Supply and Demand Uncertainty in Multi-Echelon Supply Chains

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Outline

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Supply vs. Demand Uncertainty

- Demand uncertainty (DU)
  - Randomness in demand quantity, timing, product mix, etc.
  - Purview of SCM/OM for decades
- Supply uncertainty (SU)
  - Disruptions
  - Capacity/yield uncertainty
  - Lead-time uncertainty
  - etc.
  - Increased attention only recently

Are DU and SU the Same?

- Under both DU and SU, the main issue is the same:
  - Not enough supply to meet demand
  - May be irrelevant whether the mismatch came from DU or SU
- Mitigation strategies are similar:
  - Safety stock
  - Multiple sourcing
  - Improved forecasts
  - Demand management
  - Excess capacity
  - etc.

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- Mitigation strategies are similar:
  - Safety stock
  - Multiple sourcing
  - Improved forecasts
  - Demand management
  - Excess capacity
  - etc.
- The good news:
  - We know a lot about supply chains under DU
- The bad news:
  - The "conventional wisdom" from DU is often **wrong** under SU

Literature Review

- Classical inventory models + disruptions
  - Parlar and Berkin (1991), Berk and Arreola-Risa (1994), Parlar and Perry (1995,1996), Gupta (1996), Mohebbi (2003,2004), many others
- Classical inventory models + yield uncertainty
  - Gerchak et al. (1988), Bassok and Akella (1991), Yano and Lee (1995), Wang and Gerchak (1996), many others
- Strategic questions
  - Tomlin (2006): optimal mitigation strategy
  - Tomlin and Snyder (2006): advanced warning
  - Lewis, Erera, and Whilte (2005): border closures
  - Chopra, Reinhardt, and Mohan (2005): "bundling" disruptions and yield uncertainty

Multi-Echelon Models

- Very few multi-echelon models with disruptions
  - Kim, Lu, and Kvam (2005): Yield uncertainty in 3-echelon supply chain, risk-averse objective
  - Hopp and Yin (2006): Optimal placement and size of inventory and capacity buffers in assembly network
- Must study disruptions in multi-echelon setting
  - Disruptions are never local
  - Cascading effect

A Newsboy-Style Result

Theorem (Tomlin 2006)

In a single-stage base-stock system with deterministic demand and stochastic supply disruptions, the optimal base-stock level is given by

$$S^* = d + dF^{-1}\left(\frac{p}{p+h}\right),$$

where d is the demand per period and F is the cdf of supply.

- $F(x) = P(\text{we are in a disruption lasting } x \text{ periods or fewer})$
- Cycle/safety stock interpretation
- Similar (but less sharp) result given by Güllü et al. (1997)

SU vs. DU: Roadmap

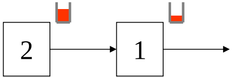
1. The cost of unreliability
2. Order frequency
3. Inventory placement
    - Centralization vs. decentralization
    - Upstream vs. downstream
4. Supply chain structure
    - Hub-and-spoke vs. point-to-point
    - Supplier redundancy
    - Supplier flexibility
5. The cost of reliability

Methodology

- Some of our results are proved analytically
- Others we demonstrate using simulation
  - BaseStockSim software
  - Rough optimization of base-stock levels

Supply Chain Assumptions

- Multi-echelon SC
  - Each stage has processing function and output buffer:



  - May represent physical location, processing activity, or SKU
- Backordered demand
- Costs h, p
- Processing (lead) time T
- Under DU, demands are $N(\mu, \sigma^2)$
- Under SU, disruption process follows 2-state Markov process
  - Disruption probability α
  - Recovery probability β

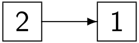
Outline

1. Introduction
2. Cost of Unreliability
3. Order Frequency
4. Inventory Placement
5. Supply Chain Structure
6. Cost of Reliability
7. Conclusions

1. Introduction
- Motivation

The Cost of Unreliability

- Two stages: supplier and retailer
  - Supplier cannot hold inventory, is subject to disruptions
- Under either DU or SU, base-stock policy is optimal at retailer
- Suppose firm fails to plan for either type of uncertainty
  - i.e., it sets $S = \mu$



Key Question:

Is this a bigger mistake under DU or SU?

Level of Uncertainty

- We need a way to compare DU and SU fairly
- Let **level of uncertainty** = % of demands backordered when $S = \mu$

$$LOU = 1 - \text{fill rate}$$

- A DU process and an SU proces are **equivalent** if they have the same LOU

Level of Uncertainty, cont'd

- Under DU, fill rate is
$$1 - \frac{\sigma \mathcal{L}(z)}{\mu},$$
where $\mathcal{L}(z)$ is standard normal loss function and $z = (S - \mu)/\sigma$.
- Therefore
$$LOU_{DU} = \frac{\sigma \mathcal{L}(0)}{\mu} \approx 0.3989 \frac{\sigma}{\mu}.$$

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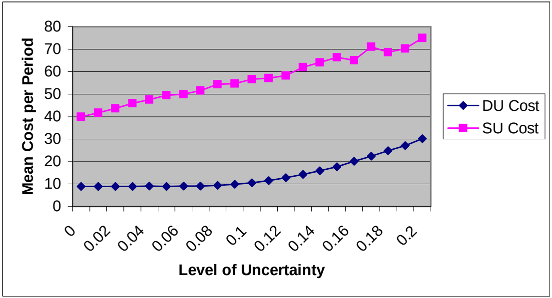
- Under SU, fill rate = % of periods in which supplier is up
- Therefore
$$LOU_{SU} = P(\text{supplier down}) = \frac{\alpha}{\alpha + \beta}.$$

Simulation Experiment

- Varied LOU from 0.0 to 0.2 (by 0.01)
- For each LOU, find σ and α that achieve it
  - (Keeping μ and β fixed)



Insights

- More costly to fail to plan for SU than for DU
- Holds under a wide range of parameters
- Cost difference is greater when
  - Holding cost is smaller
  - Stockout cost is larger
  - Recovery probability is smaller

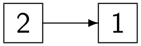
Outline

1 Introduction

- Motivation

Order Frequency

- Two-stage supply chain
- $\mu = 20$, $p = 100$ at retailer
- $T = 1$ at supplier
- Under DU, $\sigma = 5$
- Two possible cost structures:
  1. $h = 2.85$ and $K = 0$
  2. $h = 0.1$ and $K = 250$



Key Question:

Does firm prefer #1 (one-for-one ordering) or #2 (batch ordering)?

Order Frequency: DU

- Option 1: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal, with

$$S^* = \mu + \sigma\Phi^{-1}\left(\frac{p}{p+h}\right) \approx 30$$

  - $E[\text{cost}] \approx 32.8$

Order Frequency: DU

- Option 1: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal, with

$$S^* = \mu + \sigma\Phi^{-1}\left(\frac{p}{p+h}\right) \approx 30$$

  - E[cost] ≈ 32.8
- Option 2: $h = 0.1$, $K = 250$
  - (s, S) policy is optimal with

$$s^* \approx 31, \quad S^* \approx 349$$

  - E[cost] ≈ 32.8
- So the firm is indifferent between the two options under DU

Order Frequency: SU

- Option 1: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal (Tomlin 2006), with

$$S^* = \mu + \mu F^{-1}\left(\frac{p}{p+h}\right) \approx 60$$

  - E[cost] $\approx$ 497.7

Order Frequency: SU

- Option 1: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal (Tomlin 2006), with

$$S^* = \mu + \mu F^{-1}\left(\frac{p}{p+h}\right) \approx 60$$

  - E[cost] $\approx$ 497.7
- Option 2: $h = 0.1$, $K = 250$
  - Optimal policy not known (deterministic demand, stochastic disruptions, fixed cost)

Lemma

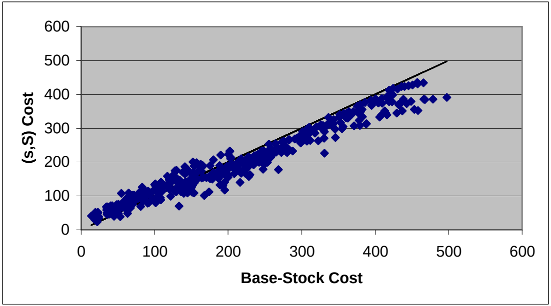
s^* and S^* are integer multiples of μ.

  - $s^* \approx 40$, $S^* \approx 340$, E[cost] $\approx$ 391.1
- So the batch ordering policy is preferred

Insights

- Why is batch policy preferred?
  - If an order is disrupted, the impact is the same under either policy
  - But the likelihood of a disruption affecting an order is smaller under batch policy

Simulation Experiment

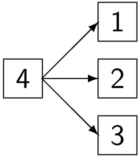


- Batch policy is usually—though not always—preferred
  - s and S may not be optimal
- Instances are generated so that batch and base-stock policies are equivalent under DU

Outline

Centralization vs. Decentralization

- One warehouse, multi-retailer (OWMR) system
- Cost of holding inventory is equal at the two echelons
- Lead times are negligible



Key Question:

Should we hold inventory at the warehouse or at the retailers?

OWMR under DU

- Let C_D, C_C be cost under decentralized and centralized systems, resp.

Theorem (Eppen 1979)

Under DU,

- $E[C_D] \propto N$
- $E[C_C] \propto \sqrt{N}$

- Therefore, centralization is optimal
  - The **risk-pooling effect**

OWMR under SU

- Under SU:
  - Disruptions affect inventory sites
  - In decentralized system, a disruption affects one retailer
  - In centralized system, a disruption affects the whole supply chain

Theorem

Under SU,

(a) $E[C_D] = E[C_C]$

(b) $V[C_D] \propto N$
$V[C_C] \propto N^2$

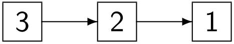
- Therefore decentralization is preferable
- We call this the **risk-diversification effect**

Implication for Facility Location

- Joint location–inventory model by Daskin et al. (2002) and Shen et al. (2003)
  - Considers DU via concave inventory costs in location model
  - Optimal # of facilities decreases because of risk-pooling effect (and inventory economies of scale)
- Reliability model by Snyder and Daskin (2005)
  - Considers SU in the form of facility failures
  - Optimal # of facilities increases—related to risk-diversification effect
- Model by Jeon et al. (working paper, 2006) balances these competing tendencies

Upstream vs. Downstream

- Serial supply chain
- Cost of holding inventory is non-increasing as we move downstream
- Lead times are negligible



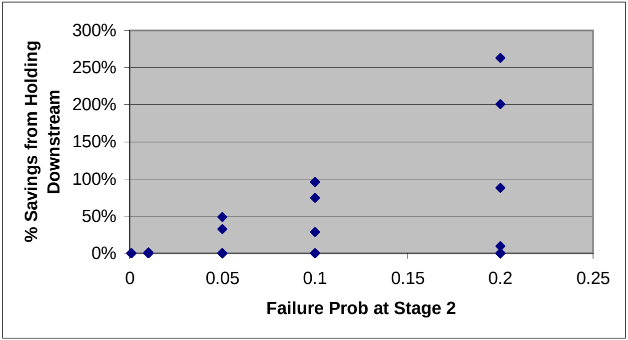
Key Question:

Should we hold inventory upstream or downstream?

Upstream vs. Downstream, cont'd

- Under DU, conventional wisdom says hold inventory upstream
    - Holding costs increase as we move downstream
- But under SU, downstream inventory may be preferable
    - Protects against stockouts anywhere in the system
    - Depends on relative holding costs

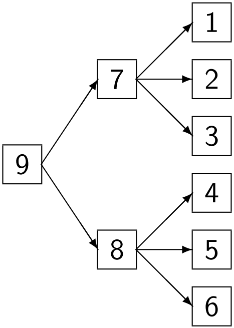
Savings Increases as Disruption Probability Increases



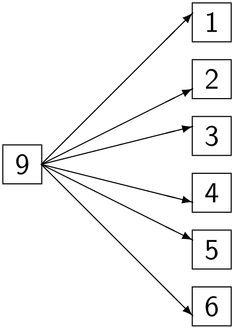
Outline

Hub-and-Spoke vs. Point-to-Point Systems

Hub-and-Spoke:



Point-to-Point:



Key Question:

Which type of network is preferred?

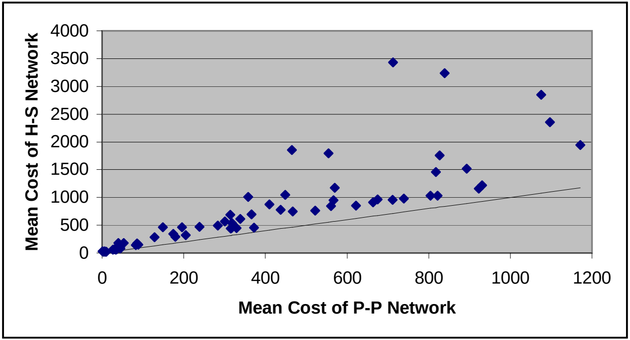
Hub-and-Spoke vs. Point-to-Point Systems, cont'd

- Under DU, hub-and-spoke systems are optimal
  - Due to **risk-pooling effect**: fewer stocking locations $\Longrightarrow$ smaller inventory requirement

Hub-and-Spoke vs. Point-to-Point Systems, cont'd

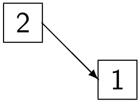
- Under DU, hub-and-spoke systems are optimal
  - Due to **risk-pooling effect**: fewer stocking locations $\Longrightarrow$ smaller inventory requirement
- Under SU, point-to-point systems are optimal
  - Related to **risk-diversification effect**: more stocking locations $\Longrightarrow$ reduced severity of disruptions

Simulation Results



Supplier Redundancy

- Single retailer with one or more suppliers
- Suppliers are identical in terms of cost, capacity, reliability



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What is the value of having backup suppliers?

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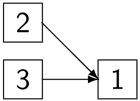

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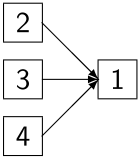

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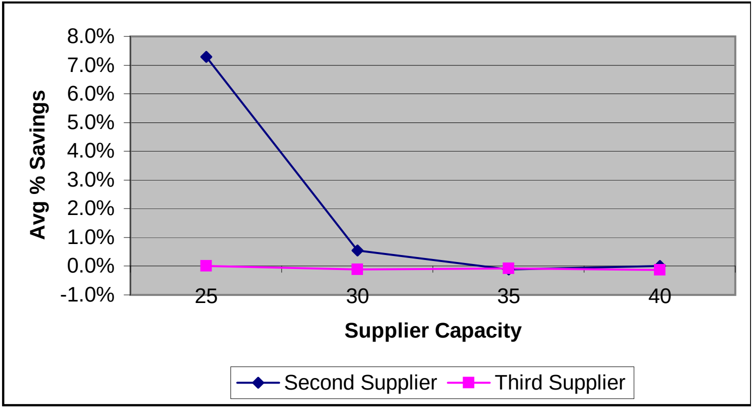
Supplier Redundancy under DU

- Under DU, second supplier provides value if capacities are tight
  - e.g., if capacity $= \mu + \sigma$
  - But value decreases quickly as capacity increases
  - Third, etc. suppliers provide little value

Value of Backup Suppliers: DU

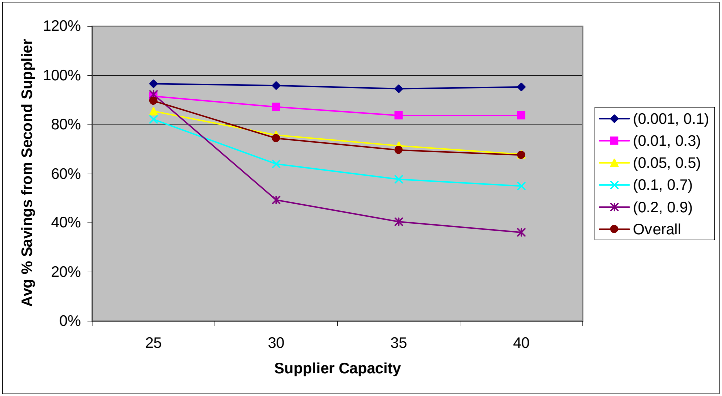


- $\mu = 20$, $\sigma = 5$

Supplier Redundancy under SU

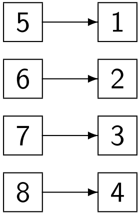
- Under SU, second supplier provides great benefit
  - Fills in when primary supplier is disrupted
  - Also helps ramp back up after disruption
  - Even third+ supplier provides some benefit

Value of Backup Suppliers: SU



Supplier Flexibility

- Related concept: supplier flexibility
- Multiple suppliers, multiple retailers
  - How many suppliers per retailer?
- Closely related to process flexibility (Jordan and Graves 1995)
  - Bipartite network of jobs and workers
  - How much cross-training is required?
  - i.e., how dense should network be?
- Results are similar



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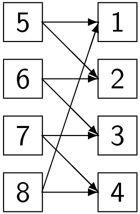

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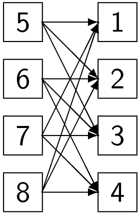

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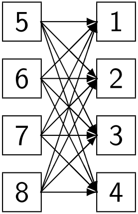

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1 Introduction

- Motivation

The Cost of Reliability

- Firms are accustomed to planning for DU
- Often reluctant to plan for SU if it requires large investment

Key Question

How much DU cost must be sacrificed to achieve a given level of reliability?

The Cost of Reliability

- Firms are accustomed to planning for DU
- Often reluctant to plan for SU if it requires large investment

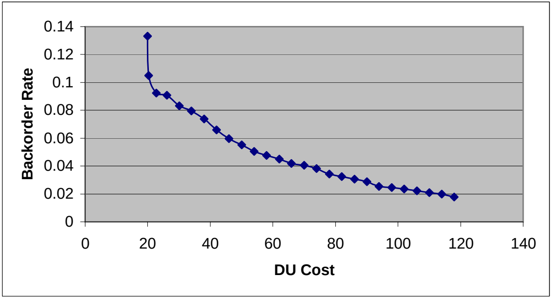
Key Question

How much DU cost must be sacrificed to achieve a given level of reliability?

- The short answer: Not much

Tradeoff Curve

- Each point represents a solution (set of base-stock levels) for serial system
  - Left-most point is "optimal" solution considering DU only
  - Second point: 21% fewer stockouts, 2% more expensive
- "Steep" left-hand side of tradeoff curve is fairly typical
  - Especially for combinatorial problems



Outline

Conclusions

- Planning for SU is critical
- Optimal strategy under SU is often exact opposite from that under DU
  - That's not to say firms are doing everything wrong
  - But SU should be accounted for more than it is
  - Strategy chosen should account for both
- Many of these results are related to risk-diversification effect
  - Disruptions are less severe when eggs aren't all in one basket
- Tradeoff between cost and reliability is often steep
  - Large improvements in reliability with small increases in cost

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- Supported by National Science Foundation grant #DMI-0522725
- Thanks to Jae-Bum Kim (Lehigh) for assistance with simulation study

- Working paper available at
  `www.lehigh.edu/~lvs2/research.html`
- BaseStockSim software available at
  `www.lehigh.edu/~lvs2/software.html`

Questions?

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