A Primal-Dual Active-Set Method for Convex Quadratic Optimization

Frank E. Curtis, Lehigh University

involving joint work with

Zheng Han, Lehigh University
Daniel P. Robinson, Johns Hopkins University

INFORMS Annual Meeting 2012
Phoenix, AZ, USA

17 October 2012

Outline

Outline

Motivation

Nonlinear optimization algorithms:

- Active-set methods (small-to-medium size)
- Interior-point methods (medium-to-large size)
- First-order methods (large-to-huge size)

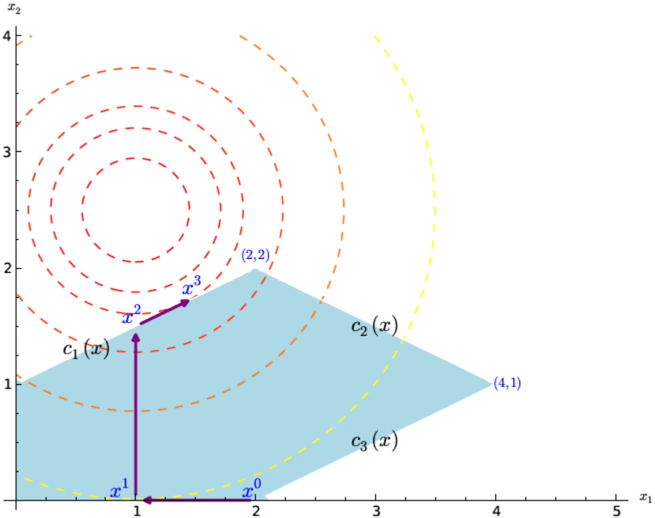
Strengths of active-set methods:

- warm-start easily
- accurate solutions despite degeneracy and ill-conditioning

Our goals:

- Active-set methods for large-scale nonlinear optimization problems
- Active-set methods for large-scale convex quadratic optimization problems (QPs)

Illustration of classical active-set method for convex QP



Purpose of this talk

Our main contributions:

- Globally convergent active-set method for convex QP
- Multiple simultaneous changes in the active-set estimate
- Cost per iteration typically only slightly more than linear system solve

Based on work by:

- Hintermüller, Ito, Kunisch (2002)
- Aganagić (1984)
- Bergournioux, Ito, Kunisch (1999)

Outline

Problem formulation and KKT conditions

Consider the strictly-convex bound-constrained QP

$$\begin{aligned}\min_x\ & c^T x + \tfrac{1}{2} x^T H x \\ \text{s.t. } & \ell \le x \le u.\end{aligned}$$

The KKT conditions are

$$\begin{aligned} c + Hx - z^\ell + z^u &= 0; \\ (x - \ell) \circ z^\ell &= 0; \\ (u - x) \circ z^u &= 0; \\ (x - \ell, u - x, z^\ell, z^u) &\ge 0. \end{aligned}$$

Active-set methods satisfy a subset of conditions and work to satisfy the rest.

Optimal partition

Associated with the optimal solution (x_*, z_*^ℓ, z_*^u), we have the optimal partition

$$\begin{array}{lcllllll}
\mathcal{L}_* & = & \{i: & \ell_i = & [x_*]_i & & \} & ([z_*^\ell]_i \geq 0, \quad [z_*^u]_i = 0); \\
\mathcal{U}_* & = & \{i: & & [x_*]_i & = u_i & \} & ([z_*^\ell]_i = 0, \quad [z_*^u]_i \geq 0); \\
\mathcal{I}_* & = & \{i: & \ell_i < & [x_*]_i & < u_i & \} & ([z_*^\ell]_i = 0, \quad [z_*^u]_i = 0).
\end{array}$$

Optimal partition

Associated with the optimal solution (x_*, z_*^ℓ, z_*^u), we have the optimal partition

$$\begin{array}{lllllll} \mathcal{L}_* & = & \{i: & \ell_i = & [x_*]_i & & \} & ([z_*^\ell]_i \geq 0, & [z_*^u]_i = 0); \\ \mathcal{U}_* & = & \{i: & & [x_*]_i & = u_i & \} & ([z_*^\ell]_i = 0, & [z_*^u]_i \geq 0); \\ \mathcal{I}_* & = & \{i: & \ell_i < & [x_*]_i & < u_i & \} & ([z_*^\ell]_i = 0, & [z_*^u]_i = 0). \end{array}$$

If $(\mathcal{L}_*, \mathcal{U}_*, \mathcal{I}_*)$ is known, then (x_*, z_*^ℓ, z_*^u) is obtained with the following steps:

1. Set
$$x_{\mathcal{L}_*} \leftarrow \ell_{\mathcal{L}_*}, \quad x_{\mathcal{U}_*} \leftarrow u_{\mathcal{U}_*}, \quad z^\ell_{\mathcal{U}_* \cup \mathcal{I}_*} \leftarrow 0, \quad \text{and} \quad z^u_{\mathcal{L}_* \cup \mathcal{I}_*} \leftarrow 0.$$
2. Compute $x_{\mathcal{I}_*}$ by solving the reduced subproblem
$$\min_{x_{\mathcal{I}_*}} \; (c_{\mathcal{I}_*} + H_{\mathcal{I}_*\mathcal{L}_*} x_{\mathcal{L}_*} + H_{\mathcal{I}_*\mathcal{U}_*} x_{\mathcal{U}_*})^T x_{\mathcal{I}_*} + \tfrac{1}{2} x_{\mathcal{I}_*}^T H_{\mathcal{I}_*\mathcal{I}_*} x_{\mathcal{I}_*}.$$
3. Set
$$z^\ell_{\mathcal{L}_*} \leftarrow [c + Hx_*]_{\mathcal{L}_*} \quad \text{and} \quad z^u_{\mathcal{U}_*} \leftarrow -[c + Hx_*]_{\mathcal{U}_*}.$$

Estimating the optimal partition

Given an estimate $(\mathcal{L}, \mathcal{U}, \mathcal{I})$, the point (x, z^ℓ, z^u) is uniquely determined by:

1. Set
$$x_{\mathcal{L}} \leftarrow \ell_{\mathcal{L}}, \quad x_{\mathcal{U}} \leftarrow u_{\mathcal{U}}, \quad z^\ell_{\mathcal{U}\cup\mathcal{I}} \leftarrow 0, \quad \text{and} \quad z^u_{\mathcal{L}\cup\mathcal{I}} \leftarrow 0.$$
2. Compute $x_{\mathcal{I}}$ by solving the reduced subproblem
$$\min_{x_{\mathcal{I}}} \ (c_{\mathcal{I}} + H_{\mathcal{I}\mathcal{L}} x_{\mathcal{L}} + H_{\mathcal{I}\mathcal{U}} x_{\mathcal{U}})^T x_{\mathcal{I}} + \tfrac{1}{2} x_{\mathcal{I}}^T H_{\mathcal{I}\mathcal{I}} x_{\mathcal{I}}.$$
3. Set
$$z^\ell_{\mathcal{L}} \leftarrow [c + Hx]_{\mathcal{L}} \quad \text{and} \quad z^u_{\mathcal{U}} \leftarrow -[c + Hx]_{\mathcal{U}}.$$

It follows that (x, z^ℓ, z^u) satisfies all KKT conditions except possibly those in

$$r(x, z^\ell, z^u) = \begin{bmatrix} \min\{[x-\ell]_{\mathcal{I}}, 0\} \\ \min\{[u-x]_{\mathcal{I}}, 0\} \\ \min\{z^\ell_{\mathcal{L}}, 0\} \\ \min\{z^u_{\mathcal{U}}, 0\} \end{bmatrix}.$$

Primal-dual active-set method

Algorithm 1 Hintermüller, Ito, and Kunisch (2002)

Input $(\mathcal{L}_0, \mathcal{U}_0, \mathcal{I}_0)$ and initialize $k \leftarrow 0$.
loop
    Compute (x_k, z_k^ℓ, z_k^u) via subspace minimization.
    If $r(x_k, z_k^\ell, z_k^u) = 0$, then **break**.
    Set

$$\begin{aligned}
\mathcal{L}_{k+1} &\leftarrow \{i : [x_k]_i < \ell_i \text{ or } i \in \mathcal{L}_k \text{ and } [z_k^\ell]_i > 0\}; \\
\mathcal{U}_{k+1} &\leftarrow \{i : [x_k]_i > u_i \text{ or } i \in \mathcal{U}_k \text{ and } [z_k^u]_i > 0\}; \\
\mathcal{I}_{k+1} &\leftarrow \{i : i \notin \mathcal{L}_{k+1} \cup \mathcal{U}_{k+1}\}.
\end{aligned}$$

end loop

Convergence guarantee and numerical results

Theorem

Suppose $H = A + B$ where A is an M-matrix (and there are only upper bounds). If $\|B\|_1$ is sufficiently small, then

- *the QP has a unique solution;*
- *for some sufficiently large k, Algorithm 1 yields $(x_k, z_k^\ell, z_k^u) = (x_*, z_*^\ell, z_*^u)$.*

The method often converges rapidly for generic convex QP:

		Algorithm 1	
n	H_{cond}	# Iter	%
1e+02	1e+02	3.78e+00	100
1e+02	1e+04	5.14e+00	100
1e+02	1e+06	6.22e+00	100
1e+03	1e+02	4.78e+00	100
1e+03	1e+04	6.64e+00	100
1e+03	1e+06	8.02e+00	100
1e+04	1e+02	5.80e+00	100
1e+04	1e+04	8.24e+00*	90
1e+04	1e+06	1.01e+01*	92

Outline

Observation

Algorithm 1 may fail to converge.

- There is no monotonicity in primal or dual objective, KKT error residual, etc.
- (If H is an M-matrix, then there is monotonicity in the primal variable values.)
- When it fails to converge, often only a few indices are cycling.

Observation

Algorithm 1 may fail to converge.

- There is no monotonicity in primal or dual objective, KKT error residual, etc.
- (If H is an M-matrix, then there is monotonicity in the primal variable values.)
- When it fails to converge, often only a few indices are cycling.

Our proposed enhancement:

- Introduce an auxiliary set $\mathcal{E}$.
- Explicitly enforce bounds on variables in $\mathcal{E}$.
- Put indices in $\mathcal{E}$ to avoid cycling (but keep it small in size!).

Subspace minimization

Given an estimate $(\mathcal{L}, \mathcal{U}, \mathcal{I}, \mathcal{E})$, the point (x, z^ℓ, z^u) is uniquely determined by:

1. Set
$$x_{\mathcal{L}} \leftarrow \ell_{\mathcal{L}}, \quad x_{\mathcal{U}} \leftarrow u_{\mathcal{U}}, \quad z^\ell_{\mathcal{U}\cup\mathcal{I}} \leftarrow 0, \quad \text{and} \quad z^u_{\mathcal{L}\cup\mathcal{I}} \leftarrow 0.$$
2. Compute $x_{\mathcal{I}}$ by solving the reduced subproblem
$$\min_{x_{\mathcal{I}}, x_{\mathcal{E}}} \begin{bmatrix} c_{\mathcal{I}} + H_{\mathcal{I}\mathcal{L}} x_{\mathcal{L}} + H_{\mathcal{I}\mathcal{U}} x_{\mathcal{U}} \\ c_{\mathcal{E}} + H_{\mathcal{E}\mathcal{L}} x_{\mathcal{L}} + H_{\mathcal{E}\mathcal{U}} x_{\mathcal{U}} \end{bmatrix}^T \begin{bmatrix} x_{\mathcal{I}} \\ x_{\mathcal{E}} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x_{\mathcal{I}} \\ x_{\mathcal{E}} \end{bmatrix}^T \begin{bmatrix} H_{\mathcal{I}\mathcal{I}} & H_{\mathcal{I}\mathcal{E}} \\ H_{\mathcal{E}\mathcal{I}} & H_{\mathcal{E}\mathcal{E}} \end{bmatrix} \begin{bmatrix} x_{\mathcal{I}} \\ x_{\mathcal{E}} \end{bmatrix}$$
$$\text{s.t.} \quad \ell_{\mathcal{E}} \le x_{\mathcal{E}} \le u_{\mathcal{E}}.$$
3. Set
$$z^\ell_{\mathcal{L}} \leftarrow [c + Hx]_{\mathcal{L}} \quad \text{and} \quad z^u_{\mathcal{U}} \leftarrow -[c + Hx]_{\mathcal{U}}.$$

Primal-dual active-set method and critical theorem

Algorithm 2 Curtis, Han, and Robinson (2012)

1: Input $(\mathcal{L}_0, \mathcal{U}_0, \mathcal{I}_0, \mathcal{E}_0)$ and initialize $k \leftarrow 0$.
2: **loop**
3: Compute (x_k, z_k^ℓ, z_k^u) via subspace minimization.
4: If $r(x_k, z_k^\ell, z_k^u) = 0$, then **break**.
5: Choose $(\mathcal{L}_{k+1}, \mathcal{U}_{k+1}, \mathcal{I}_{k+1}, \mathcal{E}_{k+1})$.
6: **end loop**

Theorem

If the updating strategy for $\{(\mathcal{L}_k, \mathcal{U}_k, \mathcal{I}_k, \mathcal{E}_k)\}$ generates $k \geq 0$ such that

$$\mathcal{L}_k \subseteq \mathcal{L}_*, \quad \mathcal{U}_k \subseteq \mathcal{U}_*, \quad \text{and} \quad \mathcal{I}_k \subseteq \mathcal{I}_*,$$

then $r(x_k, z_k^\ell, z_k^u) = 0$, i.e., $(x_k, z_k^\ell, z_k^u) = (x_*, z_*^\ell, z_*^u)$.

Partition update based on monitoring active-set changes

Indices that move between index sets are prime candidates for $\mathcal{E}$.

1. Apply the same strategy as in Algorithm 1.
2. For each index, count the number of times it changes sets.
3. Once the number of changes for an index reaches a threshold, put it in $\mathcal{E}$.

Partition update based on monitoring KKT residual

By moving elements into $\mathcal{E}$, we can force monotonic decrease in the KKT residual.

1. Apply the same strategy as in Algorithm 1.
2. If the norm of the KKT residual does not decrease, then move elements into $\mathcal{E}$ until the norm of the KKT residual does decrease.

   (We use a non-monotone method that only forces a decrease every p iteratons.)
3. (Optional) Elements can be removed from $\mathcal{E}$ if the KKT error is unaffected.

Numerical results for Algorithm 2 (monitoring active-set changes)

		Algorithm 1		Algorithm 2 (changes)				
n	H_{cond}	# Iter	%	# Iter	# SSM	Final $\lvert\mathcal{U}\rvert$	Avg $\lvert\mathcal{U}\rvert$	%
1e+02	1e+02	3.78e+00	100	3.78e+00	4.78e+00	0.00e+00	0.00e+00	100
1e+02	1e+04	5.14e+00	100	5.14e+00	6.14e+00	6.00e-02	1.42e-02	100
1e+02	1e+06	6.22e+00	100	6.20e+00	7.20e+00	2.40e-01	6.12e-02	100
1e+03	1e+02	4.78e+00	100	4.78e+00	5.78e+00	0.00e+00	0.00e+00	100
1e+03	1e+04	6.64e+00	100	6.60e+00	7.60e+00	4.20e-01	1.05e-01	100
1e+03	1e+06	8.02e+00	100	8.18e+00	9.18e+00	8.20e-01	3.00e-01	100
1e+04	1e+02	5.80e+00	100	5.80e+00	6.80e+00	1.60e-01	2.90e-02	100
1e+04	1e+04	8.24e+00*	90	9.04e+00	1.00e+01	1.14e+00	4.47e-01	100
1e+04	1e+06	1.01e+01*	92	1.93e+01	2.03e+01	3.38e+00	1.45e+00	100

Numerical results for Algorithm 2 (monitoring KKT residual)

		Algorithm 2 (no removals from $\mathcal{E}$)				
n	H_{cond}	# Iter	# SSM	Final $\lvert\mathcal{U}\rvert$	Avg $\lvert\mathcal{U}\rvert$	%
1e+02	1e+02	3.78e+00	4.78e+00	0.0e+00	0.00e+00	100
1e+02	1e+04	5.14e+00	6.14e+00	0.0e+00	0.00e+00	100
1e+02	1e+06	6.22e+00	7.22e+00	0.0e+00	0.00e+00	100
1e+03	1e+02	4.78e+00	5.78e+00	0.0e+00	0.00e+00	100
1e+03	1e+04	6.64e+00	7.64e+00	0.0e+00	0.00e+00	100
1e+03	1e+06	8.02e+00	9.02e+00	0.0e+00	0.00e+00	100
1e+04	1e+02	5.80e+00	6.80e+00	0.0e+00	0.00e+00	100
1e+04	1e+04	8.86e+00	1.00e+01	1.8e-01	3.99e-02	100
1e+04	1e+06	1.06e+01	1.18e+01	2.4e-01	5.17e-02	100

		Algorithm 2 (removals from $\mathcal{E}$)				
		# Iter	# SSM	Final $\lvert\mathcal{U}\rvert$	Avg $\lvert\mathcal{U}\rvert$	%
1e+02	1e+02	3.78e+00	4.78e+00	0.0e+00	0.00e+00	100
1e+02	1e+04	5.14e+00	6.14e+00	0.0e+00	0.00e+00	100
1e+02	1e+06	6.22e+00	7.22e+00	0.0e+00	0.00e+00	100
1e+03	1e+02	4.78e+00	5.78e+00	0.0e+00	0.00e+00	100
1e+03	1e+04	6.64e+00	7.64e+00	0.0e+00	0.00e+00	100
1e+03	1e+06	8.02e+00	9.02e+00	0.0e+00	0.00e+00	100
1e+04	1e+02	5.80e+00	6.80e+00	0.0e+00	0.00e+00	100
1e+04	1e+04	8.86e+00	1.00e+01	1.0e-01	3.51e-02	100
1e+04	1e+06	1.06e+01	1.18e+01	1.4e-01	3.89e-02	100

Outline

Equality constraints

Consider the strictly-convex generally-constrained QP

$$\min_x \; c^T x + \tfrac{1}{2} x^T H x$$
$$\text{s.t. } Ax = b, \quad \ell \le x \le u.$$

The KKT conditions are

$$\begin{aligned} c + Hx - A^T y - z^\ell + z^u &= 0; \\ Ax - b \; &= 0; \\ (x - \ell) \circ z^\ell &= 0; \\ (u - x) \circ z^u &= 0; \\ (x - \ell, u - x, z^\ell, z^u) &\ge 0. \end{aligned}$$

Subspace minimization with equality constraints

Given an estimate $(\mathcal{L}, \mathcal{U}, \mathcal{I}, \mathcal{E})$, the point (x, z^ℓ, z^u) is uniquely determined by:

1. Set
$$x_\mathcal{L} \leftarrow \ell_\mathcal{L}, \;\; x_\mathcal{U} \leftarrow u_\mathcal{U}, \;\; z^\ell_{\mathcal{U}\cup\mathcal{I}} \leftarrow 0, \;\; \text{and} \;\; z^u_{\mathcal{L}\cup\mathcal{I}} \leftarrow 0.$$
2. Compute $x_\mathcal{I}$ by solving the reduced subproblem
$$\min_{x_\mathcal{I}, x_\mathcal{E}} \begin{bmatrix} c_\mathcal{I} + H_{\mathcal{I}\mathcal{L}} x_\mathcal{L} + H_{\mathcal{I}\mathcal{U}} x_\mathcal{U} \\ c_\mathcal{E} + H_{\mathcal{E}\mathcal{L}} x_\mathcal{L} + H_{\mathcal{E}\mathcal{U}} x_\mathcal{U} \end{bmatrix}^T \begin{bmatrix} x_\mathcal{I} \\ x_\mathcal{E} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x_\mathcal{I} \\ x_\mathcal{E} \end{bmatrix}^T \begin{bmatrix} H_{\mathcal{I}\mathcal{I}} & H_{\mathcal{I}\mathcal{E}} \\ H_{\mathcal{E}\mathcal{I}} & H_{\mathcal{E}\mathcal{E}} \end{bmatrix} \begin{bmatrix} x_\mathcal{I} \\ x_\mathcal{E} \end{bmatrix}$$
$$\text{s.t.} \;\; A_\mathcal{I} x_\mathcal{I} + A_\mathcal{E} x_\mathcal{E} = b - A_\mathcal{L} x_\mathcal{L} - A_\mathcal{U} x_\mathcal{U}, \;\; \ell_\mathcal{E} \le x_\mathcal{E} \le u_\mathcal{E}.$$
3. Set
$$z^\ell_\mathcal{L} \leftarrow [c + Hx - A^T y]_\mathcal{L} \;\; \text{and} \;\; z^u_\mathcal{U} \leftarrow -[c + Hx - A^T y]_\mathcal{U}.$$

Obtaining a feasible partition

1. Given $(x_{\mathcal{L}}, x_{\mathcal{U}})$, solve the linear optimization problem (LP)

$$\min_{x_{\mathcal{I}}, x_{\mathcal{E}}, r, s} \; e^T(r+s)$$
$$\text{s.t. } A_{\mathcal{I}}x_{\mathcal{I}} + A_{\mathcal{E}}x_{\mathcal{E}} = b - A_{\mathcal{L}}x_{\mathcal{L}} - A_{\mathcal{U}}x_{\mathcal{U}} + r - s, \;\; \ell_{\mathcal{E}} \leq x_{\mathcal{E}} \leq u_{\mathcal{E}}.$$

2. If $e^T(r+s) = 0$, then a feasible partition has been obtained.
3. Else, move an element from $\mathcal{L} \cup \mathcal{U}$ to $\mathcal{I}$.

Primal-dual active-set method and critical theorem

Algorithm 3 Curtis, Han, and Robinson (2012)

Input $(\mathcal{L}_0, \mathcal{U}_0, \mathcal{I}_0, \mathcal{E}_0)$ and initialize $k \leftarrow 0$.
loop
  Transform $(\mathcal{L}_k, \mathcal{U}_k, \mathcal{I}_k, \mathcal{E}_k)$ into a feasible partition.
  Compute (x_k, z_k^ℓ, z_k^u) via subspace minimization.
  If $r(x_k, z_k^\ell, z_k^u) = 0$, then **break**.
  Choose $(\mathcal{L}_{k+1}, \mathcal{U}_{k+1}, \mathcal{I}_{k+1}, \mathcal{E}_{k+1})$.
end loop

Theorem

If the updating strategy for $\{(\mathcal{L}_k, \mathcal{U}_k, \mathcal{I}_k, \mathcal{E}_k)\}$ generates $k \geq 0$ such that

$$\mathcal{L}_k \subseteq \mathcal{L}_*, \ \ \mathcal{U}_k \subseteq \mathcal{U}_*, \ \ \text{and} \ \ \mathcal{I}_k \subseteq \mathcal{I}_*,$$

then $r(x_k, z_k^\ell, z_k^u) = 0$, i.e., $(x_k, z_k^\ell, z_k^u) = (x_*, z_*^\ell, z_*^u)$.

Numerical results for Algorithm 2

		Algorithm 2						
m	H_{cond}	# Iter	# SSM	# Feas	Feas Mod	Final $\lvert\mathcal{U}\rvert$	Avg $\lvert\mathcal{U}\rvert$	%
1e+01	1e+02	1.05e+01	1.15e+01	1.15e+01	6.80e-01	1.24e+00	5.59e-01	100
1e+01	1e+04	1.33e+01	1.43e+01	1.43e+01	2.08e+00	1.72e+00	7.87e-01	100
1e+01	1e+06	2.39e+01	2.49e+01	2.49e+01	3.32e+00	4.34e+00	1.91e+00	100
1e+02	1e+02	2.06e+01	2.16e+01	2.16e+01	1.12e+01	3.28e+00	1.52e+00	100
1e+02	1e+04	3.72e+01	3.82e+01	3.82e+01	2.45e+01	6.36e+00	3.12e+00	100
1e+02	1e+06	5.39e+01	5.51e+01	5.49e+01	2.99e+01	9.92e+00	4.80e+00	100

		Algorithm 2 (no removals from $\mathcal{E}$)						
m	H_{cond}	# Iter	# SSM	# Feas	Feas Mod	Final $\lvert\mathcal{U}\rvert$	Avg $\lvert\mathcal{U}\rvert$	%
1e+01	1e+02	1.50e+01	1.87e+01	1.87e+01	7.20e-01	2.74e+00	1.00e+00	100
1e+01	1e+04	2.28e+01	3.05e+01	3.05e+01	2.92e+00	6.68e+00	2.46e+00	100
1e+01	1e+06	2.79e+01	3.90e+01	3.90e+01	3.84e+00	1.01e+01	3.84e+00	100
1e+02	1e+02	8.32e+01	1.61e+02	1.61e+02	3.47e+01	7.69e+01	3.77e+01	100
1e+02	1e+04	1.08e+02	2.03e+02	2.03e+02	4.31e+01	9.48e+01	4.58e+01	100
1e+02	1e+06	1.27e+02	2.42e+02	2.42e+02	5.21e+01	1.14e+02	5.53e+01	100

		Algorithm 2 (removals from $\mathcal{E}$)						
m	H_{cond}	# Iter	# SSM	# Feas	Feas Mod	Final $\lvert\mathcal{U}\rvert$	Avg $\lvert\mathcal{U}\rvert$	%
1e+01	1e+02	1.53e+01	1.93e+01	1.93e+01	9.20e-01	9.40e-01	7.53e-01	100
1e+01	1e+04	2.26e+01	3.04e+01	3.04e+01	2.98e+00	1.28e+00	1.14e+00	100
1e+01	1e+06	3.57e+01	5.63e+01	5.63e+01	4.40e+00	2.32e+00	1.48e+00	100
1e+02	1e+02	9.70e+01	1.93e+02	1.93e+02	4.48e+01	2.02e+00	2.68e+00	100
1e+02	1e+04	1.26e+02	2.54e+02	2.54e+02	4.69e+01	3.28e+00	2.60e+00	100
1e+02	1e+06	1.83e+02	4.05e+02	4.05e+02	5.97e+01	4.90e+00	2.86e+00	100

Outline

Summary

- Proposed and analyzed a globally convergent framework for convex QP.
- Multiple simultaneous changes in the active-set are allowed.
- Set auxiliary to the active-set for variables whose bounds are explicitly enforced.
- Numerical results illustrate that auxiliary set often remains empty/small.
- **Reference**: F.E. Curtis, Z. Han, and D.P. Robinson, "A Globally Convergent Primal-Dual Active-Set Framework for Large-Scale Convex Quadratic Optimization," submitted to *SIAM Journal on Optimization*, 2012.