_\beta[q(\beta)] = E\left[\frac{1}{N}L^*J^{-1}L\right] && (4)\\ s.t. \quad & \\ & d_{mn} \geq d_0\lambda && \forall m,n = 1,2,..,N; m \neq n\\ & (d_n,\alpha_n) \in R && \forall n = 1,2,..,N \end{aligned}$$

where $q(\beta)$ is written so as to stress that β is changing. This is an example of a *stochastic optimization* model. The second model maximizes the worst-case solution over a range of β values and is an example of *robust optimization*:

$$\begin{aligned} \max_{(d,\alpha)} \quad & \min_{\beta_l \leq \beta \leq \beta_u} \{q(\beta)\} = \min_\beta \left\{\frac{1}{N}L^*J^{-1}L\right\} && (5)\\ s.t. \quad & \\ & d_{mn} \geq d_0\lambda && \forall m,n = 1,2,..,N; m \neq n\\ & (d_n,\alpha_n) \in R && \forall n = 1,2,..,N \end{aligned}$$

where β_l and β_u are lower and upper bounds for the range of wave direction, β. Figure 2 plots q vs. β/π for 5-WEC solutions found by optimizing these two objectives using a genetic

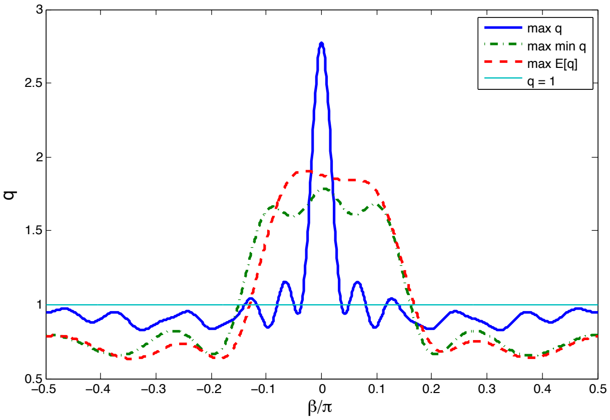


FIGURE 2. q-factor vs. β/π, for 5-WEC Layout that ● Maximize q assuming $\beta = 0$, ● Maximize $\min_\beta\{q\}$ assuming $\beta \in [-\pi/8, \pi/8]$, and ● Maximize $E_\beta[q]$ assuming $\beta \sim N(0, (\frac{\pi}{8})^2)$.

algorithm [6]. The red curve plots the stochastic solution, which maximizes $E[q(\beta)]$, while the green curve plots the robust solution, which maximizes $\min\{q(\beta)\}$. Both solutions are significantly more robust than the deterministic solution, in the sense that they perform at $q > 1$ for a much broader range of β values. Of course, this comes at some expense, since $q(\beta = 0)$ is smaller for the stochastic and robust solutions than for the deterministic solution, as is typical for optimization under uncertainty. The stochastic and robust solutions are also worse in the tails, but this is of less concern since the tails represent unrealistic wave angles such as waves headed out to sea from shore.

Finally, we use an expression for the average q-factor for spectral wave climates, similar to the average interaction factor proposed by [10]. Real ocean waves are of stochastic nature and can be considered as a superposition of a number of regular waves each with their own phase, frequency, amplitude and direction of propagation; these components are random. We consider random ocean waves as linear superpositions of regular wave components, where each component, indexed by p, has a wave amplitude, A_p, a wavenumber, k_p, a wave direction, β_p, an angular frequency, ω_p ($\omega_p^2 = gk_p \tanh(k_p h)$, where h is the water depth and g is gravity acceleration), and a phase ϕ_p. We discretize the wave spectrum by assuming that the ranges of possible values of the wavenumber and wave direction are divided into equal intervals of length Δk and $\Delta\beta$, respectively. Then the average interaction factor is given by:

$$\bar{q} = \frac{\sum_p \frac{1}{k_p} q(k_p,\beta_p) c_g(k_p) S(\omega_p,\beta_p)\Delta\beta\Delta k}{\sum_p \frac{1}{k_p} c_g(k_p) S(\omega_p,\beta_p)\Delta\beta\Delta k}, \qquad (6)$$

where $c_g(k_p)$ is the group velocity and $S(\omega,\beta)$ is the wave energy spectrum.

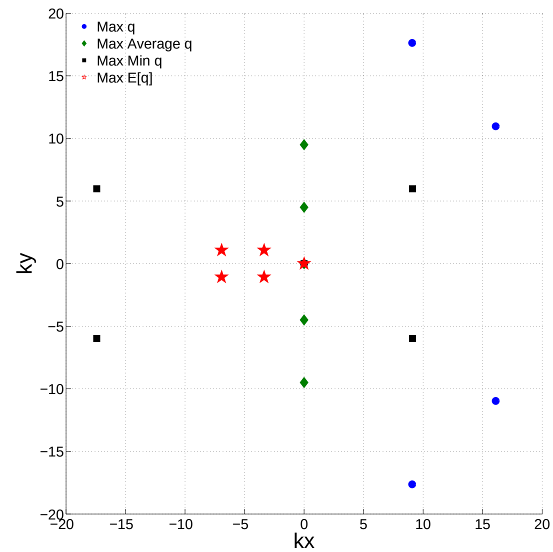


FIGURE 3. Best known configurations for a wave farm with 5 WECs under different optimization problems.

2 Max-Min Optimization Model

In the max-min (robust) model, we reduce the effect of uncertainty in the wave heading, β, by maximizing the worst case output. For convenience, we work with the expanded form of the q-factor. One can show that

$$q(\beta) = \frac{1}{N}\left(\sum_{n=1}^{N} J_{nn}^{-1} + \sum_{n=1}^{N-1}\sum_{m=n+1}^{N} 2J_{nm}^{-1}\cos(kz_{nm}(\beta))\right), \qquad (7)$$

where $z_{nm}(\beta) = d_n\cos(\beta-\alpha_n) - d_m\cos(\beta-\alpha_m)$. Here, if $\beta_u - \beta_l \geq \pi$ and $kd_n \geq \pi$ for $n = 1, 2, \cdots, N$, the minimum value of $2J_{nm}^{-1}\cos(kd_n\cos(\beta-\alpha_n) - kd_m\cos(\beta-\alpha_m))$ is equal to $-2J_{nm}^{-1}$ when $J_{nm} \geq 0$, and is $2J_{nm}^{-1}$ otherwise. After further simplification, the max-min optimization problem reduces to:

$$\max_{\boldsymbol{d}} \frac{1}{N}\left(\sum_{n=1}^{N} J_{nn}^{-1} - \sum_{n=1}^{N-1}\sum_{m=n+1}^{N} 2|J_{nm}^{-1}|\right). \qquad (8)$$

The condition $\beta_u - \beta_l \geq \pi$ means that the wave angle can vary by more than π, which is unrealistic. However, without this assumption, the optimization model is much more difficult to solve, at least analytically. Therefore, for the sake of analysis, we assume that $\beta_u - \beta_l \geq \pi$ holds and consider a 2-WEC array. For $N = 2$, without loss of generality we put the first device at the origin and the second one at the point (d, α) in polar coordinates. Figure 4(a) shows the objective function value, $\frac{1-|J_0(kd)|}{1-J_0(kd)^2}$, versus the value of kd. In this figure, the blue line is the objective function, $\frac{1-|J_0(kd)|}{1-J_0(kd)^2}$, and the green line is the Bessel function, $J_0(kd)$. The fact that $q \leq 1$ implies that the two WECs together can never perform better than if they were separated; that is, there is no synergy between the devices. This is not a desirable property of wave farms. However, for more realistic situations with $\beta_u - \beta_l < \frac{\pi}{2}$, the optimal max-min objective will be greater than one.

The optimal value of the max-min q-factor, calculated by discretization of the search space and enumeration, as a function of $\beta_u - \beta_l$ is depicted in Figure 4(b).

3 Maximum Expected Value Problem

As we observed, the solution of the max-min optimization problem can be conservative. Thus, we may use the other optimization model, the maximum expected value model. However, this problem is analytically challenging to study and in some cases obtaining a closed form solution is impossible. For the expected value optimization problem, we have:

$$\max_{d,\alpha} \left\{ E_\beta[q(\beta)] = \frac{1}{N}\sum_{n=1}^{N} J_{nn}^{-1} + \frac{1}{N}\sum_{n=1}^{N-1}\sum_{m=n+1}^{N} 2J_{nm}^{-1} E_\beta[\cos kz_{nm}(\beta)] \right\}.$$

In this problem, computing $E_\beta[\cos(kz_{nm}(\beta))]$ analytically is important but difficult. To get some insight into this problem, we start by analyzing the expected value problem for the 2-WEC array. In this case, the expected value optimization problem is:

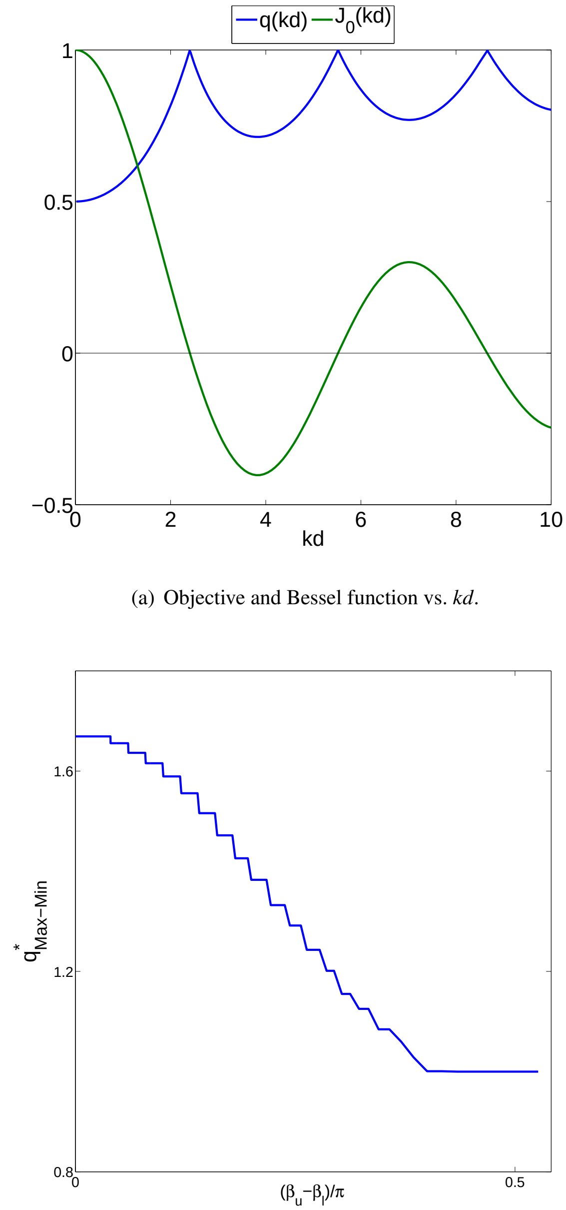
$$\max_{d,\alpha} \left\{ E_\beta[q(\beta)] = \frac{1 - J_0(kd) E_\beta[\cos(kd\cos(\beta-\alpha))]}{1 - J_0(kd)^2} \right\}. \quad (9)$$

Proposition 1. *Suppose β is normally distributed with a mean of 0 and a variance of σ^2. Then if the first WEC is located at $(0,0)$ and the second at (d,α), we have*

$$E_\beta[q(\beta)] = 1 - \frac{2J_0(kd)\sum_{r=1}^{\infty}\left((-1)^r J_{2r}(kd) e^{-2r^2\sigma^2}\cos(2r\alpha)\right)}{1 - J_0(kd)^2}. \quad (10)$$

From Proposition 1, we observe that as the variance of β increases, the expected value of the q-factor converges to one, which is the optimal value for the max-min model for large ranges of β. This indicates that the mathematical models tends to provide conservative solutions as the uncertainty increases.

Optimal layouts for the various problems discussed in this section for a 5-WEC instances are shown in Figure 3.



(a) Objective and Bessel function vs. kd.

(b) Optimal objective vs. range of β.

FIGURE 4. Max-min solution for 2-WEC problem.

4 Average q-Factor for Spectral Wave Climate

Real ocean waves are random and irregular, but there is enormous historical data and measurements for the sea surface elevation. These statistical measurement would help in estimating the wave spectrum. However, if we design the wave farm based on the significant wavenumber and wave direction, we lose farm efficiency as the ocean environment changes. Figure 5 plots the

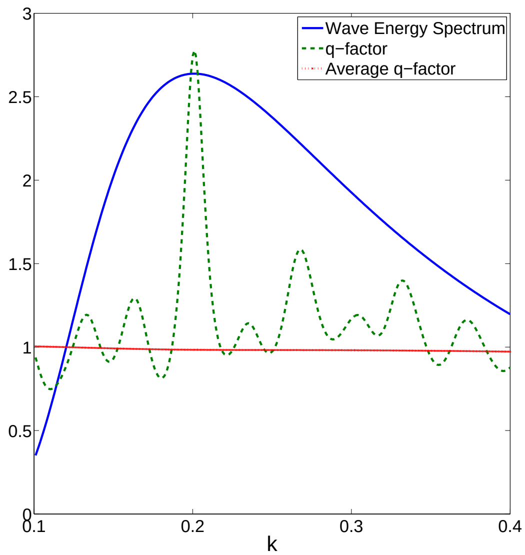


FIGURE 5. Wave farm performance designed for significant wavenumber $k = 0.2$.

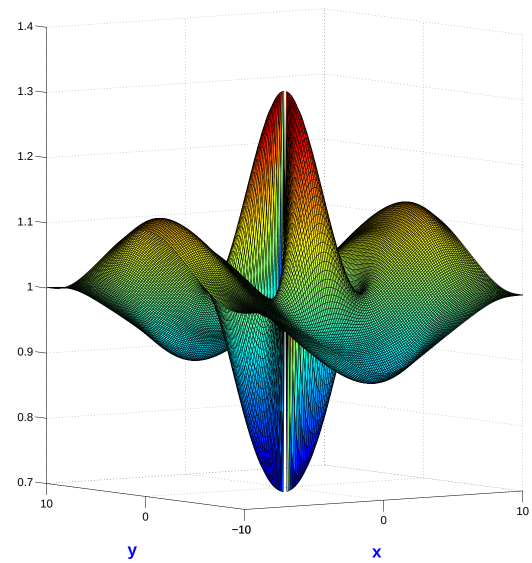


FIGURE 6. $\bar{q}$ vs. location of device 2 in 2-WEC layout.

q-factor of the 5-WEC farm optimally designed for significant wavenumber $k = 0.2$. As depicted, the farm performance is not satisfactory for a wide range of wavenumbers, even where the wave energy spectrum is high. (We note that the wave energy spectrum curve is scaled up by a constant.)

This motivates the need to design wave farms that perform well over the range in which the wave energy spectrum is high. We consider an omni-directional wave spectrum and use the modified q-factor in (6). Hence, the WECLP optimization problem for real random ocean waves is:

$$\max_{(d,\alpha)} \quad \frac{\sum_p \frac{1}{k_p} q(k_p,\beta_p) c_g(k_p) S(\omega_p,\beta_p)\Delta\beta\Delta k}{\sum_p \frac{1}{k_p} c_g(k_p) S(\omega_p,\beta_p)\Delta\beta\Delta k} \tag{11}$$

$$s.t.$$

$$d_{mn} \geq d_0\lambda \qquad \forall m,n = 1,2,..,N; m \neq n$$

$$(d_n,\alpha_n) \in R \qquad \forall n = 1,2,..,N$$

Figure 6 plots the objective function in (11) for a wave farm with $N = 2$ WECs, where one WEC is fixed at the origin. As depicted, the objective function is nonconvex, thus we need to develop heuristic algorithms in order maximize the average q-factor.

Figure 7 plots the wave energy spectrum for significant wavenumber $k = 0.2$, against the q-factor in (3) and (6) for a best solution obtained from problem (11) for $N = 5$ based on a random search heuristic.

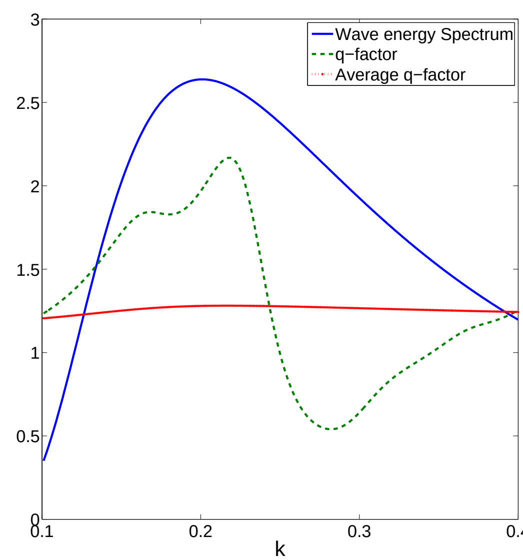


FIGURE 7. Wave farm performance designed for significant wavenumber $k = 0.2$ using the average interaction factor.

5 Conclusion

In this work, we develop two optimization models for the wave energy converter location problem under uncertainty. We propose models for ocean environments in which the wave heading, β, is stochastic. We prove structural properties of the max-min model and the maximum expected value model for a 2-WEC layout and observe that the optimal distance between the two device decreases as uncertainty increases. Moreover, we provide a performance measure for the design of the wave energy farms in irregular ocean waves and show its importance.

For future research, it is necessary to generalize the max-min and the maximum expected value results for layouts with $N \geq 3$ devices, and considering stochastic wavenumber, k. Moreover, developing efficient and effective optimization algorithms to solve the max-min model, the maximum expected value model and the average q-factor for spectral wave climate is important and should be studied.

ACKNOWLEDGMENT

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