**Proceedings of the 2nd Marine Energy Technology Symposium**
**METS2014**
**April 15-18, 2014, Seattle, WA**

# LAYOUTS FOR OCEAN WAVE ENERGY FARMS: MODELS, PROPERTIES, AND HEURISTIC

**Lawrence V. Snyder***
Dept. of Industrial and Systems Engr.
Lehigh University, Bethlehem, PA, USA

**M. Mohsen Moarefdoost**
Dept. of Industrial and Systems Engr.
Lehigh University, Bethlehem, PA, USA

## ABSTRACT

We present models and algorithms for choosing optimal locations of wave energy conversion (WEC) devices within an array, or wave farm. The location problem can have a significant impact on the total power of the farm due to the interactions among the incident ocean waves and the scattered and radiated waves produced by the WECs. Depending on the nature of the interference (constructive or destructive) among these waves, the wave energy entering multiple devices, and thus the power output of the farm, may be significantly larger or smaller than the energy that would be seen if the devices were operating in isolation. Our model chooses WEC locations to maximize the performance of a wave farm as measured by a well known performance measure called the $q$-factor, which is the ratio of the power from an array of $N$ WECs to the power from $N$ WECs operating independently, under the *point absorber approximation*. We prove bounds for the $q$-factor based on the eigenvalues of an important data matrix, and provide an analytical optimal solution for the 2-WEC problem. We propose an iterative heuristic for the general problem and discuss the WEC location problem under uncertainty.

## INTRODUCTION

In this paper, we study optimization models and algorithms for the problem of finding an *optimal configuration for an array of wave energy converter (WEC) devices*. Wave energy represents a large untapped source of energy in the world. According to the Electric Power Research Institute [1], the total potential wave energy resource along the U.S. continental shelf edge is estimated to be 1,170 TWh per year, which is almost one third of the annual electricity consumption in the U.S. Wave energy is more predictable and stable than wind and solar energy. However, uncertainties are still present and need to be investigated and mitigated in order to optimize the power output from wave energy systems.

A wave farm's configuration or layout can have a significant effect on the power output of the farm, depending on the nature of the interference (constructive or destructive) among the incident ocean waves and the scattered and radiated waves produced by the WECs. It is therefore important to consider the hydrodynamic interactions when locating the devices. This problem is the focus of the present work. For the sake of tractability, and following the existing literature on the WEC layout problem, we consider a simple model of the ocean environment that assumes the incident waves are regular (sinusoidal) waves in water of infinite depth; we also simplify the power calculations by employing the *point-absorber approximation*. In the point absorber approximation, the devices are assumed to be small enough with respect to the wavelength of incident waves that the scattered waves can be neglected. Even with these simplifying assumptions, the $q$-factor is a highly nonlinear, nonconvex function of the WEC locations. See Figure 1, which plots the $q$-factor vs. the location of device 1 in the 5-device layout S5A given by [2] assuming $\beta = 0$; the location is plotted in Cartesian coordinates scaled by the wavenumber $k$.

The first study related to wave farm layout is that of Budal [3], who investigates the power absorption of a system of multiple identical interacting bodies under the assumptions of linear wave theory. By assuming one mode of motion and equal amplitudes, [3] simplifies the calculations using the point-absorber approximation and provides the optimal power absorption. Falnes [4] and Evans [5] independently modify the assumption of identical motion amplitudes for oscillating devices and provide the optimal power absorption. [6] studies the hydrodynamic aspects of a system of interacting WEC devices and summarizes the known

*Corresponding Author: lvs2@lehigh.edu

**FIGURE 1**: $q$-FACTOR VS. LOCATION OF DEVICE 1 IN 5-WEC LAYOUT.

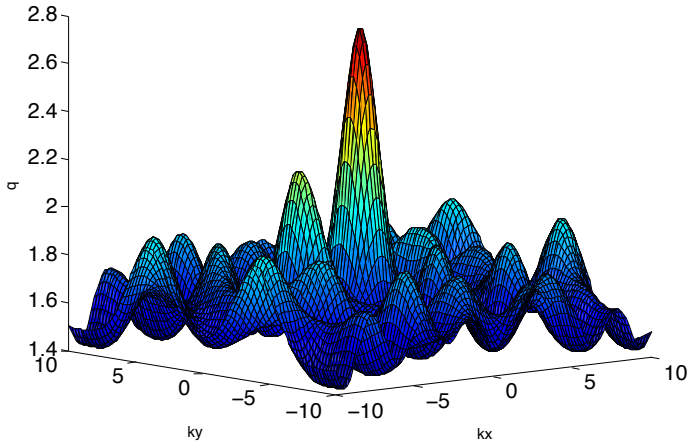
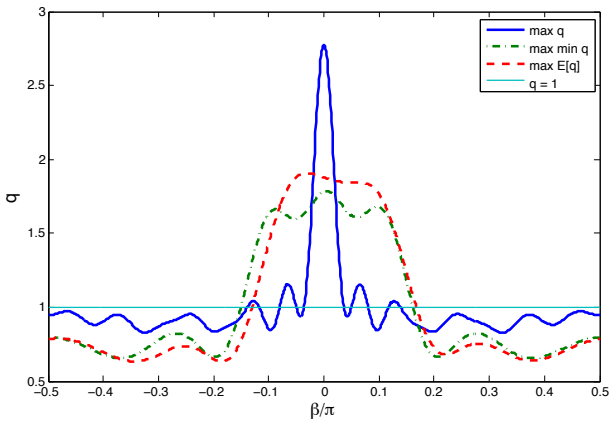



**FIGURE 2**: $q$-FACTOR VS. $\beta/\pi$, FOR 5-WEC LAYOUTS THAT ● MAXIMIZE $q$ ASSUMING $\beta = 0$, ● MAXIMIZE $\min_\beta\{q\}$ ASSUMING $\beta \in [-\pi/8, \pi/8]$, AND ● MAXIMIZE $E_\beta[q]$ ASSUMING $\beta \sim N(0, (\frac{\pi}{8})^2)$.




mathematical results for both heaving and surging devices in an array of interacting devices. These papers provide the theoretical background for modeling the power absorbed by interacting bodies in an array. They derive formulations for the optimal power absorption characteristics of an array of interacting devices under the point-absorber approximation and provide expressions for the *q-factor* to measure the performance of wave arrays. The $q$-factor is the ratio of the total power absorbed by the array to that of the same number of devices in isolation; it captures the interaction effect of the array configuration. The $q$-factor is, essentially, the objective function of the location problem studied by these references and by our own models.

The papers cited thus far do not discuss optimal configuration of the wave farm or propose algorithms for choosing layouts to maximize the $q$-factor. In contrast, [7] considers devices with simple geometry (e.g., spheres) laid out in simple arrangements (e.g., rows) and concludes that the spacing among devices has a larger impact on $q$ than the device geometry. Fitzgerald and Thomas [2] appear to be the first to consider general configurations of WECs in the plane. They employ the small-body approximation [8, 9] rather than the point-absorber approximation and solve the resulting problem using a sequential quadratic programming (SQP) solver with multiple manually chosen starting points. Cruz, et al. [10] consider a few fixed layout strategies in the context of evaluating control strategies; they do not solve a layout-optimization problem. Child and Venugopal [11] consider the layout problem for WECs with simplified geometries under the exact $q$-factor calculation. They argue it is advantageous for each device to be located at the intersection points of certain parabolas centered at the other devices. They use this to develop a heuristic they call the Parabolic Intersection (PI) method, which they find is less accurate but faster than a genetic algorithm (GA) that they also introduce. However, to compute the $q$-factor exactly requires a boundary element method code such as WAMIT [12] and is therefore computationally prohibitive within an optimization context.

From an optimization point of view, there is still a gap in the literature, as the existing algorithms to maximize the $q$-factor appear to be relatively slow and inaccurate. Moreover, the main focus of the current literature is on deterministic ocean states, even though the uncertainties in the sea environment can have a substantial degrading effect on the power output. In fact, many authors (e.g., [2, 3, 11, 13]) have lamented the fact that a wave farm optimized for a particular wave environment (wave heading angle or wavenumber) performs quite poorly when the environment changes just a little. For example, the best-known 5-device layout [2] performs quite well if the incident waves arrive at an angle of $\beta = 0$, but the performance degrades almost immediately as $\beta$ changes; see the blue curve in Figure 2. In the latter part of this paper, we discuss the WEC location problem under uncertainty. Our results demonstrate that significantly more robust solutions can be obtained by maximizing either the expected or minimum value of the $q$-factor; see the red and green curves in Figure 2. To our knowledge, this is the first demonstration of this important fact, which contradicts the conventional wisdom that the sharp drop off in $q$ depicted in the blue curve in Figure 2 is inevitable, and that most "good" layouts will perform at $q < 1$ if $\beta$ differs from the angle assumed during the optimization.

## THEORETICAL BACKGROUND

For an array of $N$ identical WECs oscillating in one mode of motion, such as heave, the total mean power absorbed by the system under the standard assumptions of linear theory is given

by [3, 5]:

$$P = \frac{1}{4}(U^*X + X^*U) - \frac{1}{2}U^*BU, \tag{1}$$

where $U$ is a column vector of complex velocity amplitudes (determined from the equations of motion); $X$ is a column vector of complex exciting forces (i.e., forces acting on a floating system due to the waves) of both the incident and scattered waves; $B$ is a matrix of real damping coefficients (i.e., parameters that quantify the reduction of oscillations in an oscillatory system); and an asterisk denotes complex conjugate transpose. The first term in (1) represents the power absorbed from the incoming waves, while the second represents the power lost to the radiated waves created by oscillating bodies.

If the WEC locations are fixed, then (1) may be maximized over the control variables $U$, and the maximum power is [5]:

$$P_{max} = \frac{1}{8}X^*B^{-1}X, \tag{2}$$

which is attained when $U = \frac{1}{2}B^{-1}X$. The expressions for power in (1) and (2) represent the mechanical power absorbed by the WEC array, which, as is typical in the literature, we use as a proxy for the electrical power output.

The expression in (2) gives the optimal power as a function of the control variables for *fixed* WEC locations. We wish to optimize the locations in order to maximize (2). Unfortunately, to calculate $P_{max}$ in (2) requires calculation of the so-called hydrodynamic coefficients in $B$ and $X$. Since these coefficients depend on the shape, geometry and location of WEC devices, they must be computed numerically using boundary element method code such as WAMIT [12]. These calculations normally are complex and time-consuming for one $q$-factor evaluation, and except for very special cases, a closed form analytical expression is out of the question. Thus, the optimization of the objective function in (2) by means of classical numerical methods is difficult and computationally expensive.

Fortunately, the *point-absorber approximation* [3, 6] provides a much more tractable expression for the absorbed power, under the assumption that the devices are small relative to the incident wavelength. Instead of using $P$ as the objective function, it is common to use the $q$-factor, which is the ratio of the total power from the array to that for the same number of devices in isolation:

$$q = \frac{\sum_{n=1}^{N} P_n}{NP_0}, \tag{3}$$

where $P_n$ is the power absorbed by the $n$th device in an array and $P_0$ is the power absorbed by a single device acting in isolation.

Under the point absorber approximation, the $q$-factor in (3) has a closed form expression [5]:

$$q = \frac{1}{N}L^*J^{-1}L, \tag{4}$$

where $L$ is an $N$-dimensional column vector with

$$L_m = e^{ikd_m\cos(\beta-\alpha_m)} \tag{5}$$

and $J$ is an $N \times N$ matrix with

$$J_{mn} = J_0(kd_{mn}); \tag{6}$$

$J_0$ is the Bessel function of the first kind with order 0; $k$ is the wavenumber ($k = 2\pi/\lambda$, where $\lambda$ is the wavelength); $\beta$ is the direction of the incident wave; $(d_m, \alpha_m)$ are the polar coordinates of device $m$; and $(d_{mn}, \alpha_{mn})$ are the local polar coordinates of device $m$ relative to device $n$. The advantage of (4) is that it does not involve the hydrodynamic coefficients and can thus be evaluated efficiently.

## OPTIMIZATION MODEL

We want to find locations of the WECs (in terms of $\alpha$ and $d$) in order to maximize the $q$-factor. The mathematical formulation of the Wave Energy Converter Location Problem (WECLP) is:

$$\max_{d,\alpha} \quad q(d,\alpha;\beta,k) = \frac{1}{N}L^*J^{-1}L \tag{7}$$

$$s.t.$$

$$d_{mn} \geq d_0\lambda \qquad \forall m,n = 1,2,..,N; m \neq n \tag{8}$$

$$(d_n,\alpha_n) \in R \qquad \forall n = 1,2,..,N \tag{9}$$

where $d_0 > 0$ is a constant, $\lambda$ is the incident wavelength, and $R$ is the region for locating WECs. In (7), the notation $q(d,\alpha;\beta,k)$ indicates the $q$-factor for solution $(d,\alpha)$ under wave angle $\beta$ and wavenumber $k$. We will often shorten this notation to simply $q$. The objective function depends on the decision variables $d$ and $\alpha$ through $L$ and $J$, as discussed above. Constraints (8) ensure a minimum level of separation between the devices, which reflect physical constraints and are also necessary for the point absorber approximation to remain valid.

**Proposition 1.** *Let $(d,\alpha)$ be a solution to the WECLP and let $\beta$ and $k$ be a wave angle and wavenumber, respectively. Then for any wave angle $\beta'$ and any wavenumber $k'$, there exists a solution $(d',\alpha')$ such that*

$$q(d,\alpha;\beta,k) = q(d',\alpha';\beta',k').$$

*Proof.* Follows from the fact that, as $\beta$ changes, we can rotate the layout, and as $k$ changes, we can scale the layout, while maintaining the same $q$.

Proposition 1 demonstrates that the WECLP is isomorphic with respect to $\beta$ and $k$. Therefore, an instance of the WECLP is completely specified by $N$, the number of devices.

### 2-WEC Case

In this section, we provide an analytical optimal solution for the special case of $N = 2$. Without loss of generality, assume WEC 1 is located at the origin and WEC 2 is located at $(d,\alpha)$. Then $(d_1,\alpha_1) = (0,0)$; $(d_2,\alpha_2) = (d,\alpha)$; $d_{12} = d_{21} = d$. Constraint (8) simplifies to

$$d \geq d_0\lambda. \tag{10}$$

Let $j_n$ be the $n$th local optimizer (min or max) of the Bessel function $J_0(\cdot)$. Approximate values are given in columns 1–3 of Table 1.

**Theorem 1.** *Let $n^*$ be the smallest integer $n$ such that $j_n \geq 2\pi d_0$. Then the optimal solution to the 2-WEC problem is to locate one WEC at $(0,0)$ and the other at $(d^*,\alpha^*)$, where*

$$d^* = \begin{cases} \frac{j_{n^*}}{k}, & \text{if } |J_0(k\lambda d_0)| \leq |J_0(j_{n^*})| \\ \lambda d_0, & \text{otherwise} \end{cases}$$

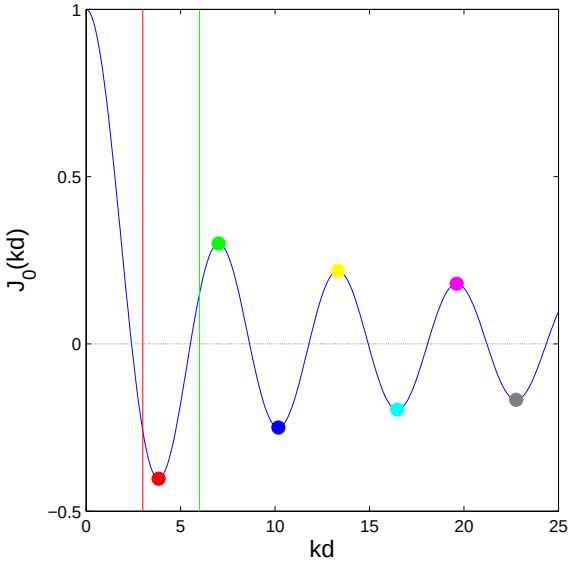
$$\alpha^* = \begin{cases} \beta - \arccos\left(\frac{\pi}{kd^*}\right), & \text{if } J_0(j_{n^*}) \geq 0, \\ \beta - \frac{\pi}{2}, & \text{otherwise.} \end{cases}$$

*The optimal solution attains a q-factor of*

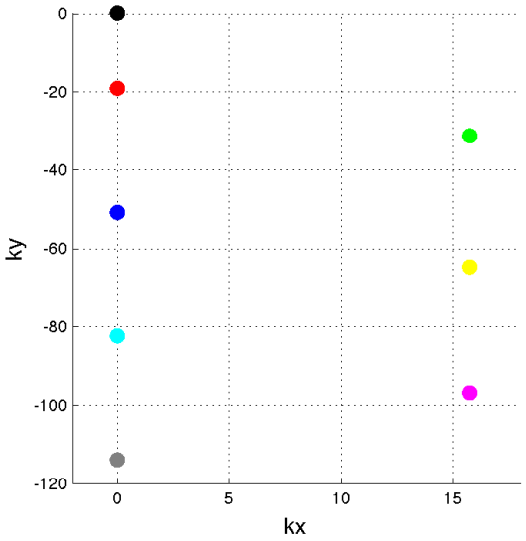
$$q^* = \frac{1}{1-|J_0(kd^*)|}.$$

*Proof.* See [14].

Roughly speaking, Theorem 1 locates the second WEC at the local optimum (min or max) with the largest absolute value, subject to (10). Figure 3(a) displays these optima (along with hypothetical constraints (10), for the first two optima), while Figure 3(b) displays the corresponding locations of WEC 2. Column 4 of Table 1 gives the $q$-factor corresponding to each of these solutions, while the last four columns give the locations of WEC 2 in polar and Cartesian coordinates, assuming $k = 0.2$ and $\beta = 0$.




(a) BESSEL OPTIMA.




(b) LOCATIONS OF WEC 2.

**FIGURE 3**: SOLUTIONS TO 2-WEC PROBLEM.

### Eigenvalue Bounds

In this section, we introduce upper and lower bounds on $q$ that provide a benchmark against which to compare feasible solutions, and also may play a role in future optimization algorithms, since the bounds are simpler to compute than $q$ itself. The first step is to show that for a feasible array configuration, if the matrix $J$ in (4) is invertible, it is also positive definite.

**TABLE 1**: LOCAL OPTIMA OF BESSEL FUNCTION, AND RESULTING OPTIMAL $q$ (FOR ANY $k, \beta$) AND OPTIMAL LOCATIONS OF WEC 2 (FOR $k = 0.2, \beta = 0$).

| $n$ | $j_n$ | $J_0(j_n)$ | $q^*$ | $d^*$ | $\alpha^*$ | $x^*$ | $y^*$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 1.0000 | $\infty$ | — | — | — | — |
| 2 | 3.8317 | -0.4028 | 1.6744 | 19.1585 | −1.5708 | 0.0000 | −19.1585 |
| 3 | 7.0156 | 0.3001 | 1.4288 | 35.0780 | −1.1065 | 15.7080 | −31.3644 |
| 4 | 10.1735 | −0.2497 | 1.3328 | 50.8675 | −1.5708 | 0.0000 | −50.8675 |
| 5 | 13.3237 | 0.2184 | 1.2794 | 66.6185 | −1.3328 | 15.7080 | −64.7401 |
| 6 | 16.4706 | −0.1965 | 1.2445 | 82.3530 | −1.5708 | 0.0000 | −82.3530 |
| 7 | 19.6158 | 0.1801 | 1.2196 | 98.0790 | −1.4099 | 15.7080 | −96.8130 |
| 8 | 22.7601 | −0.1672 | 1.2007 | 113.8005 | −1.5708 | 0.0000 | −113.8005 |

**Lemma 1.** *Matrix $J$ in* (4) *is positive definite if it is invertible.*

*Proof.* We know that $q \geq 0$, thus by (4) we can conclude that $J^{-1} \succeq 0$, which means that $det(J^{-1}) \geq 0$. Since matrix $J$ is invertible, $det(J) \neq 0$ and also $det(J^{-1}) \neq 0$. So, it is clear that $det(J^{-1}) > 0$, which means $J^{-1}$ is positive definite.

The following proposition provides bounds on the value of the $q$-factor in (4).

**Proposition 2.** *Let $0 < \lambda_{min} = \lambda_1 \leq \lambda_2 \leq .. \leq \lambda_{N-1} \leq \lambda_N = \lambda_{max}$ be the N eigenvalues of the matrix $J$. Then*

$$\frac{1}{\lambda_N} \leq q \leq \frac{1}{\lambda_1}. \tag{11}$$

*Proof.* Since matrix $J^{-1}$ is symmetric, $\frac{L^* J^{-1} L}{L^* L}$ is a Rayleigh quotient, so by the min–max theorem,

$$\mu_{min} \leq \frac{L^* J^{-1} L}{L^* L} \leq \mu_{max},$$

for $\|L\| \neq 0$, where $\mu_{min}$ and $\mu_{max}$ are the minimum and maximum eigenvalues of $J^{-1}$, respectively [15]. Thus,

$$\mu_{min} \leq \frac{Nq}{L^* L} \leq \mu_{max}.$$

The proof follows from the fact that $L^* L = N$ and $\mu_{min} = \frac{1}{\lambda_N}$ and $\mu_{max} = \frac{1}{\lambda_1}$.

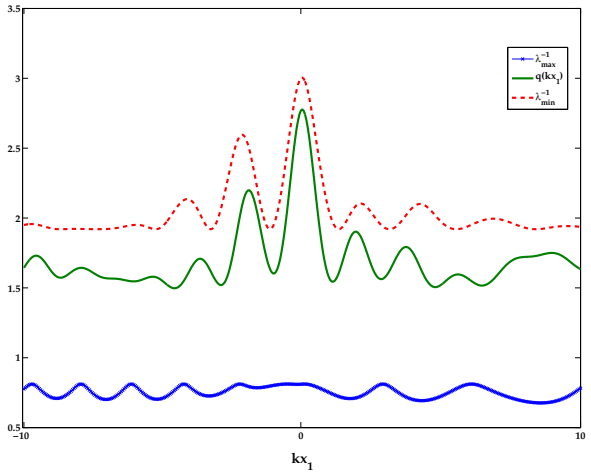
These bounds are illustrated in Figure 4 for the 5-WEC configuration in [2] as the location of the first WEC changes along the $x$- axis and $y$-axis. Note that in both figures, the upper bound and the exact $q$-factor both attain their global maxima at the same values, which suggests that the upper bound may be useful as a proxy for the $q$-factor in an optimization algorithm, since the bounds are faster to compute than the exact value of $q$. Moreover, the bounds are robust with respect to the incident wave direction, $\beta$, since they depend only on the $J$ matrix, which does not depend on $\beta$. So, by maximizing the lower bound, we can obtain a solution that is robust with respect to $\beta$.

## HEURISTIC ALGORITHM

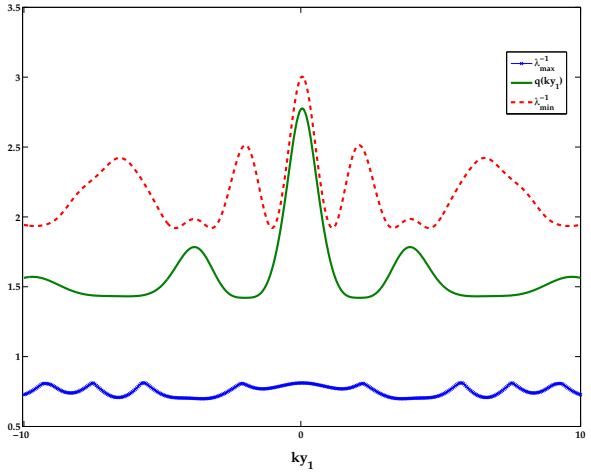
In order to develop a good heuristic algorithm, we first explore the optimality conditions and structure. Figure 5 shows near-optimal layouts for WEC location problems with 4, 5 and 6 devices, obtained by exhaustive search over a discretization of the allowable region $R$. From the figure, it appears that good layouts exhibit several geometric properties. First, they are symmetric with respect to the wave direction. Second, there are at most two WECs on each line normal to the ray of wave direction.

We use these properties to develop a preliminary two-phase greedy-type algorithm. Without loss of generality, we assume $x$ axis is parallel to the ray of wave direction. In the first phase, we add two WECs to the current layout, one above $x$ axis and one below it in order to maintain symmetry in the solution being generated. We find the best locations for the two new WECs in order to maximize the $q$-factor. Then, in the second phase, the algorithm takes the solution from the first phase as its starting point and optimizes the resulting layout locally. The process then repeats to add additional WECs. See Algorithm 1.

Note that step 3 requires us to solve a difficult nonconvex optimization problem. We choose to solve this problem by discretizing the search space and enumerating the possible locations. (Only the location of one WEC must be enumerated, since




(a) ALONG $x$-AXIS.




(b) ALONG $y$-AXIS.

**FIGURE 4**: BOUNDS AND $q$-FACTOR VS. LOCATION OF FIRST WEC ALONG $x$- AND $y$-AXES.

the location of the first determines the location of the second.) However, other approaches may be substituted instead. In step 4, the local optimization is performed using a convex optimization solver (we used MATLAB's `fmincon`). The key idea is that, once the two new WECs are added, the hydrodynamic interactions change for the existing WECs, so the locations of all WECs must be reoptimized to ensure they are located at local maxima. Step 6 is executed only if $N$ is even, in which case the last WEC must be located on the $x$-axis in order to maintain symmetry.

Table 2 shows the performance of our heuristic for $N = 2,\ldots,7$. It lists the $q$-factor for the solution returned by our heuristic, as well as from the GA by [16]. Our heuristic finds the

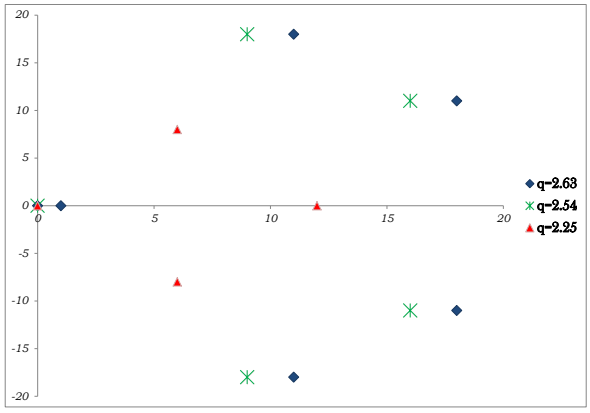



**FIGURE 5**: NEAR-OPTIMAL CONFIGURATIONS FOR THREE VALUES OF $N$.

**Algorithm 1** TWO-PHASE HEURISTIC ALGORITHM

1. Place the first WEC at the origin and let $\mathcal{N} \leftarrow N-1$; $\mathcal{N}$ will equal the number of remaining WECs.
2. If $\mathcal{N} = 0$, STOP. If $\mathcal{N} = 1$, go to 6, otherwise continue.
3. Add two WECs to the current layout by solving

$$\max_{x_1,x_2,y_1,y_2} \; q \quad s.t. \quad x_1 = x_2, \quad y_1 = -y_2.$$

4. Using the layout from step 3 as the initial solution, and optimize all locations locally using a convex optimization solver.
5. Set $\mathcal{N} \leftarrow \mathcal{N}-2$, and go to 2.
6. Add the last WEC (if any) to the current layout by solving

$$\max_{x,y} \; q \quad s.t. \quad y = 0.$$

7. Using the layout from step 6 as the initial solution, and optimize all locations locally using a convex optimization solver.
8. STOP.

optimal solution for $N = 2$ (from Theorem 1) and the best-known solution from the literature for $N = 5$ [13], as well as improving on the result by [16] for $N = 7$. (No other solutions have been reported in the literature.) The table also lists the WEC coordinates for the best solution for each value of $N$, normalized by the wavenumber $k$.

**TABLE 2**: RESULTS OF OUR HEURISTIC AND THAT OF [16] FOR $N = 2,\ldots,7$.

| $N$ | $q$ [Heuristic] | $q$ [16] | Best Solution |
|---|---|---|---|
| 2 | 1.67 | 1.67 | $kx = [0, 0]$ |
| | | | $ky = [0, -19.16]$ |
| 3 | 1.98 | 1.98 | $kx = [0, 0, 0]$ |
| | | | $ky = [0, 4.44, -4.44]$ |
| 4 | 2.28 | 2.17 | $kx = [0, 4.26, 4.26, 8.53]$ |
| | | | $ky = [0, -5.26, 5.26, 0]$ |
| 5 | 2.78 | 2.78 | $kx = [0, -9.08, -9.08, -16.01, -16.01]$ |
| | | | $ky = [0, 17.63, -17.63, 10.97, -10.97]$ |
| 6 | 2.72 | 2.79 | $kx = [0, -15.80, -15.80, -8.88, -8.88, 4.54]$ |
| | | | $ky = [0, -10.95, 10.95, 17.75, -17.75, 0]$ |
| 7 | 2.70 | 3.07 | $kx = [0, -0.27, -0.27, -6.22, -6.22, 1.67, 1.67]$ |
| | | | $ky = [0, -15.92, 15.92, 8.10, 8.65, -8.65]$ |

## THE WECLP UNDER UNCERTAINTY

The methods and analysis provided in the previous sections are based on deterministic ocean states; however, ocean environments are stochastic, and these uncertainties can have a substantial degrading effect on the power of the wave farm. As noted above, the blue curve in Figure 2 shows that the $q$-factor of the optimal 5-WEC configuration degrades drastically as the sea environment ($\beta$) changes just a little. Thus, we need optimization models that design layouts that perform well, even as the sea state changes. Mao [16] proposes two models for mitigating the effect of uncertainty on the total power. The first model maximizes the expected value of the $q$-factor when the wave direction, $\beta$, is stochastic with known distribution. The model in mathematical form is:

$$\max_{(d,\alpha)} \quad E_\beta[q(\beta)] = E\left[\frac{1}{N} L^* J^{-1} L\right] \tag{12}$$

subject to (8)–(9), where $q(\beta)$ is written so as to stress that $\beta$ is changing. This is an example of a *stochastic optimization* model. The second model maximizes the worst-case solution over a range of $\beta$ values and is an example of *robust optimization*:

$$\max_{(d,\alpha)} \quad \min_{\beta}[q(\beta)] = \min_{\beta}\left[\frac{1}{N} L^* J^{-1} L\right] \tag{13}$$

Figure 2 plots $q$ vs. $\beta/\pi$ for 5-WEC solutions found by optimizing these two objectives using a genetic algorithm [16]. The red curve plots the stochastic solution, which maximizes $E[q(\beta)]$, while the green curve plots the robust solution, which maximizes $\min[q(\beta)]$. Both solutions are significantly more robust than the deterministic solution, in the sense that they perform at $q > 1$ for a much broader range of $\beta$ values. Of course, this comes at some expense, since $q(\beta = 0)$ is smaller for the stochastic and robust solutions than for the deterministic solution, as is typical for optimization under uncertainty. The stochastic and robust solutions are also worse in the tails, but this is of less concern since the tails represent unrealistic wave angles such as waves headed out to sea from shore.

We plan to modify our proposed heuristic algorithm to design wave farm layouts that optimize the objectives (12) and (13).

## ACKNOWLEDGMENT

This research was supported in part by an Accelerator Grant from Lehigh University. This support is gratefully acknowledged.

## REFERENCES

[1] EPRI, 2011. "Mapping and Assessment of the United States Ocean Wave Energy Resource". *Electric Power Research Institute*.

[2] Fitzgerald, C., and Thomas, G., 2007. "A priliminary study on the optimal formation of an array of wave devices". In The 7th European wave and tidal energy conference, Porto, Portugal.

[3] Budal, K., 1977. "Theory for Absorption of Wave Power by a System of Interacting Bodies". *Journal of Ship Research,* ***21***(4), pp. 248–258.

[4] Falnes, J., 1980. "Radiation impedance matrix and optimum power absorbtion for interacting oscillators in surface waves". *Applied Ocean Research,* ***2***(2).

[5] Evans, D., 1980. "Some analytic results for two and three dimensional wave energy absorbers". In Power From Sea Waves, pp. 213–249.

[6] McIver, P., 1994. "Some hydrodynamic aspects of arrays of wave energy devices". *Applied Ocean Research,* ***16***, pp. 61–69.

[7] Thomas, G., and Evans, D., 1981. "Arrays of three dimensional wave energy absorbers". *Journal of Fluid Mechanics,* ***108***, pp. 67–88.

[8] Havelock, T. H., 1955. "Waves due to a floating sphere making periodic heaving oscillations". *Proceedings of the Royal Society A,* ***231***(1184), pp. 1–7.

[9] Hulme, A., 1982. "The wave forces on a floating hemisphere undergoing forced periodic oscillations". *Journal of Fluid Mechanics,* ***121***, pp. 443–463.

[10] Cruz, J., Sykes, R., Siddorn, P., and Eatock Taylor, R., 2009. "Radiation impedance matrix and optimum power absorption for interacting oscillators in surface waves". *Proceedings of the 8th European Wave and Tidal Energy Conference, Uppsala, Sweden*, pp. 736–745.

[11] Child, B., and Venugopal, V., 2010. "Optimal configurations of wave energy device arrays". *Ocean Engineering,* ***37***, pp. 1402–1417.

[12] WAMIT, Inc. WAMIT (website). `http://www.wamit.com/`.

[13] Folley, M., and Whittaker, T. J. T., 2009. "The effect of sub-optimal control and the spectral wave climate on the performance of wave energy converter arrays". *Applied Ocean Research,* ***31***, pp. 260–266.

[14] Snyder, L. V., 2013. $q$-factor optimization for 2-WEC arrays. Tech. rep., Lehigh Unviersity.

[15] Schatzman, M., 2002. *Numerical Analysis: A Mathematical Introduction*. Clarendon Press.

[16] Mao, L., 2013. "Optimizing Wave Farm Layouts Under Uncertainty". *Msc thesis, Lehigh University*.